AN ADVANCED ARITHMETIC

FOR

HIGH SCHOOLS, NORMAL SCHOOLS
AND ACADEMIES

BY

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PREFACE.

EVERY high school, normal school, and academy should allow sufficient time for a thorough review of Arithmetic. This book has been written as a text-book for that purpose, and for that purpose only. It is not intended for beginners, but assumes that pupils have previously read a more elementary Arithmetic.

The shortest and surest road to a knowledge of Arithmetic is by solving problems. This work is abundantly supplied with well-graded, practical problems, many taken from Wentworth and Hill's High School Arithmetic, but many of them are new, and of a kind to meet the requirements of the present time. These problems are designed to convey a great amount of useful information, as well as to furnish the very best mental training, the primary object of the study. They cover to a great extent the field of mercantile transactions, and so far as practicable the field of science. It is not necessary for any pupil, or any class even, to do all the problems. Every teacher can select such chapters and such parts of chapters as are suited to the needs of his pupils.

Decimals are introduced at the beginning of the book. Numbers on each side of the decimal point perform precisely the same office. The only difference is that numbers at the left of the decimal point count *whole units*, and numbers at the right count *equal parts of the unit*. Pupils learn the notation on both sides of the decimal point as

easily as on one side, provided they have a clear conception of the units counted. Dimes and cents are good examples of tenths and hundredths of a dollar; but decimeters, centimeters, and millimeters, marked on a meter stick, are the best examples of tenths, hundredths, and thousandths of a unit, in general. The Metric System is taught naturally in connection with decimals, and is easily learned. Only the units employed furnish any difficulty. The great number of problems given under the Metric System is to familiarize the learner with the units of the system, to show the simplicity of the system in its application to everyday problems, and at the same time to give practice in operations involving decimals. This system is used in the laboratories of science, and in international transactions. Though not yet adopted by the United States in the common affairs of life, it has certainly forced its way to a position requiring recognition in all secondary schools of the country.

The introduction of logarithms into the High School Arithmetic was warmly welcomed by progressive teachers; and the chapter on that subject in this book has been written with special reference to acquiring easily the practical use of a four-place table.

Every effort has been made to avoid errors in problems and answers. The author will be very grateful to any one who will call his attention to any mistake that may be discovered.

G. A. WENTWORTH.

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CONTENTS.

VOCABULARY.

Abstract numbers. Numbers used without reference to any particular unit, as 8, 10, 21. *All numbers are in themselves abstract whether the kind of thing numbered is mentioned or is not mentioned.*

Addition. The process of combining two or more numbers.

Agent. A person who transacts business for another.

Aliquot parts of a given number. Numbers that are contained an integral number of times in the given number.

Alligation. The process of finding the value of a mixture of quantities of different values; or of finding the proportion of quantities of different values to be used to make a mixture of a given value.

Amount. The sum of two or more numbers. In interest, the sum of the principal and interest.

Ampère. The unit of quantity of electricity. The current produced by a force of one volt in a circuit of a resistance of one ohm.

Analysis. The process of reasoning from the *given number* to *one*, and then from *one* to the *required number*.

Annual interest. Simple interest on the principal and on *each year's* interest from the time each interest is due until settlement.

Annuity. A sum of money to be paid at regular intervals of time.

Antecedent. The first term of a ratio.

Antilogarithm. The number corresponding to a logarithm.

Area of a surface. The number of square units in the surface.

Arithmetic. The science that treats of numbers and the methods of using them.

Arithmetical progression. A series of numbers that increase or decrease by a common difference.

Assets. All the property of an estate, individual, or corporation.

Average of numbers. The number that can be put for each of the numbers without altering their sum.

Average of payments. The average time at which several payments due at different dates may be equitably made.

Bank. An establishment for the custody, loaning, and exchange of money; and often for the issue of money.

Bank discount. An allowance received by a bank for the loan of money, paid as interest at the time of lending.

Bills. Written statements of goods sold, or services rendered, giving the price and date of each item, and the parties concerned.

Bonds. Written contracts under seal to pay specified sums of money at specified times, issued by national governments, states, cities, and other corporations.

Breakage. In customs, an allowance of a certain per cent on liquors in bottles; also on glassware and china.

Broker. A person who buys and sells stocks and bonds for another.

Brokerage. The commission charged by a broker.

Cancellation. The striking out of a common factor from the dividend and divisor.

Centrifugal force. The force that tends to make a revolving body move in a straight line.

Characteristic of a logarithm. The integral part of the logarithm.

Check. A draft upon a bank where the maker has money deposited.

Cologarithm of a number. The logarithm of the reciprocal of the number.

Commercial discount. A reduction from the list price of an article, from the amount of a bill, or from the amount of a debt.

Commission. Compensation for the transaction of business, reckoned at some per cent of the money employed in the transaction.

Common denominator. A denominator common to two or more fractions.

Common factor. A factor common to two or more numbers.

Common fractions. Fractions expressed by two numbers, one under the other, with a line between them.

Common measure of two or more numbers. A number that exactly divides each of them.

Common multiple of two or more numbers. A number exactly divisible by each of them.

Complex fraction. A fraction that has a fraction in one or both of its terms.

Composite number. The product of two or more integral factors, each factor being greater than unity.

Compound fraction. A fraction of an integer or of a fraction.

Compound interest. Interest not paid when due, but added to the principal at regular intervals.

Compound quantities. Quantities expressed in two or more denominations.

Concrete numbers. Numbers applied to specified things.

Consequent. The second term of a ratio.

Consignee. The person or firm to whom goods are sent.

Consignor. The person or firm who sends goods to another.

Continued fraction. A fraction whose numerator is 1, and whose denominator is an integer plus a fraction whose numerator is 1, and whose denominator is an integer plus a fraction, and so on.

Coöperative bank. A mutual corporation for the accumulation of a capital to be loaned to its members.

Corporation. An association of individuals authorized by law to transact business as a single person.

Couplet. The two terms of a ratio taken together.

Coupon. A certificate of interest attached to a bond, to be cut off when due and presented for payment.

Creditor. A person or firm to whom money is due.

Cube root of a number. One of its three equal factors.

Currency. The medium of exchange employed in business.

Customs. Duties or taxes imposed by law on merchandise imported, and sometimes on merchandise exported.

Debtor. A person or firm who owes money to another.

Decimal point. A dot placed between the number that counts whole units and the number that counts decimal units.

Decimals. Fractions of which only the numerators are written, and the denominators are ten or some power of ten.

Decimal system. The common system of numbers founded on their relations to *ten*.

Demand notes. Notes that are payable *on demand*.

Denominator. The number that shows into how many equal parts a unit is divided.

Difference. The number found by subtraction.

Discount. An allowance made for the payment of money before it becomes due. Also, the difference between the market value and the face value when the market value is *below* the face value.

Dividend. In division, the number to be divided. In business, the sum paid on each share of stock from the profits of the business.

Division. The operation of finding the other factor, when a product and one of its factors are given.

Divisor. The number by which a given dividend is to be divided.

Draft. A written order directing one person or firm to pay a specified sum of money to another.

Drawee of a draft. The person on whom the draft is drawn.

Drawer of a draft. The person who signs the draft.

Duties. Taxes required by the government to be paid on goods imported, exported, or put on the market for consumption.

Endorser of a note. A person who writes his name on the back of the note. The endorser is responsible for the payment of the note unless he writes above his name the words *without recourse.*

Equation. A statement that two expressions of number are equal.

Equivalents. Equals in value, area, or volume.

Even numbers. Numbers exactly divisible by 2.

Evolution. The process of finding the root of a number.

Exact interest. Simple interest reckoning 365 days to the year.

Example. A question to be solved.

Exchange. A system of paying debts, due to persons living at a distance, by transmitting drafts instead of money.

Exponent. A small figure placed at the right of a number to show how many times the number is taken as a factor.

Extremes. The first and last terms of a proportion.

Face of a note or draft. The sum of money named in it.

Factors. A set of numbers whose product is the given number. In commerce, agents employed by merchants to transact business.

Figures. Symbols used to represent numbers in the Arabic system. Also diagrams used to represent geometrical forms.

Firm. The name under which a company transacts business.

Foot-pound. The unit of work. The work done in raising a weight of one pound to a height of one foot.

Force. That which tends to produce motion.

Fractions. One or more of the equal parts of a unit.

Fulcrum. The point or line on which a lever turns.

Gain. The selling price minus the cost price.

Geometrical progression. A series of numbers, each term of which after the first is obtained by multiplying the preceding term by a constant multiplier.

Grace. An allowance of three days, after the date a note becomes due, within which to pay the note.

Gram. The unit of weight in the metric system.

Gravitation. The force by which all bodies attract each other.

Greatest common measure of two or more numbers. The greatest number that will exactly divide each of them.

Harmonical progression. A series of numbers, the reciprocals of which form an arithmetical progression.

Holder of a note. The person who has legal possession of it.

Horse power. The power to do 33,000 foot-pounds of work a minute.

Improper fraction. A fraction whose numerator equals or exceeds its denominator.

Index. A figure written at the left and above the radical sign to show what root of the number under the radical sign is required. A fraction written at the right of a number, of which the numerator shows the required power of that number, and the denominator the required root of that power.

Instalment. A payment in part.

Insurance. A guarantee by an insurance company of a specified sum of money to the person insured in the event of loss of property by fire, by storm at sea, or by other specified disaster; or in the event of the death of the person insured, or of accident to him.

Integral numbers. Numbers that denote whole units.

Interest. Money paid for the use of money.

Involution. The process of finding a power of a number.

Latitude of a place. The distance north or south from the equator, expressed in degree measures.

Leakage. In customs, an allowance of a certain per cent on liquors in barrels or casks.

Least common denominator of two or more fractions. The least common multiple of their denominators.

Least common multiple of two or more numbers. The least number that is exactly divisible by each of them.

Lever. A rigid bar that will move freely about its fulcrum.

Liability. A debt, or obligation to pay.

Like numbers. Numbers applied to the same unit.

Line. Length without breadth or thickness.

Liter. The unit of capacity in the metric system.

Logarithm of a number. The exponent of the power to which the *base* must be raised to obtain the given number.

Long division. The method of dividing in which the processes are written in full.

Longitude of a place. The distance east or west from a *standard meridian*, expressed in degree measures.

Loss. The cost price minus the selling price.

Mantissa of a logarithm. The decimal part of the logarithm.

Maturity of a note. The date a note legally becomes due.

Mean proportional. A number that is both the second and third terms of a proportion.

Means. The terms of a proportion between the extremes.

Measures of a number. The exact divisors of the numbers.

Meridians. Imaginary lines drawn straight around the earth through both poles.

Meter. The unit of length in the metric system.

Metric system. A system of weights and measures expressed in the decimal scale.

Minuend. The number from which the subtrahend is taken.

Mixed number. A whole number and a fraction.

Money of a country. The legal currency of the country.

Multiple of a number. The product obtained by taking the given number an integral number of times.

Multiplicand. The number to be multiplied by another.

Multiplication. The operation of taking a number of units a number of times.

Multiplier. The number by which the multiplicand is multiplied.

Net proceeds. The money that remains of the money received for property after all expenses and discounts are paid.

Notation. A system of expressing numbers by symbols.

Note. A written agreement to pay, for value received, a specified sum of money on demand or at a specified time.

Numbers. Expressions applied to a unit to show how many times the unit is taken.

Numeration. A system of naming numbers.

Numerator. The number that shows how many are taken of the equal parts into which a unit is divided.

Obligation. A debt, or liability to pay.

Odd numbers. Numbers not exactly divisible by 2.

Ohm. The unit of resistance to electricity. The resistance of a column of mercury, 1^{qmm} in cross section and 106^{cm} long at $0°$ C.

Order of units. The number of things in a group, as *tens*, *hundreds*.

Partial payment. Part payment on a note.

Partnership. An association of persons to carry on business.

Par value. Face or nominal value.

Pendulum. A body suspended by a straight line from a fixed point, and moving freely about that point as a centre.

Percentage of a number. One or more hundredths of the number.

Perimeter. The length of the boundary of a plane figure.

Period. A group of three figures.

Policy. A written contract of insurance.

Poll tax. A tax levied by the head or poll.

Power. The product of two or more equal factors.

Premium. Money paid for insurance. Also the excess of market value above par value.

Present worth. The present value of a debt due at a future time.

Prime number. A number that cannot be exactly divided by any number except itself and one.

Principal. Money drawing interest.

Problem. A question to be solved.

Proceeds or avails of a note or draft. The amount of the note or draft less the discount and exchange.

Product. The number found by multiplication.

Proof. The evidence that establishes the accuracy of any result.

Proper fraction. A fraction whose numerator is less than its denominator.

Proportion. A statement that two ratios are equal.

Protest. A notice in writing by a notary public to the endorsers of a note that it has not been paid.

Pulley. A grooved wheel that turns freely within a block, fixed or movable, by means of a rope that passes over the groove.

Quotient. The number found by division.

Rate per cent. Rate by the hundred.

Ratio. The *relative magnitude* of two numbers or of two quantities, when expressed by the quotient of the first divided by the second.

Receipt. A written acknowledgment of money or goods received.

Reciprocal of a number. *One* divided by that number.

Reduction. The process of changing the *unit* in which a quantity is expressed without changing the *value* of the quantity.

Remainder. The number found by subtraction.

Repeating or circulating decimals. Decimals that contain a constantly recurring figure or series of figures.

Repetend. The figure or series of figures that constantly recurs in a repeating decimal.

Root of a number. One of the equal factors of the number.

Rule. The statement of a prescribed method.

Savings bank. A bank to receive deposits and pay compound interest to the depositors.

Screw. A cylinder that has on its surface a uniform projection in the form of a spiral curve, called the thread.

Security. Property used to guarantee the payment of any debt.

Share. One of a certain number of equal parts into which the capital of a company is divided.

Short division. The method of dividing in which the operations of multiplying and subtracting are performed mentally.

Similar fractions. Fractions that have a common denominator.

Simple fractions. Fractions whose terms are integral numbers.

Simple quantities. Quantities expressed in a single denomination.

Sinking fund. The final value of sums of money put at interest at regular intervals of time, to pay a debt due at a stated time.

Solid. A magnitude that has length, breadth, and thickness.

Solution. The process by which the answer to a question is obtained.

Specific gravity of a substance. The ratio of the weight of a given volume of it to the weight of an equal volume of water.

Square root of a number. One of its two equal factors.

Stock. Capital invested in business.

Stock company. An association of persons under the laws of the state for the purpose of carrying on a specified business.

Subtraction. The process of taking one number from another.

Subtrahend. The number that is taken from the minuend.

Sum. The number found by addition.

Surd. An indicated root, that cannot be exactly found.

Surface. A magnitude that has length and breadth.

Tare. In customs, an allowance for the weight of the box, cask, bag, or other wrapping.

Taxes. Money required of persons and corporations for the support of the government and for other public purposes.

Thermometer. An instrument for measuring temperature.

Time notes. Notes that are payable at a specified time.

Units. The standards by which we count separate objects or measure magnitudes.

Velocity. Rate of motion, measured by the number of units of space passed over in a unit of time.

Verify. To establish by trial the truth of any statement.

Volt. The unit of force of electricity. The force required to send one ampère of electricity through a circuit of a resistance of one ohm.

Volume of a solid. The number of cubic units in the solid.

Wheel and axle. A simple machine, consisting of a wheel firmly attached to an axle.

Work. The act of changing the position of a body by overcoming resistance to the change.

Yard. The unit of length in the common system.

Short Processes.

NOTE. These processes should be learned as fast as they can be utilized in the ordinary work of Arithmetic. The time to learn each process is left to the discretion of the teacher.

1. To multiply by 25 ($\frac{1}{4}$ of 100),
Multiply by 100 and divide the product by 4.

2. To divide by 25,
Multiply by 4 and divide the product by 100.

3. To multiply by 2.5 or $2\frac{1}{2}$ ($\frac{1}{4}$ of 10),
Multiply by 10 and divide the product by 4.

4. To divide by 2.5 or $2\frac{1}{2}$,
Multiply by 4 and divide the product by 10.

5. To multiply by 50 ($\frac{1}{2}$ of 100),
Multiply by 100 and divide the product by 2.

6. To divide by 50,
Multiply by 2 and divide the product by 100.

7. To multiply by 75 ($\frac{3}{4}$ of 100),
Multiply by 100 and subtract from the product $\frac{1}{4}$ of it.

8. To divide by 75,
Divide by 100 and add to the quotient $\frac{1}{3}$ of it.

9. To multiply by $33\frac{1}{3}$ ($\frac{1}{3}$ of 100),
Multiply by 100 and divide the product by 3.

10. To divide by $33\frac{1}{3}$,
Multiply by 3 and divide the product by 100.

11. To multiply by $3\frac{1}{3}$ ($\frac{1}{3}$ of 10),
Multiply by 10 and divide the product by 3.

12. To divide by $3\frac{1}{3}$,
Multiply by 3 and divide the product by 10.

13. To multiply by $333\frac{1}{3}$ ($\frac{1}{3}$ of 1000),
Multiply by 1000 and divide the product by 3.

14. To divide by $333\frac{1}{3}$,
Multiply by 3 and divide the product by 1000.

15. To multiply by $16\frac{2}{3}$ ($\frac{1}{6}$ of 100),
Multiply by 100 and divide the product by 6.

16. To divide by $16\frac{2}{3}$,
Multiply by 6 and divide the product by 100.

17. To multiply by $166\frac{2}{3}$ ($\frac{1}{6}$ of 1000),
Multiply by 1000 and divide the product by 6.

18. To divide by $166\frac{2}{3}$,
Multiply by 6 and divide the product by 1000.

19. To multiply by $66\frac{2}{3}$ ($\frac{2}{3}$ of 100),
Multiply by 100 and subtract from the product $\frac{1}{3}$ of it.

20. To divide by $66\frac{2}{3}$,
Divide by 100 and add to the quotient $\frac{1}{2}$ of it.

21. To multiply by $12\frac{1}{2}$ ($\frac{1}{8}$ of 100),
Multiply by 100 and divide the product by 8.

22. To divide by $12\frac{1}{2}$,
Multiply by 8 and divide the product by 100.

23. To multiply by $14\frac{2}{7}$ ($\frac{1}{7}$ of 100),
Multiply by 100 and divide the product by 7.

24. To divide by $14\frac{2}{7}$,
Multiply by 7 and divide the product by 100.

25. To multiply by a number that is a little less than some multiple of 10, as 100, 1000, etc.,

Multiply the multiplicand by the multiple of 10 that differs little from the given multiplier. Then multiply the multiplicand by the difference between this multiple of 10 and the given multiplier, and find the difference of the two products.

Thus, to multiply by 998 (1000 — 2), multiply by 1000 and then by 2, and take the difference of the products.

NOTICE TO TEACHERS.

Pamphlets containing the answers will be furnished without charge to teachers for their classes, *on application to GINN & COMPANY, Publishers.*

ADVANCED ARITHMETIC.

CHAPTER I.

NOTATION AND NUMERATION.

1. Units. The standards by which we count or measure are called *units.*

The unit may be a single thing or a definite group of things. Thus, in counting the eggs in a nest the unit is an egg; in selling eggs by the dozen the unit is a dozen eggs; in selling bricks by the thousand the unit is a thousand bricks; in measuring short distances the unit is an inch, a foot, or a yard; in measuring long distances the unit is a rod or a mile.

2. Numbers. Expressions applied to a unit to show *how many times the unit is taken* are called *numbers.*

Thus, if we put an apple into an empty fruit dish, then another, and then another, we shall have *three* apples in the dish. Here an apple is the unit, and *three* is the *number of times the unit is taken.*

3. Integral Numbers. Numbers applied to *whole units* are called *whole numbers, in'tegral numbers,* or *integers.*

4. Figures. The following symbols, called *figures,* or *digits,* are used to represent the numbers of Arithmetic:

0	1	2	3	4	5	6	7	8	9
Zero	One	Two	Three	Four	Five	Six	Seven	Eight	Nine

The first figure, 0, is called *zero, naught,* or *cipher,* and stands for the words *no number.* Each of the other figures stands for the number whose name is written below it.

5. Tens. The next number, *ten,* is expressed by writing 0 at the right of 1. Thus, ten is written 10. In this position 1 signifies not *one*, but *one group of ten ones.*

Figures signifying *tens* are written in the *second* place from the right.

In the same way twenty (2 tens) is expressed by 20; thirty (3 tens) by 30; forty (4 tens) by 40; fifty (5 tens) by 50; sixty (6 tens) by 60; seventy (7 tens) by 70; eighty (8 tens) by 80; ninety (9 tens) by 90.

6. Tens and Ones. A number containing tens and ones is expressed by writing the figure for the tens in the *second* place from the right, and the figure for the ones in the *first* place.

Eleven,	one	ten	and one,	is expressed by 11.
Twelve,	one	ten	and two,	is expressed by 12.
Thirteen,	one	ten	and three,	is expressed by 13.
Fourteen,	one	ten	and four,	is expressed by 14.
Fifteen,	one	ten	and five,	is expressed by 15.
Sixteen,	one	ten	and six,	is expressed by 16.
Seventeen,	one	ten	and seven,	is expressed by 17.
Eighteen,	one	ten	and eight,	is expressed by 18.
Nineteen,	one	ten	and nine,	is expressed by 19.
Twenty-one,	two	tens	and one,	is expressed by 21.
Twenty-two,	two	tens	and two,	is expressed by 22.
Forty-three,	four	tens	and three,	is expressed by 43.
Fifty-four,	five	tens	and four,	is expressed by 54.
Sixty-five,	six	tens	and five,	is expressed by 65.

And so on to ninety-nine (nine tens and nine) which is expressed by 99.

7. Hundreds. A group of 10 tens is called a *hundred,* and figures signifying hundreds are written in the *third* place from the right.

Thus, one hundred, two hundreds, three hundreds, etc., are expressed by 100, 200, 300, etc.

8. To Write Hundreds, Tens, and Ones. We write first the hundreds, then the tens and ones.

Thus, two hundred seventy-six is written 276.

Express in figures the following numbers:

Seven hundred sixty-five.	Nine hundred.
One hundred twenty-three.	Five hundred eighty-one.
Six hundred ninety-four.	Four hundred thirty.
Nine hundred forty-six.	Seven hundred nine.
Two hundred twenty-nine.	Seven hundred ninety.
One hundred ten.	Seven hundred ninety-nine.

9. To Read Hundreds, Tens, and Ones. We read first the hundreds, then the tens and ones.

Thus, the number 359 has 3 hundreds, 5 tens, and 9 ones, and is read three hundred fifty-nine; the number 807 has 8 hundreds, no tens, and 7 ones, and is read eight hundred seven.

NOTE. In reading 359, 807, or any other integral number, do not introduce the word *and*; that is, do not say three hundred *and* fifty-nine, eight hundred *and* seven; but simply three hundred fifty-nine, eight hundred seven.

Read the following numbers, and state the number of hundreds, tens, and ones in each:

507	469	101	260	206	301	808	888
321	694	929	300	185	340	671	999

10. Notation and Numeration. The method of writing numbers is called *notation*, and the method of reading numbers is called *numeration*.

11. The system of notation here explained is called the **Arabic** system of notation.

12. Thousands. A group of 10 hundreds is called a *thousand*, and figures signifying thousands are written in the *fourth* place from the right.

Thus, one thousand, two thousands, three thousands, etc., are expressed by 1000, 2000, 3000, etc.

13. Numbers expressed by Four Figures. Numbers expressed by four figures may be read as thousands, hundreds, tens, and ones; or as hundreds, tens, and ones.

The shortest way of reading numbers is the best way. The best way to read 1896 is eighteen hundred ninety-six. The best way to read 7005 is seven thousand five.

Read in the best way the following numbers:

1776	1924	1907	2359	5050	3627
7006	7076	2706	6010	5500	2036

14. Orders of Units. The ones of a number are called *units of the first order.* The tens of a number are called *units of the second order.* The hundreds of a number are called *units of the third order.* The thousands of a number are called *units of the fourth order.*

Figures in the *fifth* place signify ten-thousands, called *units of the fifth order.* Figures in the *sixth* place signify hundred-thousands, called *units of the sixth order.*

NOTE. The ones of a number are commonly called *units*, the word *units* standing for the phrase *units of the first order.* Thus, we say the number 459 has 4 hundreds, 5 tens, and 9 units.

15. Decimal System. Since *ten* units of any order are equal to *one* unit of the next higher order, this system of notation is called the *decimal system; decimal* being derived from the Latin word *decem*, meaning ten.

16. Periods. When the figures of a number are five or more, we separate them into groups of three figures each by commas, beginning at the right. The right-hand group is called the *period of units;* the second group is called the *period of thousands;* the third group is called the *period of millions;* the fourth group is called the *period of billions.*

The unit of any period is equal to 1000 units of the next lower period.

One million is equal to 1000 thousands, and is written 1,000,000; one billion is equal to 1000 millions, and is written 1,000,000,000.

The left-hand period may have one, two, or three figures; every other period must have three figures.

17. To Read an Integral Number expressed in Figures.
Read the number 26217320416.

We begin at the right and point off the figures into periods of three figures each. Thus,

26,217,320,416.

We begin at the left and read each period as if it stood alone, adding the *name* of the period. The *fourth* period from the right is the period of *billions*. Hence we read the number,

Twenty-six billion, two hundred seventeen million, three hundred twenty thousand, four hundred sixteen. Therefore,

Beginning at the right, we separate the figures by commas into periods of three figures each. We begin at the left and read each period as if it stood alone, adding the name of each period except the name of the period of units.

NOTE. The names of periods above billions are in order: trillions, quadrillions, quintillions, sextillions, septillions, octillions, etc.

EXERCISE 1.

Write in periods, and read:

1. 7000.	11. 234567.	21. 703101.
2. 7842.	12. 34561.	22. 870890.
3. 5043.	13. 123456.	23. 21978564.
4. 8375.	14. 654089.	24. 17756423.
5. 2020.	15. 600897.	25. 300200100.
6. 1753.	16. 704608.	26. 707303202.
7. 18757.	17. 350709.	27. 3125476890.
8. 75764.	18. 240682.	28. 79501346081.
9. 22003.	19. 682000.	29. 3000872696.
10. 70856.	20. 753110.	30. 72727000000.

18. To Write an Integral Number in Figures. Write in figures the number two hundred sixty-three million, six hundred thirty-five thousand, two hundred one.

We consider first the *periods* of the number.

This number has the period of millions, the period of thousands, and the period of units.

We write first the period of millions, and put a comma after it; then the period of thousands, and put a comma after it; and then the period of units. Thus, 263,635,201.

NOTE. Every period except the one at the left must have three figures (§ 16). If any order of units of a period is lacking, we put a cipher in its place; and if any entire period is lacking we put three ciphers in its place. Thus, four million, sixteen thousand, four is written 4,016,004; sixteen million, four hundred sixteen is written 16,000,416. Therefore,

We begin at the left and write the hundreds, tens, and units of each period, putting zeros in all vacant places, and putting a comma between each period and the period that follows it.

EXERCISE 2.

Write in figures, arranged in periods:

1. Six hundred thousand, six.
2. Seven hundred thirteen thousand, three hundred twenty-nine.
3. Seven thousand, eight hundred fifty-four.
4. Four million, three thousand, three hundred thirty.
5. One hundred ten million, two hundred seventy-nine.
6. Nineteen trillion, four million, three hundred nine.
7. Seven million, six hundred seventy-six thousand, four hundred sixty-six.
8. Three hundred forty-seven million, six hundred fifty-one thousand, seven hundred eighty-five.
9. Two hundred million, two hundred seven.
10. Four hundred billion, four hundred thousand, four.

19. The unit of measure of any kind of quantity may be divided into *ten* smaller measures.

Thus, a dollar, as a measure of value, is divided into ten dimes, or ten *tenths* of a dollar; each dime into ten cents, or ten *hundredths* of a dollar; each cent into ten mills, or ten *thousandths* of a dollar.

20. The *dollar sign*, $, is written before the number.

21. If dollars and cents are written, a dot called the *decimal point* is placed between the dollars and cents.

Thus, $17 is 17 dollars; $18.20 is 18 dollars 2 dimes, or 18 dollars 20 cents; $35.875 is 35 dollars 87 cents 5 mills; and $0.08 is 8 cents.

22. Read as dollars, cents, and mills: $76.375; $163.58; $12.50; $0.875; $1.01; $2.10; $3.08; $0.75; $0.125.

23. In reading United States money, we read the number to the left of the decimal point as dollars; the number in the first two places to the right of the point as cents; and the number in the third place as mills.

24. In writing United States money, we must have the cents occupy *two* places. If the number of cents is less than ten, we write a cipher in the first place at the right of the decimal point.

25. Parts of other measures may be expressed as tenths, hundredths, etc.; just as dimes, cents, and mills are respectively tenths, hundredths, and thousandths of a dollar.

Thus, if we omit the dollar sign from $5.375, the expression may stand for 5 yards, quarts, bushels, or any other full measures, and 375 thousandths of another measure.

26. Parts thus written are called **Decimals.** We write a number to the right of the units' place just as we do to the left, first marking the units' place with a decimal point to its right.

27. In the number

9,876,543,210.123,456,789

the full point after 0 shows that 0 stands in the **units'** place. The 1 to the left is 1 *ten*, the 1 to the right is 1 *tenth;* the 2 to the left is 2 *hundreds*, the 2 to the right is 2 *hundredths;* the 3 to the left is 3 *thousands*, the 3 to the right is 3 *thousandths;* the 4 to the left is 4 *ten-thousands*, the 4 to the right is 4 *ten-thousandths;* the 5 to the left is 5 *hundred-thousands*, the 5 to the right is 5 *hundred-thousandths;* the 6 to the left is 6 *millions*, the 6 to the right is 6 *millionths;* and so on.

Also, the 210 is the *units'* period; the 543 is the *thousands'*, the 123 the *thousandths'* period, etc.; so that the number may be read 9 billions, 876 millions, 543 thousands, 210, and 123 thousandths, 456 millionths, 789 billionths.

28. To Read a Decimal expressed in Figures. The usual way of reading a decimal is:

Read the decimal precisely as if a whole number, and add the fractional name of the lowest place.

Thus, 5.17 is read 5 and 17 hundredths; 5.0017, five and 17 ten-thousandths; 6.0203107, six and 203 thousand 107 ten-millionths.

The word *and* is distinctly pronounced at the decimal point and carefully omitted in all other places.

Thus, one hundred forty-seven means 147; but one hundred *and* forty-seven thousandths means 100.047; and 0.147 must be read one hundred forty-seven thousandths.

29. Practical computers often introduce the word *decimal* at the place of the point, and then pronounce the name of each digit in succession to the right.

Thus, 203.07051 may be read two hundred three, decimal, naught, seven, naught, five, one.

EXERCISE 3.

Read:

1. 6,728,642.
2. 3.24658.
3. 49,568.4782.
4. 34,598,492,212.
5. 4,002,000.02.
6. 1872.17.
7. 94.658265.
8. 0.0307.
9. 100.01.
10. 1,872,563.372.
11. 17.008.
12. 143.00143.
13. 29.00081.
14. 5,262,873.
15. 8.7854.
16. $182.275.
17. $0.086.
18. $0.075.
19. $463.87.
20. $20,542.02.
21. $0.75.
22. 428,428.428.
23. 1542.087.
24. 642.873654.
25. 400.00004.
26. 3,543,362,338.
27. 0.0000009.
28. 52.02.
29. 56,482.56.
30. 87,865,842.87866.

EXERCISE 4.

Write in figures:

1. Eighty-one thousand and three hundred forty-five thousandths.
2. Thirty-seven hundred forty-one and six hundred seventy-five thousandths.
3. Four hundred thirteen and eight hundredths.
4. Ninety-six and ninety-six thousandths.
5. Nine and forty-three millionths.
6. One hundred fifty-four and thirty-two ten-thousandths.
7. Seventy-five thousandths.
8. Three tenths.
9. Forty-four million, forty-four thousand, forty-four, and forty-four thousandths.

10. One hundred and forty-three millionths.

11. One hundred forty-three millionths.

12. One hundred forty and three millionths.

13. Nine hundred forty-three thousand and nine hundred forty-three thousandths.

14. Seven hundred twenty-two ten-millionths.

15. Thirteen, decimal, naught, one, four, six, eight.

16. Four and one thousand nine ten-thousandths.

17. One hundred one and one hundred one ten-thousandths.

18. Seventeen million, six hundred forty-nine thousand.

19. Twelve billion, twelve thousand.

20. Twelve billion and twelve thousandths.

21. Eight dollars and twelve cents.

22. One hundred twenty-seven dollars and one cent.

23. Fourteen thousand, two hundred seventy-eight dollars and twenty-seven cents, five mills.

24. One thousand dollars and one cent, one mill.

25. Two hundred thirty-four dollars and fifty-five cents.

26. Twenty-five cents ; three cents, four mills.

27. One million, four hundred eighty-nine thousand, five hundred ninety and five hundred ninety thousandths.

28. Forty-three thousand, six hundred seventy-seven and four thousand six hundred-thousandths.

29. Three thousand sixty-nine and seventy-eight thousand four hundred sixteen ten-millionths.

30. The Roman System of Notation. The Roman system employs seven letters, as follows:

Letters,	I	V	X	L	C	D	M
Values,	1	5	10	50	100	500	1000

The first nine numbers are expressed by

I	II	III	IV	V	VI	VII	VIII	IX
1	2	3	4	5	6	7	8	9

The tens are expressed by

X	XX	XXX	XL	L	LX	LXX	LXXX	XC
10	20	30	40	50	60	70	80	90

Tens and ones are expressed by writing the expressions for units at the right of the expressions for tens. Thus,

XI	XII	XIV	XV	XIX	XXII	LVI	XCIX
11	12	14	15	19	22	56	99

The hundreds are expressed by

C	CC	CCC	CD	D	DC	DCC	DCCC	DCCCC
100	200	300	400	500	600	700	800	900

Writing M at the left of each of these expressions we have

MC	MCC	MCCC	MCD	MD	MDC	MDCC	MDCCC	MDCCCC
1100	1200	1300	1400	1500	1600	1700	1800	1900

Hundreds, tens, and ones are expressed by writing the hundreds, then the tens, and then the ones.

Thus, eighteen hundred ninety-six is written: MDCCC for eighteen hundred, then XC for ninety, and VI for six, making MDCCCXCVI.

Exercise 5.

Read:

XXXVI; XL; XLVI; LVIII; LIX; LXXXI; XCI; XCIII; CIX; CCIX; CCXX; CLIX; MDCCCLXXXVI; MDCLXVI; MDCCLXXVI; MCDLIX; MDLXXXIX.

Express in the Roman system:

43; 55; 81; 77; 99; 113; 128; 514; 724; 630; 1020; 1040; 1088; 1131; 1218; 1492; 1776; 1899; 1319; 1556; 1897; 1620; 1783; 1812; 1861; 1872.

CHAPTER II.

ADDITION AND SUBTRACTION.

31. The sign $+$ is called *plus*, and means that the number after it is to be *counted with* the number before it; that is, *added* to the number before it.

32. The sign $=$ is called the sign of *equality*, and stands for the word *equals;* so that $5+4=9$ is read: 5 plus 4 equals 9.

33. Addition. The operation of finding a number equal to two or more numbers taken together is called *addition;* and the result is called their *sum.*

34. The sum of two or more numbers is the same in whatever order the numbers are added.

Thus, $3+2+5=10$, or $5+3+2=10$.

35. Abstract Numbers. Numbers not applied to any particular unit, as 7, 17, 25, are called *abstract numbers.*

36. Concrete Numbers. Numbers applied to a particular unit, as 7 horses, 17 apples, are called *concrete numbers.*

37. Like Numbers. Numbers applied to *the same unit*, expressed or understood, are called *like numbers.*

38. *Only like numbers, and units of the same order, can be added.*

Exercise 6.

Count to 100 or more:

1. By 2's, beginning 0, 2, 4; beginning 1, 3, 5.

2. By 3's, beginning 0, 3, 6; beginning 1, 4, 7; beginning 2, 5, 8.

3. By 4's, beginning 0, 4, 8; beginning 1, 5, 9; beginning 2, 6, 10; beginning 3, 7, 11.

4. By 5's, beginning 0, 5, 10; beginning 1, 6, 11; beginning 2, 7, 12; beginning 3, 8, 13; beginning 4, 9, 14.

5. By 6's, beginning 0, 6, 12; beginning 1, 7, 13; beginning 2, 8, 14; beginning 3, 9, 15; beginning 4, 10, 16; beginning 5, 11, 17.

6. By 7's, beginning 0, 7, 14; beginning 1, 8, 15; beginning 2, 9, 16; beginning 3, 10, 17; beginning 4, 11, 18; beginning 5, 12, 19; beginning 6, 13, 20.

7. By 8's, beginning 0, 8, 16; beginning 1, 9, 17; beginning 2, 10, 18; beginning 3, 11, 19; beginning 4, 12, 20; beginning 5, 13, 21; beginning 6, 14, 22; beginning 7, 15, 23.

8. By 9's, beginning 0, 9, 18; beginning 1, 10, 19; beginning 2, 11, 20; beginning 3, 12, 21; beginning 4, 13, 22; beginning 5, 14, 23; beginning 6, 15, 24; beginning 7, 16, 25; beginning 8, 17, 26.

Find the sum of:

9.	10.	11.	12.	13.	14.	15.	16.	17.	18.	19.	20.
3	2	3	5	3	2	5	5	4	5	3	1
5	1	6	6	3	7	3	6	8	5	6	8
7	9	7	7	4	7	2	4	7	3	7	8
6	8	8	8	5	3	1	7	3	6	3	7

21.	22.	23.	24.	25.	26.	27.	28.	29.	30.	31.	32.
6	9	6	4	4	3	6	7	5	8	2	9
8	5	4	5	4	7	2	5	5	2	9	6
7	4	3	6	3	5	1	8	9	2	9	5
9	3	7	7	7	5	8	3	3	7	4	4

NOTE. In adding, begin at the bottom, and give *results* only. Thus, in example 9, say 13, 18, 21; do *not* say 6 and 7 are 13, 13 and 5 are 18, and 18 and 3 are 21.

39. Examples. 1. Find the sum of 45.32, 27.21, and 68.55.

SOLUTION. Since only units of the same order can be added, we write units under units, tens under tens, tenths under tenths, and hundredths under hundredths, so that the decimal points stand in a vertical line; and draw a line beneath.

We first find the sum of the hundredths and write this sum, 8, under the column of hundredths. We next find the sum of the tenths and write the units' figure, 0, of this sum under the column of tenths, but the tens' figure, 1, of this sum we add with the figures of the next column, making $1 + 8 + 7 + 5 = 21$. We write the units' figure, 1, of this sum under the column of units, but the tens' figure, 2, we add with the figures of the next column, making $2 + 6 + 2 + 4 = 14$. We write the entire sum of the last column on the left. We put the decimal point in the sum directly under the column of decimal points.

$$\begin{array}{r} 45.32 \\ 27.21 \\ \underline{68.55} \\ 141.08 \end{array}$$

2. Add 41.38, 37.89, 26.67, and 58.21.

$$\begin{array}{r} 41.38 \\ 37.89 \\ 26.67 \\ \underline{58.21} \\ 164.15 \end{array}$$

In adding, the following wording, and no more, should be used: one, eight, seventeen, twenty-five (emphasize five, and write it while pronouncing it), carry two; four, ten, eighteen, twenty-one, carry two; ten, sixteen, twenty-three, twenty-four, carry two; seven, nine, twelve, sixteen.

40. Hence, we have the following

RULE FOR ADDITION. *Write the numbers so that units of the same order shall stand in the same column.*

Add the right-hand column; write the units of this sum beneath, and add the tens, if any, to the next column.

So proceed with each column. Write the entire sum of the last column.

If any of the numbers contain a decimal, write the decimal point in the sum directly under the column of decimal points in the numbers.

PROOF. *Add each column in reverse order. If the results agree, the work may be assumed to be correct.*

Exercise 7.

Find the sum of :

1. 231 + 764.
2. 341 + 57.8.
3. 430.31 + 58.61.
4. 512.87 + 36.84.
5. 12.78 + 711.56 + 415.86.
6. 1543.1 + 164.7.
7. 1728 + 402.56.
8. 1897.3 + 675.34 + 6897.65.
9. 475.34 + 6897.65 + 1728.
10. 402.56 + 164.7 + 0.5236.
11. 0.7854 + 3.1416 + 2.71828.
12. 2.71828 + 402.56 + 1897.3.
13. 0.7854 + 4.12 + 30.103.
14. 2.7113 + 27.53 + 341.586.
15. 230.8 + 223 + 2.63 + 373.8.
16. 32.358 + 821.9 + 23.04 + 73.7.
17. 202.3031 + 71.575 + 65.813.
18. 0.0078 + 7.377 + 653.03.
19. 653.03 + 65.303 + 6.5033.
20. 939.303 + 65.746 + 8.2794.

21.	22.	23.
2.7182818	0.4342945	1.6093295
3.1415927	0.2098882	15.4323487
0.7853982	4.8104774	3.785

24.	25.	26.
0.4771213	1.6093295	0.6213768
0.2908882	3.2808693	3.785
4.8104774	0.3937043	0.264
2.5399772	0.5235988	15.4323487
0.3937043	0.4342945	1.7320508

27.	28.	29.
0.6213768	0.3937043	1.4142136
1.4142136	0.3047973	1.6093295
3.2808693	1.7320508	0.30103
0.3047973	2.236068	0.381966
4.8104774	0.381966	3.2808693

41. A simple test of the correctness of an addition is to add a second time, beginning at the top instead of the bottom of the columns, or to add two columns at once.

It is of great advantage to educate the eye to take in at a glance digits enough to make 10 or more, and then these *sums* can be added instead of the separate digits.

To illustrate, take the example in the margin. Adding from the bottom, the computer says to himself (seeing $8 + 2 = 10$, and $9 + 3 = 12$), 10, 12, **22**; **8**; 8, 12, **20**; 11, 4, **15**; 11, 10, **21**.

61,803
43,429
47,712
62,138
215,082

In the two-column mode he says, 50, 32, **82**; and writes down the 82. Then he continues, 98, 52, **150**, and writes the 50 at the left of 82; then he proceeds, 11, 10, **21**, and writes the 21.

Many bookkeepers and merchants strongly recommend the addition of two columns at once, as the most expeditious and the least liable to error.

42. Many computers begin at the bottom of the right-hand column in adding, and write on a piece of waste-paper the full sum of each column or double column; then they begin at the top of the left-hand column, and add each column or double column, also writing the full sum; finally, they add the sums obtained in the first addition, and the sums obtained in the second addition, and compare the results.

Thus, in the example above,

By Single Columns:

22	20
6	13
20	20
13	6
20	22
215082	215082

By Double Columns:

82	213
150	206
20	22
215082	215082

EXERCISE 8.

Add the following by double columns, and test by adding by single columns:

1.	2.	3.
$ 45.68	$154.31	$ 73.86
73.91	296.85	453.71
78.54	736.48	137.64
534.69	345.19	98.87
134.70	782.34	643.48
581.43	78.43	462.71

4.	5.	6.
$498.50	$ 65.42	$621.65
17.37	638.34	167.32
684.29	763.43	856.96
231.56	809.31	718.83
210.10	798.83	501.49
671.54	835.78	315.72
643.53	356.47	768.44

7.	8.	9.
$791.52	$ 32.54	$763.89
504.83	254.63	78.23
879.26	63.27	345.61
243.97	131.56	26.73
732.86	506.72	489.56
47.95	283.54	812.35
856.43	345.83	607.28
497.65	643.46	219.07
541.26	708.91	68.72
616.72	463.73	216.78
857.94	67.74	436.74

10.	11.	12.
$8400.07	$1873.33	$2336.29
3212.17	6170.24	336.00
1716.41	4813.25	2456.25
1020.08	662.25	641.25
1452.44	622.64	1174.50
1829.51	692.82	326.03
1929.96	2457.75	1219.87
114.78	2126.76	226.78
89.75	5391.25	276.75
173.67	7349.86	5936.40
17.45	1422.75	1914.78
112.44	9667.50	311.87
1098.75	6000.00	7956.00
6170.24	572.80	1919.66

13.	14.	15.
$1482.40	$ 773.72	$2406.08
2575.71	442.37	3101.24
3364.27	454.86	1452.09
689.81	358.61	3693.91
1533.61	2003.17	2054.76
735.58	179.56	1231.25
105.69	8493.75	1828.35
261.64	4179.54	1562.50
1516.56	3493.54	6937.50
2197.23	178.17	1987.57
1317.71	727.53	943.27
408.30	2889.42	2312.11
609.53	992.92	1409.28
1679.47	1183.08	2759.94

43. The sign — is called *minus*, and means that the number after it is to be *taken from* the number before it; that is, *subtracted* from the number before it.

The expression $9 - 5 = 4$ is read: 9 minus 5 equals 4.

44. Subtraction. The process of taking one number from another is called *subtraction.*

45. The number taken away is called the *subtrahend;* the number from which the subtrahend is taken, the *minuend;* and the number remaining, the *remainder* or *difference.*

46. *The sum of the remainder and the subtrahend is always equal to the minuend.* Hence,

47. *To test the accuracy of the work in subtraction,* we add the remainder and the subtrahend. The sum will be equal to the minuend, if the work is correct.

48. *The minuend, subtrahend, and remainder must all be like numbers; and from units of any order units of the same order only can be subtracted, ones from ones, tens from tens, tenths from tenths, hundredths from hundredths, etc.*

Exercise 9.

1. Subtract by 2's from 20 to 0; from 21 to 1.
2. Subtract by 3's from 20 to 2; from 21 to 0.
3. Subtract by 4's from 30 to 2; from 31 to 3; from 32 to 0; from 33 to 1.
4. Subtract by 5's from 32 to 2; from 33 to 3; from 34 to 4; from 35 to 0; from 36 to 1.
5. Subtract by 6's from 33 to 3; from 34 to 4; from 35 to 5; from 36 to 0; from 37 to 1; from 38 to 2.
6. Subtract by 7's from 42 to 0; from 43 to 1; from 44 to 2; from 45 to 3; from 46 to 4; from 47 to 5.
7. Subtract by 8's from 42 to 2; from 43 to 3; from 44 to 4; from 45 to 5; from 46 to 6; from 47 to 7.
8. Subtract by 9's from 55 to 1; from 56 to 2; from 57 to 3; from 59 to 5; from 61 to 7; from 62 to 8.

49. Examples. 1. From 359.7 take 186.3.

Solution. We write units under units, tens under tens, tenths under tenths, and so on. Then 3 tenths from 7 tenths leaves 4 tenths, and we write 4 under the column of tenths; 6 units from 9 units leaves 3 units, and we write 3 under the column of units; since we cannot take 8 tens from 5 tens, we change one of the 3 *hundreds* to 10 *tens* and add them to the 5 *tens*, making 15 *tens*; then 8 tens from 15 tens leaves 7 tens, and we write 7 under the column of tens. As we have taken one of the 3 *hundreds*, we have only 2 hundreds remaining; and 1 hundred from 2 hundreds leaves 1 hundred. The remainder, therefore, is 173.4.

Minuend,	359.7
Subtrahend,	186.3
Remainder,	173.4

2. From 50 take 27.65.

Solution. Since the subtrahend contains tenths and hundredths, and the minuend has neither tenths nor hundredths, we put zeros in the place of tenths and hundredths in the minuend. As there are no hundredths, no tenths, and no units in the minuend, 1 of the 5 tens is taken, leaving 4 tens, and changed to 10 units; then 1 of the 10 units is taken, leaving 9 units, and changed to 10 tenths; then one of the 10 tenths is taken, leaving 9 tenths, and changed to 10 hundredths. That is, 50.00 is changed to 4 tens, 9 units, 9 tenths, and 10 hundredths. Then, subtracting, we have 22.35.

$$\begin{array}{r} (4)(9)(9)(10) \\ 5\ 0.0\ 0 \\ \underline{2\ 7.6\ 5} \\ 2\ 2.3\ 5 \end{array}$$

50. Hence, we have the following

Rule for Subtraction. *Write the subtrahend under the minuend, placing units of the same order in the same column.*

Begin at the right and subtract each order of units of the subtrahend from the corresponding order of the minuend. Write the result beneath, step by step, and put in the decimal point when reached.

If any order of the minuend has fewer units than the same order of the subtrahend, increase the units of this order of the minuend by 10 and subtract; then diminish by one the units of the next higher order of the minuend.

Proof. *Add the remainder and subtrahend. If the sum equals the minuend, the work may be assumed correct.*

Exercise 10.

Find the remainder and prove:

1. 234 — 123.
2. 343 — 123.
3. 424 — 123.
4. 555 — 123.
5. 676 — 123.
6. 725 — 123.
7. 839 — 123.
8. 999 — 123.
9. 1000 — 123.
10. 5120 — 123.
11. 789 — 456.
12. 879 — 456.
13. 978 — 456.
14. 6378 — 456.
15. 6855 — 456.
16. 6853 — 456.
17. 7797 — 456.
18. 7006 — 456.
19. 3542 — 456.
20. 4000 — 456.
21. 974 — 779.
22. 368 — 249.
23. 2301 — 479.
24. 2731 — 929.
25. 708 — 394.
26. 1123 — 1072.
27. 891 — 773.
28. 8103 — 5621.
29. 19,001 — 3456.
30. 2180 — 792.
31. $183.45 — $76.47.
32. $716.43 — $628.74.
33. $647.51 — $549.64.
34. $270.04 — $128.31.
35. $125 — $101.50.
36. $247.93 — $129.47.
37. $641.87 — $333.95.
38. $56.27 — $29.89.
39. 3.1415927 — 2.7182818.
40. 0.7853982 — 0.5235988.
41. 4.8104774 — 0.4342945.
42. 2.5399772 — 0.3937043.
43. 0.3937043 — 0.3047973.
44. 3.2808693 — 0.3047973.
45. 3.2808693 — 1.6093295.
46. 3.785 — 0.6213768.
47. 15.4323487 — 0.264.
48. 1.7320508 — 1.4142136.
49. 2.236068 — 1.7320508.
50. 2.236068 — 0.618034.
51. 0.381966 — 0.30103.
52. 3.1415927 — 0.7853982.
53. 2.3561945 — 0.7853982.
54. 1.5707963 — 0.7853982.
55. 3.1415927 — 0.5235988.
56. 2.6179939 — 0.5235988.
57. 2.0943951 — 0.5235988.
58. 1.5707963 — 0.5235988.
59. 1.0471975 — 0.5235988.
60. 1 — 0.381966.
61. 1.4142136 — 0.618034.
62. 0.618034 — 0.381966.
63. 9,873,210 — 8,765,420.
64. 8010.101 — 4187.94.
65. 1,000,000 — 817,259.
66. 729,434 — 613,488.
67. 6532.18 — 1916.47.
68. 1718.754 — 1389.328.
69. 21,205 — 1787.563.
70. 42,786.95 — 4278.695.

Exercise 11.

1. In a till are $391 in bills, $67.50 in gold, $39.75 in silver, and $2.77 in copper and nickel. How much money is in the till?

2. Starting out with $315.75 in one wallet and $54.37 in another, I pay the grocer $127.38; the butcher, $64.17; the shoemaker, $21.40; the landlord, $50; the tailor, $35. What ought I to have left?

3. On a bill of $753.43, I pay $517.87. How much do I still owe? If I owe $817.87, and have but $637.50, how much do I lack of being able to pay?

4. If a man was born January 1, 1812, how old was he January 1, 1878?

5. America was discovered in 1492. How many years after its discovery was each of the following events?

Settlement of Florida, 1565; of Virginia, 1607; of Massachusetts, 1620; of Quebec, 1608; French and Indian War, 1756; Declaration of Independence, 1776; Inauguration of Washington, 1789; War with England, 1812; Mexican War, 1846; Civil War, 1861.

6. The minuend is one hundred million two hundred fifty-six thousand three hundred seventy-two, and the subtrahend is nineteen million nine hundred thousand nine hundred ninety-nine. Find the remainder.

7. If the minuend is 9874, and remainder 3185, what is the subtrahend? The subtrahend being 7659, and remainder 675.68, what is the minuend?

8. The smaller of two numbers is 7.95764328; their difference is 0.00087692. What is the larger number?

9. The larger of two numbers is 7.95764328, and their difference is 7.153485. What is the smaller number?

10. If the subtrahend is 10,542, and the difference 544.2, what is the minuend?

11. A man pumps out of a cistern in one hour 243.75 gallons; in the next hour, 227.5 gallons; in 45 minutes more, 137.75 gallons; and the cistern is empty. How many gallons of water were in it?

12. From what number must I subtract 5 to leave 7? 8 to leave 9? From what number must I subtract 5.1736 to leave 8.1964? 6.231 to leave 9.6648? 74.213 to leave 25.787?

13. What must be subtracted from 1 to leave 0.5? to leave 0.53? to leave 0.532? to leave 0.5236? to leave 0.5235988?

14. I start on a journey of 3433 miles. The first day I make 428 miles; the second day, 511 miles; the third, 497 miles; the fourth, 513. How many miles of my journey remained for me at the close of each day? How many miles had I gone at the close of each day?

15. Subtract 76,343 from the sum of 61,932, 51,387, 5193, 4674, and 8199; then subtract 23,657 from the remainder.

16. Jones bought a farm and stock for $7633.90; sold the stock for $305.75; then sold the farm for $7325. How much did he lose?

17. If I gave $4375 for my land, and paid for house, barn, sheds, and fences, $2789.50; also $973.75 for horses, cattle, tools, etc.; what did my farm and stock cost?

18. If I paid $8138.25 for land and cattle, and sold part of the land for $675, and part of the cattle for $217.50, what is the cost of the land and the cattle left?

19. John has 158 cents, James has 271 cents; James gives John 56 cents. Which has then more than the other, and how many cents more?

20. A cattle dealer had 228 oxen, 475 sheep, and 49 lambs; he sold 17 oxen, 64 sheep, and 7 lambs. How many animals of each kind did he then have, and how many all together?

CHAPTER III.

MULTIPLICATION.

51. Multiplication. The process of taking a *number of units* a *number of times* is called *multiplication.*

52. Multiplicand. The number of units taken is called the *multiplicand.*

53. Multiplier. The number that shows how many times the multiplicand is taken is called the *multiplier.*

54. Product. The number found by multiplication is called the *product.*

55. *The multiplier always signifies a number of times, and is, therefore, an abstract number.*

56. *The multiplicand and product are like numbers.*

57. Factors. The numbers used in making a product are called *factors* of the product.

58. The product of two factors is the same whichever factor is taken as the multiplier.

● ● ● ● Thus, 3 times 4 = 4 times 3. The dots in the mar-
● ● ● ● gin read across the page make 3 fours; read up and
● ● ● ● down the page they make 4 threes.

NOTE. The multiplicand always signifies a *number of units*, whether *the kind of units is stated or not.* The only difference between 15 and 15 horses is that in the first case the kind of units counted is not stated, and in the second case the kind is stated.

We may interchange the multiplicand and multiplier if we refer to the *numbers only.* Thus, in the example 3 times 4 horses, we cannot say 4 horses times 3, but we may interchange the 3 and 4, and have 4 times 3 horses. The product in either case is 12 horses. With this understanding, we may always use the smaller number as multiplier.

59. The sign of multiplication is ×. When the multiplier *precedes* the multiplicand, the sign × is read *times.*

Thus, 6 × $7 = $42 is read : 6 *times* $7 equals $42.

When the multiplier *follows* the multiplicand, the sign × is read *multiplied by.*

Thus, $7 × 6 = $42 is read: $7 *multiplied by* 6 equals $42, and means $7 taken 6 *times* equals $42.

60. The products, in all cases in which neither factor exceeds twelve, should be thoroughly committed to memory. They will be found in the following

MULTIPLICATION TABLE.

1	2	3	4	5	6	7	8	9	10	11	12
2	4	6	8	10	12	14	16	18	20	22	24
3	6	9	12	15	18	21	24	27	30	33	36
4	8	12	16	20	24	28	32	36	40	44	48
5	10	15	20	25	30	35	40	45	50	55	60
6	12	18	24	30	36	42	48	54	60	66	72
7	14	21	28	35	42	49	56	63	70	77	84
8	16	24	32	40	48	56	64	72	80	88	96
9	18	27	36	45	54	63	72	81	90	99	108
10	20	30	40	50	60	70	80	90	100	110	120
11	22	33	44	55	66	77	88	99	110	121	132
12	24	36	48	60	72	84	96	108	120	132	144

61. In this table, take the multiplicand in the upper line, and the multiplier in the left-hand column; the product will be found directly under the multiplicand, and opposite the multiplier; as, 12 × 7 is 84.

62. *A change in the position of the decimal point* of a number expressed in figures affects the value of the number.

Thus, if in 79.253 we move the decimal point one place to the right, so that the number becomes 792.53, we increase the value of each figure tenfold; the 7 tens become 7 hundreds, the 9 units become 9 tens, the 2 tenths become 2 units, the 5 hundredths become 5 tenths, and the 3 thousandths become three hundredths. The value of the entire number, therefore, is increased tenfold. But multiplying a number by 10 increases its value tenfold. Hence, moving the decimal point of a number one place to the right has the same effect as multiplying the number by 10. In the same way, moving the decimal point two places to the right multiplies the number by 100, and so on. Hence,

63. To Multiply a Number by 10, 100, 1000, etc.,
Move the decimal point in the multiplicand as many places to the right, annexing zeros if necessary, as there are zeros in the multiplier.

Again, if in 79.253 we move the decimal point one place to the left, so that the number becomes 7.9253, we decrease the value of each figure tenfold; the 7 tens become 7 units, the 9 units become 9 tenths, the 2 tenths become 2 hundredths, the 5 hundredths become 5 thousandths, and the 3 thousandths become 3 ten-thousandths. The value of the resulting number, therefore, is one tenth that of the original number. But multiplying a number by 0.1 means to find one tenth of the number. Hence, moving the decimal point of a number one place to the left has the same effect as multiplying the number by 0.1. In the same way, moving the decimal point two places to the left has the same effect as multiplying the number by 0.01, and so on. Hence,

64. To Multiply a Number by 0.1, 0.01, 0.001, etc.,
Move the decimal point in the multiplicand as many places to the left, prefixing zeros if necessary, as there are decimal places in the multiplier.

65. Since multiplication, as defined in § 51, is the process of taking the multiplicand the number of times indicated by the multiplier, the multiplier must be an integral number ; but the meaning of multiplication is *extended* to cover the case in which the multiplier contains a decimal.

To multiply by a decimal is to take such a part of the multiplicand as the decimal is of one.

To multiply by an integral number and a decimal is to take the multiplicand as many times as is indicated by the integral number, and such a part of the multiplicand as is indicated by the decimal.

66. Examples. 1. Find the product of 6×34.87.

$$\begin{array}{r} 34.87 \\ \underline{6} \\ 209.22 \end{array}$$

SOLUTION. We write the multiplier, 6, under the multiplicand, as in the margin, and begin at the right to multiply. 6 times 7 hundredths equals 42 hundredths, or 4 tenths and 2 hundredths. We write the 2 hundredths in the place of hundredths, and reserve the 4 tenths to add to the product of the tenths. 6 times 8 tenths equals 48 tenths, and 48 tenths plus the 4 tenths reserved makes 52 tenths, or 5 units and 2 tenths. We write the 2 tenths in the place of tenths and reserve the 5 units to add to the product of the units. 6 times 4 units equals 24 units, and 24 units plus the 5 units reserved makes 29 units, or 2 tens and 9 units. We write the 9 units in the place of units, and reserve the 2 tens to add to the product of the tens. 6 times 3 tens equals 18 tens, and 18 tens plus the 2 tens reserved makes 20 tens. We write 20 to the left of the 9 units.

2. Find the product of 0.6×34.87.

$$\begin{array}{r} 34.87 \\ \underline{0.6} \\ 20.922 \end{array}$$

SOLUTION. The multiplier, 0.6, equals 6×0.1. We therefore multiply first by 6, and this product by 0.1. But multiplying by 0.1, simply moves the decimal point in the product one place to the left. Hence, the product has two decimal places for the decimal in the multiplicand, and one more place for the decimal in the multiplier, or three in all.

3. Find the product of 263×74.782.

Solution. The multiplier is $200 + 60 + 3$. We obtain the product by multiplying the multiplicand by 3, then by 60, then by 200, and adding these products.

	(1)	(2)
	74.782	74.782
	263	263
3 times the multiplicand =	224.346	224346
60 times the multiplicand =	4486.920	448692
200 times the multiplicand =	14956.400	149564
263 times the multiplicand =	19667.666	19667.666

Since zeros at the right of the partial products do not affect the result of the addition, they may be omitted as in (2). *Care must be taken, however, to put the right-hand figure of each partial product directly under the figure of the multiplier used in obtaining it.*

4. Find the product of 2.63×74.782.

74.782
2.63
224346
448692
149564
196.67666

Solution. The multiplier, 2.63, equals 263×0.01. We therefore multiply first by 263 and this product by 0.01. But multiplying by 0.01 simply moves the decimal point in the product two places to the left. Hence, the product has three decimal places for the decimal in the multiplicand, and two more places for the decimal in the multiplier, or five in all.

5. Find the product of 2007×4587.

4587
2007
32109
9174
9206109

Solution. The partial products corresponding to the zeros in the multiplier will be zero, and therefore they need not be written.

67. In each of these examples, if we interchange the multiplier and multiplicand and multiply, we find that we obtain the same product.

68. Hence, we have the following

Rule for Multiplication. *Write the multiplier under the multiplicand with their right-hand figures in a vertical line, and draw a line beneath.*

Begin at the right and multiply each order of units of the multiplicand by each order of the multiplier. Write the units of each product, and add the tens, if any, to the next product.

Place the right-hand figure of each partial product under the figure of the multiplier used in obtaining it, and add the partial products.

Point off from the right of the product as many figures for decimals, prefixing zeros if necessary, as there are decimal places in the multiplicand and multiplier together.

PROOF. *Interchange the multiplier and multiplicand, and multiply. If the results agree, the work may be assumed to be correct.*

EXERCISE 12.

Find the product of :

1. 6×0.5235988.
2. 4×0.7853982.
3. $3.14159265 \times 5 \times 5$.
4. 30×8.75.
5. 0.07×6.975.
6. 700×7.81.
7. 8000×65.432.
8. 300×7.85.
9. $0.009 \times 10{,}356.78$.
10. 7.37×0.785398.
11. 8.56×0.785398.
12. 1001×0.785398.
13. 0.083×2150.42.
14. 0.75×2150.42.
15. 0.075×2150.42.
16. 0.7071×1.4142136.
17. 1.41421×1.4142.
18. 1.732×1.732.
19. 2.23607×2.236.
20. 0.618×618.
21. 0.618034×0.618035.
22. 12×0.12936.
23. 7.92801×0.9.
24. 58.383×0.39.
25. 0.08×0.28744.
26. 0.065×491.205.
27. 68.325×6.25.
28. 0.732×1.6.
29. 0.438×1208.88.
30. 498×0.0125.
31. 7×0.007.
32. 1000×0.0001.
33. 0.235×10.24.
34. 0.00507702×0.0283.
35. 89.3×0.00752.
36. 74.1×0.0256.

69. Powers and Roots. If a product consists of *equal* factors, it is called a *power* of that factor; and one of the equal factors is called a *root* of the product.

The power is named from the *number* of equal factors. Thus,

25 (5×5) is the *second power*, or **square**, of 5.
125 $(5 \times 5 \times 5)$ is the *third power*, or **cube**, of 5.
625 $(5 \times 5 \times 5 \times 5)$ is the *fourth power* of 5.

5 is called the *second root*, or **square root**, of 25.
5 is called the *third root*, or **cube root**, of 125.
5 is called the *fourth root* of 625.

To avoid writing long rows of equal factors, a figure, called the **exponent**, is written at the right of a number to show how many times the number is taken as a factor.

Thus, 5^5 means the same as $5 \times 5 \times 5 \times 5 \times 5$, and is read the fifth power of 5. 3^3 times 3^4 means $3 \times 3 \times 3$ times $3 \times 3 \times 3 \times 3 = 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 = 3^7$. Hence,

The product of two or more powers of the same number may be expressed by writing the number with an exponent equal to the sum of the exponents of the given powers.

EXERCISE 13.

Express the product of:

1. $7^5 \times 7^3$; $8^2 \times 8$; $2^8 \times 2$; $5^4 \times 5^2$.
2. $3.01^2 \times 3.01$; $0.67^2 \times 0.67^3$; 0.208×0.208^3.
3. $2.003^2 \times 2.003^4$; $20.03^3 \times 20.03$; 20.03×20.03^2.

70. Casting out Nines. The product of two integral factors is called a *multiple* of either of its factors.

Every power of 10 is one more than some multiple of 9.

Thus, $10 = 9 + 1$; $10^2 = 11 \times 9 + 1$; $10^3 = 111 \times 9 + 1$, etc.

Every multiple of a power of 10 by a single digit is, therefore, some multiple of 9, plus that digit.

Thus, $500 = 55 \times 9 + 5$; $7000 = 777 \times 9 + 7$, etc.

But as every number consists of the sum of such multiples of powers of 10, every number is a multiple of 9, plus the sum of its own digits.

Thus, 24,573 is a multiple of 9 plus $2 + 4 + 5 + 7 + 3$. In casting out the nines from a number, the remainder, therefore, is the same as that arising from casting out the nines from the sum of its digits.

In finding the remainder from *casting out the nines* from a number, we may, of course, omit the nines, or any two or three digits which we see at a glance will make 9.

Thus, to cast out the nines from 1,926,754, we see at once that 1, 2, 6, and 5, 4, make nines, and the single 7 will be the remainder. So in 254,786, we reject 5, 4, and 2, 7, and add only 8, 6, from the sum of which reject 9, and there is left 5.

71. Example. Multiply 761 by 147.

SOLUTION. The remainder after the *nines are cast out*

From	761	. . . . is 5	Multiply.
From	147	. . . . is 8	
	5327	From 15 . . is 6	
	3044		
	761		
From	111867	. . . . is 6	

The product of the two given *numbers* has 6 remaining, and the product of the two *remainders* has 6 remaining after the nines are cast out. Therefore, the work may be assumed to be correct.

72. Casting out Elevens. Even powers of 10 are multiples of 11, plus 1; odd powers of 10 are multiples of 11, minus 1.

Thus, 10^2, or $100 = 9 \times 11 + 1$; 10^4, or $10{,}000 = 909 \times 11 + 1$, etc.; $10 = 11 - 1$; 10^3, or $1000 = 91 \times 11 - 1$; $10^5 = 9091 \times 11 - 1$, etc.

Hence, if the elevens are cast out from a number expressed by two digits, the remainder equals the digit in the odd place minus the digit in the even place, (the digit in the odd place, if less than that in the even place, being

increased by 11); and, if the elevens are cast out from *any* number, the remainder equals the sum of the digits in the odd places (increased by a multiple of 11 if necessary) minus the sum of the digits in the even places.

73. The proof by casting out elevens is similar to that by casting out nines; and if a process stands both tests, the only possible error is a multiple both of 9 and of 11.

Multiply 67,853 by 2976, and test by casting out the elevens.

$$\begin{array}{r l}
67853 & \ldots\ldots\ 5 \\
\underline{2976} & \ldots\ldots\ \underline{6} \Big\}\text{ Multiply.} \\
407118 & 30 \ldots\ldots\ 8 \\
474971 & \\
610677 & \\
\underline{135706} & \\
201930528 & \ldots\ldots\ldots\ldots\ldots\ldots\ 8
\end{array}$$

74. In applying either test to decimals, disregard the decimal point. In questions involving large numbers, *always* apply one or both tests to the work.

Exercise 14.

Find the following products, and test the accuracy by casting out the nines and by casting out the elevens:

1. $21.3706 \times 15.243 \times 1.8954$.
2. $0.026891 \times 5.328 \times 29.74$.
3. $0.0012 \times 5.8281 \times 0.6827$.
4. $23.9875 \times 12.4764 \times 0.017$.
5. $39.801 \times 1.44 \times 17.9645$.
6. $0.0165 \times 5.2817 \times 0.8469$.
7. $0.54237 \times 16 \times 0.00176$.
8. $24.271 \times 3.6485 \times 15.271$.
9. $13.256 \times 14.125 \times 30.254$.

75. Contracted Multiplication of Decimals. In ordinary calculations we seldom use a decimal smaller than 0.00001 of the unit.

Multiply 0.123456789 by 1.23456789.

	0.123456789
	1.23456789
	1111111101
	987654312
	864197523
	740740734
6	17283945
49	3827156
370	370367
2469	13578
12345	6789
0.15241	578750190521

The 6 at the left of the vertical line is obtained by multiplying the 1 in the multiplicand by 5 in the multiplier, and carrying 1 from 5 × 2. The 9 below the 6 is obtained by multiplying the 2 by 4, and carrying 1 from 4 × 3. The 0 below the 9 is obtained by multiplying the 3 by 3, and carrying 1 from 3 × 4. The 9 below the 0 is obtained by multiplying the 4 by 2, and carrying 1 from 2 × 5. The 5 below the 9 is obtained by multiplying the 5 by 1.

It is evident that, if five decimal places only are required in the product, all the work to the right of the vertical line is wasted.

To shorten the work, disregard the decimal point in the multiplier, and write the multiplier under the multiplicand, so that each digit of the multiplier shall fall directly under the digit of the multiplicand into which it is multiplied to produce the first figure to the left of the vertical line in each partial product. Thus :

0.123456789
987654321
12346
2469
370
49
6
1
0.15241

Notice that the figures of the multiplier are reversed; that the units' figure of the multiplier falls under the last decimal of the multiplicand which is to be retained, and that the decimals in the result are correct. In order, however, to have the required number of decimal places correct, it will generally be necessary to take one more than the required number of decimal places. Hence,

76. To Multiply Decimals by the Contracted Method,

Reverse the multiplier, and put the units' figure under the last place of decimals to be retained.

Multiply each figure of the multiplier into the figure next to the right above it. Do not write this result, but carry the nearest ten to the next result, multiplying as usual.

Write the first figures of the partial products in a vertical column.

Add the products, and point off from the sum as many decimal places as were taken in the multiplicand.

If the multiplier has no units' figure, supply its place with a zero.

To insure accuracy, take one decimal place more than the required number.

To detect errors that may arise from displacement of the decimal point, or from an erroneous arrangement of the factors, test the result by a rough estimate of what the product should be.

Exercise 15.

Find to the fifth decimal the value of:

1. $0.49714987 \times 1.75812263$.
2. $0.79817987 \times 0.99429975$.
3. $1.09920986 \times 0.24857494$.
4. $0.62208861 \times 0.16571662$.
5. $1.75812263 \times 2.05915963$.
6. $0.55630251 \times 0.33445375$.
7. $0.75142506 \times 9.98998569$.
8. $0.05245506 \times 0.16571662$.
9. $0.33143325 \times 1.79263713$.
10. $0.90633287 \times 0.6154551376$.
11. $2.84657842 \times 0.96695542$.
12. $0.546794489 \times 2.847697495$.

CHAPTER IV.

DIVISION.

77. Division. If the product and one factor are given, the process of finding the other factor is called *division.*

78. Dividend. The given product, that is, *the number to be divided,* is called the *dividend.*

79. Divisor. The given factor, that is, *the number by which the dividend is divided,* is called the *divisor.*

80. Quotient. The required factor, that is, *the number found by division,* is called the *quotient.*

81. To divide 35 apples by 7 apples is to find the *number of times* we must take 7 apples to obtain 35 apples. Hence,

If the divisor and dividend denote the same kind of units, the quotient is an abstract number.

82. To divide 35 apples by 7 is to find the *number of apples* in each part, when 35 apples are divided into 7 equal parts. Hence,

If the divisor is an abstract number, the quotient denotes units of the same kind as the dividend.

83. What is one of the parts called, if a number is divided into 2 equal parts? 3? 4? 5? 6? 7? 8? 9?

One half is written $\frac{1}{2}$; one third, $\frac{1}{3}$; one fourth, $\frac{1}{4}$; one fifth, $\frac{1}{5}$; one sixth, $\frac{1}{6}$; one seventh, $\frac{1}{7}$; one eighth, $\frac{1}{8}$; one ninth, $\frac{1}{9}$; and so on.

To divide 35 apples by 7 is to find $\frac{1}{7}$ of 35 apples.

84. Division is indicated by the *sign of division* ÷, or by writing the dividend over the divisor with a line between them.

Thus, $42 \div 6$ and $\frac{42}{6}$ have the same meaning, and each is read: forty-two *divided by* six.

85. Remainder. If the divisor does not exactly divide the dividend, the part of the dividend left from the division is called the *remainder.*

Thus, $42 ÷ 4 = $10 *with remainder* $2.

86. Principles of Division. The value of the quotient depends upon the *relative* values of the dividend and divisor.

Suppose we have $36 \div 6 = 6$.

If we multiply the dividend 36 by 2, what effect will this have on the quotient?

If we divide the dividend 36 by 2, what effect will this have on the quotient?

If we multiply the divisor 6 by 2, what effect will this have on the quotient?

If we divide the divisor 6 by 2, what effect will this have on the quotient?

If we multiply both the dividend and divisor by 2, what effect will this have on the quotient?

If we divide both the dividend and divisor by 2, what effect will this have on the quotient?

From the answers to these questions, we have the following important principles of division:

Multiplying the dividend or dividing the divisor by a number multiplies the quotient by that number.

Dividing the dividend or multiplying the divisor by a number divides the quotient by that number.

Multiplying both dividend and divisor by the same number, or dividing both by the same number, does not change the quotient.

Short Division.

87. If the divisor is so small that the work can be performed mentally, the process is called **short division.**

88. Examples. **1.** Divide 976 by 3.

3)976
325⅓

SOLUTION. We write the divisor to the left of the dividend with a curved line between them, and the quotient under the dividend, as in the margin. 3 is contained in 9 three times; and we write 3 in the quotient under the 9 of the dividend. 3 is contained in 7 twice with remainder 1; and we write 2 in the quotient in the place of tens. The remainder 1 is 1 ten or 10 units, and 10 units + the 6 units of the dividend makes 16 units. 3 is contained in 16 five times with remainder 1. The answer is 325 with remainder 1. We may write the remainder over the divisor as a part of the quotient; thus, 325⅓.

The following wording and no more should be used: 3 in 9, **3**; in 7, **2**; in 16, **5** remainder 1.

2. Divide 72.56 by 8.

8)72.56
9.07

SOLUTION. 8 in 72, **9**; in 5, **0**; in 56, **7**. Here the 72 to be divided by 8 is 72 units, and 72 units divided by 8 gives 9 units; we place the decimal point in the quotient after the units' figure 9. The next figure 5 is 5 tenths, and 5 will not contain 8. We write 0 in the quotient and annex the 6 hundredths to the 5 tenths, making 56 hundredths; and 56 hundredths divided by 8 is 7 hundredths. The quotient is 9.07. Hence,

If the divisor is an integral number, we write the decimal point in the quotient as soon as we reach the decimal point in the dividend.

3. Divide 72.56 by 0.08.

8)7256
907

SOLUTION. Since we do not change the quotient if we multiply both divisor and dividend by the same number (§ 86), we make the divisor an integral number by multiplying it by 100, and multiply the dividend by the same number. We then divide as before. Hence,

If the divisor contains decimal places, we multiply both divisor and dividend by 10, or some power of 10, so as to make the divisor an integral number.

4. Divide 78.52 by 8000.

$$\begin{array}{r} 8\cancel{0}\cancel{0}\cancel{0}\overline{)0.07852} \\ 0.009815 \end{array}$$

Solution. We first divide the divisor by 1000 by cutting off the three zeros at its right; and move the decimal point in the dividend three places to the left, prefixing one zero. When we reach the last figure of the dividend, we mentally supply a zero, and continue dividing. Hence,

If the divisor is an integral number ending in one or more zeros, we cut off the zeros, and move the decimal point in the dividend as many places to the left as we cut off zeros.

89. If we add the remainder to the product of the divisor and quotient, we obtain the dividend. Hence,

We find the product of the divisor and quotient, and to this product add the remainder. If the result equals the dividend, the work may be assumed to be correct.

Exercise 16.

Find the quotient of:

1. 126.409 ÷ 9.
2. 13.31 ÷ 10.
3. 13.31 ÷ 11.
4. 1.728 ÷ 12.
5. 37.632 ÷ 30.
6. 42,631 ÷ 20.
7. 96,464 ÷ 400.
8. 58.775 ÷ 600.
9. 75,230 ÷ 700.
10. 8956 ÷ 80.
11. 98,254 ÷ 900.
12. 82,610 ÷ 7000.
13. 83,690 ÷ 500.
14. 96,464 ÷ 110.
15. 87,585 ÷ 1200.
16. 27,485 ÷ 200.
17. 10.01 ÷ 0.02.
18. 0.04 ÷ 0.05.
19. 7.432 ÷ 0.04.
20. 31 ÷ 0.005.
21. 480 ÷ 0.012.
22. 980 ÷ 0.0007.
23. 10.98 ÷ 0.00009.
24. 10.98 ÷ 0.09.
25. 0.1098 ÷ 0.00009.
26. 0.1098 ÷ 0.09.
27. 1441 ÷ 0.11.
28. 18.92 ÷ 1.1.

Long Division.

90. The process of Long Division is the same as that of Short Division, except that the work is written in full, and the quotient is written *over* the dividend, the first quotient figure being written over the right-hand figure of the partial dividend used in obtaining it.

NOTE. The quotient may be written *at the right* of the dividend. The advantage, however, of writing the quotient *over* the dividend is easily seen when the quotient contains a decimal.

91. Each quotient figure may be *estimated* by taking for a trial divisor the nearest number of tens or hundreds, etc., represented by the divisor, and by taking for a trial dividend the nearest number of tens or hundreds, etc., represented by the partial dividend.

92. *In each step of division, the product must be less than the partial dividend, and the remainder less than the divisor.*

93. Examples. 1. Divide 4199 by 78.

SOLUTION. As 78 is more than 41, it is necessary to take *three* figures of the dividend for the first partial dividend. As the nearest number of *tens* represented by the divisor is 8, we take 8 for the trial divisor. As the nearest number of tens represented by the partial dividend is 42, we take 42 for the trial dividend. 8 is contained 5 times in 42. Hence, the first quotient figure is 5, and we write 5 in the quotient over the right-hand figure, 9, of the partial dividend. We multiply the divisor 78 by 5 and subtract the product 390 from 419. We annex the next figure 9 of the dividend to the remainder 29. The nearest number of tens represented by the second partial dividend is 30, and 8 is contained 3 times in 30. We place 3 as the second figure of the quotient and multiply the divisor by 3. This product subtracted from the second partial dividend leaves for a remainder 65. The *complete quotient* may be written $53\frac{65}{78}$.

$$\begin{array}{r} 53 \\ 78\overline{)4199} \\ \underline{390} \\ 299 \\ \underline{234} \\ 65 \end{array}\ \textit{remainder.}$$

2. Divide 2791.163 by 394.

```
     7.084
394)2791.163
    2758
     3316
     3152
      1643
      1576
        67 remainder.
```

SOLUTION. The first partial dividend is 2791. As the nearest number of *hundreds* represented by the divisor 394 is 4, we take 4 for the trial divisor. As the nearest number of hundreds represented by the partial dividend 2791 is 28, we take 28 for the trial dividend. 4 is contained 7 times in 28. We write the 7 over the 1, the right-hand figure of the partial dividend. *We place the decimal point in the quotient directly over the decimal point in the dividend*, that is, directly after the 7. We multiply the divisor 394 by 7. We subtract the product 2758 from 2791 and have for a remainder 33, to which we annex the 1 of the dividend. As 331 is less than 394, the next quotient figure is 0. To 331 we annex the next figure, 6, of the dividend. 4 is contained 8 times in 33. We therefore write 8 for the next quotient figure, and find the product of 8×394 to be 3152. The remainder obtained by subtracting 3152 from 3316 is 164, to which we annex the 3 of the dividend. 4 is contained 4 times in 16. The product of 4×394 is 1576, and this subtracted from 1643 leaves 67 for the final remainder. The *complete quotient* may be written $7.084\frac{67}{394}$.

```
     17
394)670
    394
    2760
    2758
```

When the quotient contains a decimal, it is not customary to write the remainder over the divisor, but to continue the division by annexing zeros to the dividend. In this example, if we carry the division two decimal places further, as in the margin, we have the quotient 7.08417.

94. Decimals are seldom carried to more than *five* places. If we wish to find the value of a decimal correct to the nearest *tenth, hundredth, thousandth, etc.*, we add 1 to the last required figure if the next figure would be *five or more.*

Thus, the value of the answer to the last example, 7.08417, correct to the nearest tenth is 7.1; correct to the nearest hundredth is 7.08; correct to the nearest thousandth is 7.084; correct to the nearest ten-thousandth is 7.0842.

95. As in Short Division, if the divisor contains decimal places, we multiply both divisor and dividend by a power of 10, so as to make the divisor an integral number.

Examples. 1. Divide 28.3696 by 1.49.

```
     (1)                              (2)
      19.04                             19.04
149)2836.96                     1.49)28.36‸96
    149                              149
    1346                             1346
    1341                             1341
       596                              596
       596                              596
```

In form (1) we multiply both dividend and divisor by 100.

The same result is obtained by counting to the right from the decimal point in the dividend as many places as there are decimal places in the divisor, inserting a caret as shown in form (2), and putting the decimal point in the quotient directly over the caret.

2. Divide 0.381876 by 2.63; 739.4112 by 0.1728.

```
         0.1452                        4279.
2.63)0.38‸1876            0.1728)739.4112‸
     263                         6912
     1188                         4821
     1052                         3456
      1367                        13651
      1315                        12096
        526                        15552
        526                        15552
```

3. Divide 34.2 by 18,000.

```
      0.0019
18000)‸034.2
          18
          162
          162
```

Solution. Here we cut off the three zeros from the divisor, and put a caret three places to the left of the decimal point in the dividend; that is, we *divide* both the divisor and the dividend by 1000.

96. From these examples we have the following

RULE FOR LONG DIVISION. *Write the divisor to the left of the dividend, with a curved line between them.*

If the divisor contains decimal places, remove the decimal point from the divisor, and move the decimal point in the dividend to the right as many places as there are decimal places in the divisor.

Take for the first partial dividend the fewest left-hand figures that will contain the divisor, and write the quotient over the right-hand figure of this partial dividend.

Multiply the divisor by this quotient, and place the product under the partial dividend used.

Subtract this product, and to the remainder annex the next figure of the dividend.

Proceed as before, and continue the process until all the figures of the dividend have been used; putting the decimal point in the quotient as soon as the decimal point in the dividend is reached.

PROOF. *Find the product of the divisor and quotient, and to this product add the remainder, if any. If the result equals the dividend, the work may be assumed to be correct.*

NOTE. If the divisor is an integral number ending in one or more zeros, cut off the zeros, and move the decimal point in the dividend to the left as many places as there are zeros cut off, prefixing zeros if necessary.

EXERCISE 17.

Find the quotient of:

1. 7553 ÷ 91.	5. 35,372 ÷ 35.	9. 370,406 ÷ 843.
2. 4593 ÷ 73.	6. 834,561 ÷ 408.	10. 978,217 ÷ 498.
3. 89,713 ÷ 76.	7. 341,586 ÷ 247.	11. 543,816 ÷ 357.
4. 53,691 ÷ 88.	8. 861,345 ÷ 395.	12. 604,730 ÷ 1289.

13. $326.7 \div 132.$
14. $79.72552 \div 1.121.$
15. $0.4077 \div 9.06.$
16. $961.9476 \div 106.8.$
17. $\$27,121.50 \div \$387.45.$
18. $123.684792 \div 39.37.$
19. $1203.75 \div 19.26.$
20. $256 \div 0.0016.$
21. $24.1802 \div 3.19.$
22. $200 \div 0.3125.$
23. $7.704256 \div 0.08302.$
24. $1.6093295 \div 0.479.$
25. $1.6093295 \div 0.917.$
26. $1.6093295 \div 0.017.$
27. $1.6093295 \div 0.0087.$
28. $3 \div 1.7.$
29. $3 \div 1.73.$
30. $3 \div 1.732.$
31. $3 \div 1.7321.$
32. $1.6093295 \div 5280.$
33. $2 \div 1.4142.$
34. $5 \div 2.236.$
35. $\$25,000 \div 117.$
36. $162 \div 14.72.$
37. $1.27 \div 19,800.$
38. $0.5632 \div 1.6382.$
39. $1,872,760 \div 42,360.$
40. $1897 \div 192.93.$
41. $0.0001 \div 0.0872.$
42. $14.1658 \div 1.8246.$
43. $\$10,150.75 \div \$303.77.$
44. $\$312.12 \div 24.73.$
45. $\$3115.20 \div 176.$
46. $\$2840 \div 5.135.$
47. $125,457.64 \div 354.2.$
48. $0.75 \div 0.866.$
49. $96,800 \div 311.13.$
50. $2,534,000 \div 6.4037.$
51. $6.4037 \div 2,534,000.$
52. $0.7407 \div 5.504.$
53. $54.44 \div 1.7359.$
54. $0.3658 \div 2.322.$
55. $\$25.12 \div 1.43.$
56. $17 \div 0.036.$
57. $\$143 \div 1.7892.$
58. $18.7 \div 121.$
59. $495,872.1765 \div 1728.$
60. $186,517.725 \div 5280.$
61. $56,287.625 \div 231.$
62. $782,847.375 \div 43,560.$
63. $18,520 \div 272.25.$
64. $36,581.17 \div 2150.42.$
65. $10,000 \div 196.$
66. $\$219.12 \div 1.025.$
67. $22,000 \div 5645.376.$
68. $0.0165 \div 1.331.$
69. $75,555 \div 1152.$
70. $12.2 \div 5.5056.$
71. $77 \div 10.7716.$
72. $66 \div 7.2426.$
73. $54.55 \div 1728.$
74. $46.88 \div 44.723.$
75. $0.874 \div 444.$
76. $54,728 \div 5280.$
77. $2.034 \div 0.0018.$
78. $0.08748 \div 10.8.$
79. $4443.33 \div 0.0037.$
80. $0.032048 \div 20.03.$

97. The Parenthesis. If numbers are included in a parenthesis ( ), the first step is to reduce these numbers to a single number as the signs direct.

$(9+7-1)\div 5=15\div 5.$ $(4\times 6-9)\div 5=15\div 5.$
$48\div(4\times 3)=48\div 12.$ $48\div(24\div 4)=48\div 6.$

NOTE. Instead of a parenthesis we sometimes use brackets [ ], braces { }, or a vinculum —. Thus, $(8-3)$, $[8-3]$, $\{8-3\}$, $\overline{8-3}$, all have the same meaning.

In reducing expressions containing the signs +, —, ×, ÷, we first perform the operations indicated by the signs × and ÷ in *the order in which they stand;* then the operations indicated by + and —.

Thus, $48\div 8\times 2-3\times 2+6\times 5\div 2=12-6+15=21.$

EXERCISE 18.

Reduce to a single expression :

1. $(16-11+2)\times 5.$
2. $(4\times 15)\div(2\times 3).$
3. $(84\div 7)+(4+5-6).$
4. $(44-31)\times(14-11).$
5. $(96\div 6+5)-(6\times 8\div 16).$
6. $(52-5\times 7)+(4\times 5)-16\div 2.$
7. $52-5\times 7+4\times 5-16\div 2.$
8. $(62+3-15)\div 10+(6\times 7-30)\div 3.$

98. Cancellation. The work of division may often be shortened by dividing out, or *cancelling,* equal factors from the divisor and dividend.

Divide 1596 by 84.

$$\begin{array}{r} 12\underline{)1596} \\ 7\underline{)133} \\ 19 \end{array}$$

SOLUTION. It is readily seen that 12 is contained in 84 and in 1596. Dividing each by 12, we have 7 and 133; and we find the quotient 19 by dividing 133 by 7 by short division.

99. Reciprocals. If the product of two numbers is 1, each of the numbers is called the *reciprocal* of the other.

Thus, $2 \times 0.5 = 1$; hence, 2 is the reciprocal of 0.5, and 0.5 is the reciprocal of 2. Again, $0.8 \times 1.25 = 1$; hence, 0.8 is the reciprocal of 1.25, and 1.25 is the reciprocal of 0.8. So 4 is the reciprocal of 0.25; 3 is the reciprocal of 0.33333; and so on.

100. When the dividend is 1, the quotient is the reciprocal of the divisor; and when the dividend is any other number than 1, the quotient is the reciprocal of the divisor multiplied by that number. Hence,

101. *To divide a number by a divisor gives the same result as to multiply the number by the reciprocal of the divisor.*

Also, to multiply a number by a multiplier gives the same result as to divide the number by the reciprocal of the multiplier.

102. The processes of multiplication and division are often made much simpler by using the reverse process with the reciprocal of the multiplier or of the divisor.

Thus, to divide 2.71827 by 37.5, since $37.5 = 3 \times 12.5$, we may divide 2.71827 by 3, and multiply the quotient by 0.08.

EXERCISE 19.

By the use of reciprocals, find the value of:

1. 8×0.25.	**10.** 1764×0.025.
2. $171 \div 0.25$.	**11.** $5381 \div 0.025$.
3. 876×1.25.	**12.** $7452 \div 0.875$.
4. 132×2.5.	**13.** 651×0.33333.
5. $591 \div 2.5$.	**14.** 456×6.66667.
6. $756 \div 0.125$.	**15.** 1554×0.16667.
7. 268×25.	**16.** $432 \div 1.33333$.
8. $753 \div 25$.	**17.** $375 \div 16.66667$.
9. $567 \div 625$.	**18.** $225 \div 6.66667$.

103. Contracted Division of Decimals. Annexing a digit to a partial *dividend* multiplies the partial dividend by 10 and adds to the product the number expressed by the digit.

Cutting off a digit from the right of the *divisor* subtracts from the divisor the number expressed by the digit and divides the remainder by 10. The quotient figure, therefore, will be the same whichever we do; and this principle can be applied to shorten the labor of division in examples involving long decimals.

Divide 15.4323487 by 1.4142136, to four decimal places.

```
                 10.9123
14142136)154323487.
         14142136
          129021270
          127279224
            17420460
            14142136
             32783240
             28284272
              44989680
              42426408
```

It is necessary to determine, first, the number of significant figures required in the quotient.

Since 1.4 is contained in 15 ten times, it is evident that the quotient will have two integral places. The two integral figures and the four decimal figures make six, which will be the number of figures required in the quotient.

104. In general, to determine the number of significant figures required in the quotient, move the decimal point of the divisor to the right of the first significant figure, and move the decimal point of the dividend as many places, and in the same direction, as the decimal point of the divisor has been moved. The number of integral places in the quotient can then easily be determined.

It is best not to cut off figures from the right of the divisor until the number of figures still required in the quotient is *two less* than the number of digits in the divisor. In multiplying the divisor by each quotient figure, multiply the figure of the divisor cut off, and carry the nearest ten.

The work may be arranged as follows:

$$\begin{array}{r} 10.9123\\ 14\cancel{1}\cancel{4}\cancel{2}\cancel{1}\cancel{3}\cancel{6})\overline{154323487.} \\ \underline{1414214} \\ 129021 \\ \underline{127279} \\ 1742 \\ \underline{1414} \\ 328 \\ \underline{283} \\ 45 \\ \underline{42} \end{array}$$

Cut off the 6. The first product is increased by 1 for the 1 × 6 omitted. The first remainder is increased by 1 for the 8 in the dividend. Cut off the 3. As the divisor is not contained in the partial dividend, we also cut off the 1. The product by 9 is increased by 1 for the 9 × 1 omitted. Cut off the 2. As 1 × 2 is less than 5, the product by 1 is not increased. Cut off the 4. The product by 2 is increased by 1 for 2 × 4 omitted. Cut off the 1. The product by 3 is not increased, for 3 × 1 is less than 5. Hence,

105. To Divide Decimals by the Contracted Method,

Determine the number of significant figures required in the quotient. Begin to cut off figures from the right of the divisor, when the number of figures still required in the quotient is two less than the number of digits in the divisor. In multiplying the divisor by each quotient figure, multiply the figure of the divisor cut off, carrying the nearest ten.

Exercise 20.

Divide by the contracted method:

1. 11.4285285 by 3.1415927 to six decimal places.
2. 0.004239239 by 3.2783278 to five decimal places.
3. 437 by 215.253 to five decimal places.
4. 0.0053 by 72.654 to eight decimal places.
5. 6 by 0.1573 to three decimal places.
6. 0.11 by 1937.43 to eight decimal places.
7. 44.2 by 0.768547 to five decimal places.
8. 0.6587465 by 0.5475869 to five decimal places.
9. 46 by 0.00751515151 to three decimal places.

106. Equations. A statement that two expressions of number have the same value is called an *equation.*

Every equation consists of two expressions of number connected by the sign of equality, $=$; the two expressions are called the *sides* or *members* of the equation.

Thus, $2 \times 4 = 8$, and $6 + 4 + 5 = 18 - 3$ are equations.

107. *Since the two members of an equation are equal, if both members are increased by, diminished by, multiplied by, or divided by equal numbers, the results are equal.*

108. Division of Powers of the Same Number.

Since $$\frac{5^5}{5^3} = \frac{5 \times 5 \times 5 \times 5 \times 5}{5 \times 5 \times 5} = 5 \times 5 = 5^2,$$

and $$\frac{5^3}{5^5} = \frac{5 \times 5 \times 5}{5 \times 5 \times 5 \times 5 \times 5} = \frac{1}{5^2}, \text{ therefore,}$$

If the exponent of the power in the dividend is greater than the exponent of the power in the divisor,

The quotient is that number with an exponent equal to the exponent of the dividend minus that of the divisor.

If the exponent of the power in the divisor is greater than the exponent of the power in the dividend,

The quotient is one divided by the number with an exponent equal to the exponent of the divisor minus that of the dividend.

Exercise 21.

Express the value of:

1. 10^1; 10^2; 10^3; 10^4; 10^5; 10^6; 10^7; 10^8.
2. $10^3 \div 10^2$; $10^8 \div 10^5$; $10^5 \div 10^8$; $10^9 \div 10^4$.
3. $9.99^4 \div 9.99^2$; $9.99^{108} \div 9.99^{110}$; $9.99^{16} \div 9.99^{18}$.
4. $1.01^{25} \div 1.01^{22}$; $1.01^{12} \div 1.01^{15}$; $1.01^{19} \div 1.01^{16}$.

109. Proof of Division by Casting out Nines. Divide 1,348,708 by 498.

If we divide 1,348,708 by 498, we have for quotient 2708 and for remainder 124.

That is,	$1,348,708 = 498 \times 2708 + 124.$
But, § 70,	$1,348,708 =$ a number of nines $+ 4$,
	$498 \times 2708 =$ a number of nines $+ 6$,
and	$124 =$ a number of nines $+ 7$.
Hence,	$498 \times 2708 + 124 =$ a number of nines $+ 6 + 7$
	$=$ a number of nines $+ 4$.

Therefore, the two members of the equation $1,348,708 = 498 \times 2708 + 124$, when divided by 9, give the same remainder 4.

We may arrange the work as follows:

```
            ( 8 . . . . . . . . 2708
   Multiply {
            ( 3 . . .  498)1348708 . . . . . . . . . . 4
     ( 6 . . . . 24         996
     |                      3527
     |                      3486
 Add {                       4108
     |                       3984
     ( 7 . . . . . . . . . .  124
      13 . . . . . . . . . . . . . . . . . . . . . . . 4
```

In examples involving large numbers *always* apply this test.

Exercise 22.

Find the following quotients, and test the accuracy of the work by casting out the nines:

1. $157,056.41692 \div 2150.42.$
2. $49,652.7896 \div 5645.376.$
3. $2000 \div 3.1416.$
4. $18.97657 \div 0.7854.$
5. $28,762.762 \div 14.39874.$
6. $8747.119 \div 6.58298.$
7. $28,752.1 \div 149.796.$

Exercise 23. — Review.

Express in words :

1. 327.244.
2. 80.9056.
3. 0.390012.
4. 20,000.002.
5. 0.0000008.
6. 41.27105.

Write in figures :

7. Two hundred thirty-five and eight hundred thirty-five thousandths.
8. Seventy-four and two hundred three thousand six millionths.
9. Twelve hundred and eight thousand three ten-millionths.
10. Five thousand sixty-four millionths.
11. One million and four tenths.
12. Six hundred-millionths.

Multiply and divide :

13. 789.365 by 10; by 100; by 100,000.
14. 0.004 by 100 ; by 10,000 ; by 1000.
15. 436 by 1,000,000 ; by 1000 ; by 10.
16. 0.1 by ten ; by ten millions.

Find the value of :

17. $21.3706 + 15.243 + 1.8954 + 0.026891 + 5.328 + 29.74$.
18. $57 + 0.0057 + 6.8 + 1200 + 0.847 + 159.2 + 3$.
19. $0.0012 + 10 + 5.8281 + 5 + 39.43 + 0.6827 + 1$.
20. $23.9875 - 12.4764$; $35.14732 - 27.62815$.
21. $102.1274 - 83.072$; $39.801 - 17.9645$.
22. $30 - 5.2817$; $1.7 - 0.8469$.
23. $1 - 0.54237$; $100 - 0.00176$.
24. $24.271 - 3.6485 + 15.271 - 13.256 - 14.125$.
25. $52 + 0.52 - 17.8946 - 30.254 - 0.5 + 21.12$.

26. 41.289×0.5 ; 0.268×0.9 ; 0.112×0.2.
27. 2.435×4.23 ; 71.651×3.37 ; 0.251×0.04.
28. 0.0012×0.005 ; 2.26823×200 ; 5.6125×0.0768.
29. $0.7 \times 7 \times 0.07$; $0.15625 \times 23.7 \times 0.00192 \times 5$.
30. $(2.465 + 1.21) \times (3.2 - 2.89)$.
31. $(3.01)^2$; $(0.045)^2$; $(0.0081)^2$; $(5.1004)^3$; $(0.76)^3$.
32. $(0.125)^2 \times (0.32)^3$.

Divide :

33. 291.84 by 6 ; 0.12936 by 12 ; 7.92801 by 0.9.
34. 58.383 by 0.39 ; 0.28744 by 0.08 ; 491.205 by 0.065.
35. 68.325 by 6.25 ; 0.732 by 1.6 ; 1208.88 by 0.438.
36. 498 by 0.0125 ; 7 by 0.007 ; 1000 by 0.0001.
37. 0.235 by 10.24 ; 27 by 12 ; 0.00507702 by 0.0283.
38. 89.3 by 0.00752 ; 74.1 by 0.0256 ; 1 by 0.128.
39. 0.39842 by 3.7164 ; 281.5 by 13.789 ; 0.0005 by 0.0028.
40. 63.04128 by 912.85 ; 287.209 by 0.00493 ; 2000 by 0.0059.

Exercise 24. — Review.

Find the value of :

1. $1.4 + 2.08 + 3.895$.
2. $2.8 + 2.08 + 0.28 + 0.028 + 0.812$.
3. $1.667 + 0.4 + 0.286 + 6.08 + 0.636 + 0.931$.
4. $6.125 - 0.57$.
5. $(4.625 + 1.146) - (1.2 + 3.571)$.
6. $6.913 - (2.85 - 0.937)$.
7. $24 - 2.4 + (5 - 3.508) - 3.092$.
8. $10 - (4.25 - 2.5 + 2 - 0.625 - 0.4 - 2.02) - 0.295$.
9. $1.5 \times 0.08 \times 0.5$.
10. $0.1204 \times 0.0168 \times 100$.
11. $0.04 \times 3.25 \times 0.06$.
12. $36 \times 0.002 \times 2.05 \times 0.00765$.
13. $0.139 \times 28 + 42 \times 0.002 + 6 \times 0.004 - 0.05 \times 20$.

14. $(10 - 1.25) \times 0.2 + 0.02 \times 2.8 + (80.3 \times 0.1 - 5.3) \times 10 - 805.3 \times 0.02$.

15. $28.3696 \div 1.49$.

16. $0.27 \div 0.00225$.

17. $8.8779 \div 175.8$.

18. $0.0427 \div 92.3$.

19. $0.28744 \div 800$.

20. $491.205 \div 650$.

21. $68.325 \div 6250$.

22. $0.732 \div 16{,}000$.

23. $1208.88 \div 0.438$.

24. $2 \div 0.01 - (0.2 \div 0.02 + 0.8 \div 10) + 36.48 \div 8 - (4 \div 0.05 - 2 + 0.6 \div 1.25)$.

25. $72.2 \div 10 - 2 \div (0.5 \div 1.60) + 2.125 \div (1.75 - 0.5)$.

Exercise 25. — Review.

1. What number subtracted 88 times from 80,005 will leave 13 as a remainder?

2. If 7 men can build a wall in 16 days, how many men will it take to build a wall three times as long in half the time?

3. How many minutes are there between 25 minutes past 8 in the morning and midnight?

4. If the velocity of sound is 1090 feet per second, at what distance is a gun fired the report of which I hear 11 seconds after seeing the flash? (5280 feet make a mile.)

5. How long will it take to travel 30.2375 miles at the rate of 8.85 miles per hour?

6. If the circumference of a circle is 3.1416 times the diameter, find the circumference of a circle whose diameter is 6.8 feet; also, find the diameter of a circle whose circumference is 20 inches.

7. How much wire will be required to make a hoop 30 inches in diameter, allowing 2 inches for the joining?

8. How many times would the hoop of Ex. 7 turn in going half a mile?

9. Cork, whose weight is 0.24 of the weight of water, weighs 15 pounds per cubic foot. What is the weight of 6 cubic feet of oak, if the weight of oak is 0.934 of the weight of water?

10. From what number can 847 be subtracted 307 times and leave a remainder of 49?

11. What is the 235th part of 141,235?

12. What will 343 barrels of flour cost at $6.37 a barrel?

13. Twelve makes a dozen, and 12 dozen makes a gross. How many steel pens in 28 gross? What will a gross of eggs cost at 27 cents a dozen?

14. How much must be added to $4429 to make the sum equal to 43 × $241?

15. What number deducted from the 26th part of 2262 will leave the 87th part of the same number?

16. At the ordinary rate, 123 words a minute, how long will it take a man to deliver a speech of 15 pages, if each page contains 28 lines, and each line 11 words? How long would it have taken Daniel Webster to deliver the same speech, whose rate was 93 words a minute?

17. How long will it take a railway train to go from New York to San Francisco, 3310 miles, at the rate of 1973 feet a minute?

18. How many hours will it take to count a million, at the rate of 67 a minute?

19. If you put into a box 17 cents a day, including Sundays, beginning January 1 and ending July 4, how much money will there be in the box?

20. If a man's income is $3000 a year, and his daily expenses average $7.68, what does he save in a year?

21. In a question of division the quotient was 87.83, the divisor, 759. What was the dividend?

22. What is the nearest number to 7196 that will contain 372 without a remainder?

23. It is 3.1416 times as far round a wheel as across it. How many times will a wheel 4.5 feet across turn in going 23 miles of 5280 feet each?

24. How many gallons of 231 cubic inches are contained in a cubic foot of 1728 cubic inches? in a bushel of 2150.42 cubic inches? How many cubic feet in a bushel? How many bushels in 31.5 gallons?

25. Seven children had left to them $7186 apiece; one died, and his share was divided among the surviving six. How much had each then?

26. How long will it take 2 men to do what 1 man can do in 6 days? what 4 men can do in 3 days? what 3 men can do in 4 days?

27. Divide $1.80 among Thomas, Richard, and Henry in such a way that Henry shall receive 3 cents for every 5 cents that Thomas gets, and Richard shall receive 2 cents for every 3 cents that Henry gets.

28. Divide $87.84 between B and C so that C shall get $19 as often as B gets $17.

29. Three partners received for goods: one, $371.63; the second, $285.40; the third, $411.91. They paid for the goods $879.34, and divided the profit equally among them. How much did each receive?

30. If there are 12 inches in a foot, how many inches long is a wall 35 feet in length? If a brick and its share of mortar is 8.4 inches long, how many bricks in length is the wall?

31. If a brick and its mortar is 2.4 inches high, how many bricks are required to build a wall 12 feet high, 35 feet long, if the width of the wall is the width of two bricks?

32. What is the total weight of the wall of Ex. 31, if a brick with its share of the mortar weighs 4.13 pounds? What is the weight after a long rain, when the weight is increased to 4.27 pounds for each brick?

33. How many pounds does each foot in length of the wall of Ex. 31 weigh?

34. If 60.98 cubic inches of brick weigh 4 pounds, how many cubic inches of brick weigh 1 pound? How many pounds will a cubic foot (1728 cubic inches) weigh?

35. If a cubic foot of water weighs 62.5 pounds, how many times as heavy as water is brick?

36. Light moves through the air at the rate of 186,500 miles a second. How many times can it go around the earth in a second, if the distance round the earth is 24,897.714 miles?

37. Light moves through the air at the rate of 300,190 kilometers a second. How many times can it go around the earth in a second, if the distance round the earth is 40,007.5 kilometers?

38. A minute is 60 seconds. How many miles and how many kilometers can light travel through air in a minute?

39. An hour is 60 minutes. How many miles and how many kilometers can light travel in an hour?

40. The distance round the earth, given in Ex. 37, is measured on a north and south line. Around the equator the distance is 40,075.45 kilometers. How many times could light move round the equator in one minute?

41. Find the reciprocal of the difference between 31.24 and 31.23768.

42. The Hanoverian mile is 25,400 Hanoverian feet long, and each foot is 0.9542 of an English foot. Find to four places of decimals the fraction that an English mile of 5280 English feet is of a Hanoverian mile.

43. Express in inches the length of a meter, given that a meter is one ten-millionth of a quarter of the earth's circumference, that the circumference is 3.14159 times the diameter, that the diameter of the earth is 7911.7 miles, and that a mile is 5280×12 inches.

44. How must a number be altered that its reciprocal may be doubled ?

45. What effect is produced on the sum of two numbers, if the same number is added to each of them? What effect on the difference ?

46. What effect is produced on the product of two numbers, if both numbers are multiplied by the same number ? What effect on the quotient ?

47. What effect is produced on the *remainder*, if both divisor and dividend are multiplied by the same number ? If both are divided by the same number ?

48. In going from one planet to another, light probably moves faster than in air. Suppose it moves at the rate of 309,800 kilometers a second, how long would it take light to perform each of the following journeys :

Moon to Earth	375,500	kilometers.
Sun to Earth	147,250,000	"
Sun to Mercury	56,900,000	"
Sun to Venus	106,400,000	"
Sun to Mars	224,100,000	"
Sun to the Asteroids	400,000,000	"
Sun to Jupiter	765,400,000	"
Sun to Saturn	1,403,000,000	"
Sun to Uranus	2,817,000,000	"
Sun to Neptune	4,421,000,000	"
Sun to the nearest star	24,000,000,000,000	"

49. A kilometer is about 0.6214 of a mile. How many miles is each of the planets from the sun ?

50. If 11.75 tons of coal cost $82.25, what will 21.4 tons cost ?

51. Find the number of hours it will take a locomotive running at the rate of 27 miles an hour to make the distance passed over in 13.25 hours by another locomotive that has a velocity of 43.5 miles an hour.

Review Questions.

What are units? numbers? integral numbers? decimal numbers? abstract numbers? concrete numbers? like numbers?

What is notation? numeration? Where is the decimal point placed? What do figures in the first place to the left of the decimal point represent? in the second place? in the third place? in the fourth place? in the first place to the right of the decimal point? in the second place? in the third place? in the fourth place? Which place do the units of a number occupy? the tens? the hundreds? the thousands? the tenths? the hundredths? the thousandths? At what rate does the value of a figure increase from right to left? In separating a row of figures into periods where do we begin? How many figures in each period? What is the period on the right called? the second period? the third period? Which period do we write first? Which period do we read first? How do we write a decimal? How do we read a decimal? In reading, by what word do we connect the integral and decimal parts of a number?

What is addition? What is the result called? What kind of numbers only can be added? What is the sign of addition? Does it make any difference in what order the numbers are added? In addition how do we arrange the decimal points? How do we prove that the work of addition is correct?

What is subtraction? What is the greater number called? the smaller? the result? What kind of numbers must the minuend, subtrahend, and remainder be? What is the sign of subtraction? In subtraction, how do we arrange the decimal points? How do we prove that the work of subtraction is correct?

What is multiplication? the multiplicand? the multiplier? the product? What kind of a number must the multiplier be? What are the factors of a product? Does it make any difference in what order the factors are multiplied? What is the sign of multiplication? How many decimal places must the product have? How do we prove that the work of multiplication is correct?

What is division? the dividend? the divisor? the quotient? If the divisor is not an integer how can we make it an integer without altering the quotient? If the divisor is an integer where do we place the decimal point in the quotient? What is the advantage of writing the quotient over the dividend in long division? What is the sign of division? How do we prove that the work of division is correct?

CHAPTER V.

METRIC MEASURES.

110. To **measure** a quantity is to find the number of times it contains a *known quantity* of the same kind, called the **unit of measure.**

111. The **metric system** is a system of weights and measures expressed in the *decimal scale.*

112. The **standard meter,** as defined by law, is the length of a bar of very hard metal, carefully preserved at Paris, accurate copies of which are furnished the governments of all civilized countries.

The meter was intended to be one ten-millionth of the distance from the equator to the north pole, but more careful measurements show that this distance is 10,001,887 meters.

113. The principal units of the metric system are :

The **meter** (^{m}) for lengths ;
The **square meter** (^{qm}) for surfaces ;
The **cubic meter** (^{cbm}) for large volumes ;
The **liter** (^{l}) (*lee'ter*) for smaller volumes ;
The **gram** (^{g}) for weights.

114. All these units are divided and multiplied decimally, and the size of the measures thus produced is shown by a prefix ; namely, *deka,* meaning 10 ; *hekto,* meaning 100 ; *kilo,* meaning 1000 ; *myria,* meaning 10,000 ; and *deci,* meaning 0.1 ; *centi,* meaning 0.01 ; *milli,* meaning 0.001.

As in United States money we seldom speak of anything except dollars and cents, so in metric measures only measures printed in **black letter** are in common use.

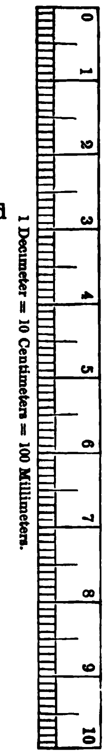
Measures of Length.

115. The principal unit of length is the **meter.**

TABLE.

10 **millimeters** (mm)	= 1 **centimeter** (cm).
10 centimeters	= 1 decimeter (dm).
10 decimeters	= 1 **meter** (m).
10 **meters**	= 1 dekameter (dkm).
10 dekameters	= 1 hektometer (hm).
10 hektometers	= 1 **kilometer** (km).
10 **kilometers**	= 1 myriameter.

NOTE. The names of all compound units are accented on the first syllable; thus, *mil'limeter*, *kil'ometer.*



116. A length given in one unit may be expressed in another unit by *simply moving the decimal point.*

Thus, 17,856,342^{mm} may be written as kilo-meters by observing that milli-meters are changed to meters by moving the point *three* places to the left; and these meters into kilo-meters by carrying it *three* places further, making in all *six* places. Therefore,

$$17{,}856{,}342^{mm} = 17.856342^{km}.$$

Again, 4.876326^{km} may be written as centi-meters by observing that kilo-meters are changed to meters by moving the point *three* places to the right, and meters to centi-meters by moving it *two* places further, making in all *five* places. Therefore,

$$4.876326^{km} = 487{,}632.6^{cm}.$$

117. The rule, therefore, for this conversion is:

First count the number of places needed to convert the given measure into terms of the principal measure; then the number needed to convert the principal into the required measure.

118. Before adding or subtracting, the quantities must be written in terms of the **same unit of measure.**

EXERCISE 26.

1. Change 5427^{m} to kilometers; to millimeters; to centimeters.

2. How many meters in 6853^{mm}? how many centimeters? what part of a kilometer?

3. Write 49.7^{m} as centimeters; as millimeters; as the decimal of a kilometer.

4. How many centimeters in 12.4^{km}? how many millimeters?

5. Change 1230^{m} to kilometers; to centimeters.

6. Write 1230^{cm} as meters; as millimeters.

Find in meters the value of the following:

7. $0.435^{m} + 852^{cm} + 4263^{mm} + 0.1595^{km}$.

8. $0.927^{km} - 6495^{cm}$; $4.37^{cm} - 42.87^{mm}$.

9. 8×0.0457^{km}; 3.04×60.93^{cm}; 5.43×67.2^{mm}.

10. $38{,}019^{mm} \div 0.097$; $0.41^{km} \div 25.625$.

11. At \$1.87 a meter, what is the cost of 6.20^{m} of cloth?

12. At \$0.75 a meter, what is the cost of 60^{m} of cloth?

13. From a piece of cloth containing 47.60^{m} a tailor cuts off three pieces: the first of 3.80^{m}, the second of 1.30^{m}, and the third of 45^{cm}. How many meters of the cloth are left?

14. What is the value of 60^{cm} of cloth at \$5.20 a meter?

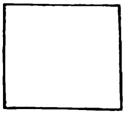
15. If \$6.00 is paid for a railroad ticket to travel 440^{km}, what is the fare per kilometer?

16. If a train goes 288^{km} in 9 hours, how many meters does it go in a minute? (1 hour = 60 minutes.)

17. If a man walks at the rate of 6^{km} an hour, what part of an hour will it take him to walk 420^{m}?

18. A railroad carried 412 passengers 18^{km} for \$88.992; at the same rate, what will it receive for carrying 350 passengers 35^{km}?

Measures of Surface.

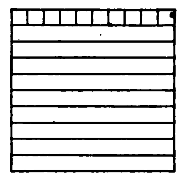


Square.

119. A **square** is a flat surface with four square corners and four equal straight sides.

120. A **square meter** is a square one meter on a side.

The principal unit of surface is the **square meter.**



121. Suppose the square in the margin to represent a **square meter.** It is divided into ten equal horizontal bands, and each band is one tenth of the square meter. Each band can be divided, as the upper one is, into ten little squares measuring one tenth of a meter on a side. Each of these squares will be 0.1 of the band, or 0.01 of the whole square. The **square meter,** therefore, contains 10 × 10, or **100 square decimeters.**

In like manner, a **square decimeter** contains 100 **square centimeters,** and therefore a **square meter** contains 100 × 100, or 10,000 **square centimeters.**

In like manner, a **square meter** contains 1000 × 1000, or 1,000,000 **square millimeters.**

TABLE.

100 square millimeters (^{qmm})	=	1 **square centimeter** (^{qcm}).
100 **square centimeters**	=	1 square decimeter (^{qdm}).
100 square decimeters	=	1 **square meter** (^{qm}).
100 **square meters**	=	1 square dekameter (^{qdkm}).
100 square dekameters	=	1 **square hektometer** (^{qhm}).
100 **square hektometers**	=	1 **square kilometer** (^{qkm}).

122. In **measures of length** each unit is 10 times as large as the next smaller unit, but in **measures of surface** each unit is 100 times as large as the next smaller unit.

123. In the measurement of land, the **square meter** is called a **centar,** the **square dekameter** is called an **ar,** and the **square hektometer** is called a **hektar.**

TABLE OF LAND MEASURES.

100 centars (^{ca})	=	**1 ar** (^{a}).
100 ars	=	**1 hektar** (^{ha}).

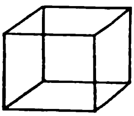
124. In surface measures, when the ar is the unit, 100 ars make a hektar; but when the square meter is the unit, 100 × 100, or 10,000 square meters make a square hektometer or hektar.

EXERCISE 27.

1. Change $1,854,276^{qm}$ to hektars; to square kilometers.
2. How many hektars in 2.7856^{qkm}?
3. Write 1.7431^{qm} as square centimeters; as square millimeters.
4. How many square kilometers in $17,467.5^{ha}$?
5. How many square meters in 1.3614^{qkm}?
6. How many square meters in 2.25^{ha}?
7. How many square centimeters in 0.0137^{qm}?
8. Write 3.571^{qcm} as square millimeters.
9. If a field contains 7500^{ca}, how many ars does it contain? What part of a hektar?
10. How many square meters must be added to $22,612^{qm}$ to make $4^{ha}\ 62^{a}\ 17^{ca}$?
11. A field containing 72.4^{a} is sold at 15 cents a square meter. What is received for the field?
12. If $62^{a}\ 12^{ca}$ of land is sold for $1366.64, what is the price per square meter?
13. How many square centimeters must be taken from $12,473^{qcm}$ to leave $1^{qm}\ 14^{qdm}\ 53^{qcm}$?

Measures of Volume.

125. A **cube** is a solid bounded by six equal squares. Each bounding square is called a *face*, and the intersection of two faces is called an *edge*.

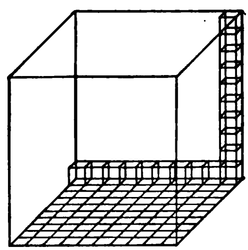


Cube.

126. A **cubic meter** is a cube one meter on an edge.

The principal unit of volume is the **cubic meter.**

127. The cubic meter can be divided into 10 layers, each a meter square and a decimeter thick. Each layer will, therefore, be 0.1 of a cubic meter. Again, each layer can be divided into 10 equal parts. Each part will, therefore, be 0.1 of the layer, or 0.01 of the meter, and will be a decimeter square and a meter long. Also, each one of these parts can be divided into 10 equal parts, each of which will be a cubic decimeter, and will be 0.1 of 0.01, or 0.001 of the cubic meter.



The **cubic meter,** therefore, contains 1000 **cubic decimeters.**

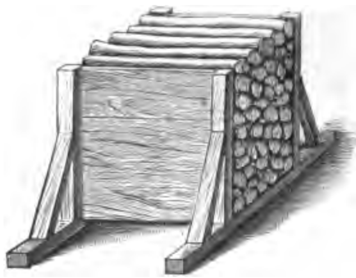
In like manner, a **cubic decimeter** contains 1000 **cubic centimeters,** and a **cubic centimeter** contains 1000 **cubic millimeters.**

TABLE.

1000 cubic millimeters (^{cmm})	= 1 **cubic centimeter** (^{ccm}).
1000 **cubic centimeters**	= 1 cubic decimeter (^{cdm}).
1000 cubic decimeters	= 1 **cubic meter** (^{cbm}).

128. In **measures of volume** each unit of measure is 1000 times as large as the next smaller unit of measure.

129. In the measurement of wood, the **cubic meter** is called a **ster.**



Ster of Wood.

TABLE OF WOOD MEASURES.

10 decisters (^{dst})	**= 1 ster (st).**
10 sters	**= 1 dekaster (^{dkst}).**

EXERCISE 28.

1. Write 2.25^{cbm} as cubic centimeters.
2. Change $2,162,875^{ccm}$ to cubic meters.
3. Change 0.0175^{cbm} to cubic millimeters.
4. Change $46,164^{ccm}$ to cubic decimeters.
5. What is the equivalent of 0.875^{dkst} in cubic meters? in cubic centimeters?
6. How many sters are there in 14.75^{dkst} of wood? how many decisters?
7. What is the cost of 28.25^{dkst} of wood at $1.25 a ster?
8. Find the cost of an oak beam containing 1250^{cdm} at $25 a cubic meter.
9. How many cubic centimeters must be added to $1,262,376^{ccm}$ to make $2^{cbm}\ 2^{cdm}\ 2^{ccm}$?
10. How many cubic millimeters must be taken from $22,350,000,000^{cmm}$ to leave $20^{cbm}\ 22^{cdm}\ 222^{ccm}$?

Measures of Capacity.

130. The principal unit of capacity is the **liter.**

131. In measuring liquids, grain, etc., the **cubic decimeter** is called a **liter.**

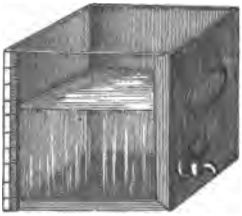
TABLE.

10 milliliters (^{ml})	= 1 **centiliter** (^{cl}).
10 **centiliters**	= 1 deciliter (^{dl}).
10 deciliters	= 1 **liter** (^{l}).
10 **liters**	= 1 dekaliter (^{dkl}).
10 dekaliters	= 1 **hektoliter** (^{hl}).
10 **hektoliters**	= 1 kiloliter (^{kl}).

132. In **measures of capacity** each unit of measure is **10** times as large as the next smaller unit of measure.

EXERCISE 29.

1. How many liters in 1.7^{cbm}? in $157,854^{ccm}$?
2. How many cubic centimeters in 9.5^{l}? in 0.015^{l}?
3. Change 1.25^{hl} to cubic centimeters; to the fraction of a cubic meter.
4. Change 431.88^{l} to hektoliters; to the fraction of a cubic meter.
5. Write 0.375^{cbm} as liters; as cubic centimeters.
6. Write $734,159.651^{ccm}$ as liters; as hektoliters; as cubic meters.



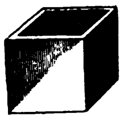
Liter = Cubic Decimeter.

7. How many cubic meters in $8,573,412.867^{ccm}$?
8. Change 0.734578912^{cbm} to cubic centimeters; to liters.
9. Change 1731.5^{l} to cubic meters; to cubic centimeters.

Measures of Weight.

133. The units of weight are the weights of units of pure water taken at its greatest density, that is, a little above the freezing point.

The principal unit is the **gram,** which is the weight of a **cubic centimeter** of water.



Cubic Centimeter.

Gram Weight.

TABLE.

10 **milligrams** (^{mg})	= 1 centigram (^{cg}).
10 centigrams	= 1 decigram (^{dg}).
10 decigrams	= 1 **gram** (^{g}).
10 **grams**	= 1 dekagram (^{dkg}).
10 dekagrams	= 1 hektogram (^{hg}).
10 hektograms	= 1 **kilogram** (^{kg}).
1000 **kilograms**	= 1 **metric ton** (^{t}).

NOTE. The kilogram is often called a *kilo*.

134. In **measures of weight** each unit of weight is 10 times as great as the next smaller unit of weight.

135. A **cubic centimeter** of water weighs a **gram.**
A **liter** of water weighs a **kilogram.**
A **cubic meter** of water weighs a **ton.**

EXERCISE 30.

1. How many kilos in 1.73^{t}? in 0.341 of a ton?
2. How many kilos will a hektoliter of water weigh?
3. Change $13,756^{mg}$ to grams; to the fraction of a kilo.
4. What is the weight in grams of 346.1^{ccm} of water?
5. Find the weight in kilograms of 0.37615^{cbm} of water.
6. Change 0.6778^{kg} to milligrams.
7. How many milligrams in the third part of 17.4^{g}?

Exercise 31.

1. Add 17.3^{m}, 87.41^{m}, 271^{cm}, 380^{mm}, and 1.79^{m}.

2. Add 15.87^{m}, 394.6^{dm}, 47.52^{m}, 7538^{cm}, and 75.89^{m}.

3. Add 187^{cm}, 49.3^{m}, 317^{mm}, and 6.138^{m}.

4. In a room the doorsill is 3^{cm} high; the door, 2.34^{m}; the finish over the door, 13.7^{cm}; and the distance from the finish to the ceiling is 93^{cm}. What is the height of the room?

5. The distance to the post office is 3.31^{km}; thence to the mill, 1.711^{km}; thence to the store, 3.718^{km}; thence home, 2.543^{km}. How long is the circuit?

6. The distance from Portland, Me., to Boston is 174^{km}; Boston to Albany, 317^{km}; Albany to Buffalo, 478^{km}; Buffalo to Chicago, 863^{km}; Chicago to Omaha, 789^{km}; Omaha to Cheyenne, 830^{km}. How far is it from Cheyenne to Portland? from Cheyenne to Albany? from Boston to Chicago? from Boston to Cheyenne?

7. If I travel 789.7^{km} a day, how far shall I go in 7 days? in 8.5? in 19.6? in 27.8? in 365?

8. How much will 3^{m} of cloth cost at $1.37 a meter? How much will 5.38^{m} cost at $2.63 a meter?

9. How much will 13.4^{kg} of opium be worth at $8.48 a kilo? 28.79^{kg} at $7.96 a kilo?

10. If one barrel of flour weighs 88.9^{kg}, how many barrels can be filled from 444.5^{t} of flour?

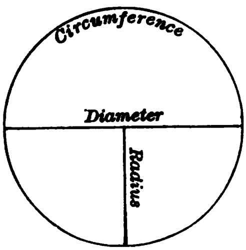
11. How many steps 80^{cm} long will a man take in walking a kilometer?

12. At 16 cents a liter, what is the cost of 52.4^{hl} of olive oil?

13. What is the cost of 6^{dkst} 4^{st} of oak wood at $1.75 per ster?

14. If a pasture contains $22{,}408^{ca}$, how many ars does it contain? how many hektars?

136. A flat surface bounded by straight lines or by a curved line is called a **plane figure.**



Circle.

137. A **circle** is a plane figure bounded by a curved line called the *circumference,* all points of which are equally distant from a point within called the *centre.*

A straight line drawn through the centre, having its ends in the circumference, is called a *diameter;* and half a diameter is called a *radius.*

138. *If the diameter of a circle is multiplied by 3.1416, the product is the length of the circumference.*

15. Find the circumference of a circle 1^{m} in diameter.

16. Find to the nearest tenth of a millimeter the circumferences of circles whose diameters are respectively 83^{m}; 3.71^{m}; 32.8^{m}; 10.4^{cm}; 11.8^{cm}; 167.1^{mm}; 39.3^{mm}.

17. What is the length of the earth's orbit, to the nearest meter, if the diameter of the orbit is $294{,}481{,}217^{km}$?

18. What is the circumference of a carriage wheel, 1.31^{m} in diameter? How far will it go in turning once? 17 times?

19. How many times must the wheel of Ex. 18 turn in going 69.429^{m}? 73.513^{m}? 17.27^{km}?

20. Find the reciprocal of 3.1416 to the fifth place.

139. *From Ex. 20, if a circumference is multiplied by 0.31831, the product is the diameter.*

21. How thick through is a tree whose girth is 2.97^{m}?

22. What is the diameter of a wheel that turns 19.5 times in going 107.25^{m}?

23. What is the diameter of a rope of which the circumference is 20^{cm}?

Areas.

140. A **surface** has two **dimensions,** *length* and *breadth.*

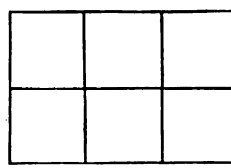
141. The **unit of surface is a square,** each side of which is a unit of length.

142. The **area of a surface** is the **number of square units** it contains.

143. The **perimeter** of a plane figure is the distance round it.

144. A **rectangle** is a plane figure with four straight sides and four square corners.

145. Suppose the rectangle in the margin is 3^{cm} long and 2^{cm} wide. If lines are drawn as represented, the surface is divided into **square centimeters.** There are 2 horizontal rows of 3^{qcm} each; that is, in all, 2×3^{qcm}. Hence,



To Find the Area of a Rectangle,

Express the length and breadth of the rectangle in the same linear unit; take the product of these two numbers for its area in square units of the same name as the linear unit.

The number of square units in a rectangle divided by the number of linear units in one dimension gives the number of linear units in the other dimension.

Exercise 32.

1. Find the area of a rectangle 17^{cm} by 19^{cm}.

2. In a rectangular township 16^{km} by 7^{km}, how many hektars? If there are in it 47.3^{km} of highway, averaging 11.7^{m} wide, how much land is left for other uses?

3. In a rectangular field 751.3^{m} long and 189.3^{m} wide is a rectangular garden 31.4^{m} by 17.8^{m}. How many hektars in the field? How many, exclusive of the garden?

4. If my garden contains 941.65^{qm} and my neighbor's 747.37^{qm}, what is the area in hektars of both taken together?

5. If a painter can cover 8.786^{qm} in an hour, how many square meters can he cover in 1.78 hours? in 3.86 hours? in 4.57 hours?

6. How many hektars in each of three rectangular fields: one measuring 315.71^{m} by 78.91^{m}; a second, 293.6^{m} by 84.84^{m}; the third, 346.8^{m} by 71.82^{m}? How many in the three together?

7. Find the price of a rectangular field, 346.8^{m} by 71.82^{m}, at $67.50 a hektar; at $384 a hektar; and at $2.375 a square meter.

8. Find the length of a rectangle 17^{cm} wide that contains 306^{qcm}. What length of carpet 75^{cm} wide is required to make 27^{qm}?

9. A room is 16^{m} long, 8^{m} wide, and 8^{m} high; another room is 7^{m} long, 7^{m} wide, and 3^{m} high. How many square meters of painting on the walls of both rooms, if no allowance is made for doors and windows? How many more square meters of painting on the walls of the larger room than on those of the smaller?

146. To Find the Area of a Circle,

Multiply the square of the radius by 3.1416; or, *multiply the square of the diameter by 0.7854 ($\frac{1}{4}$ of 3.1416).*

10. What is the area of a circle 27^{cm} in diameter? of a circle 1^{m} in diameter?

11. What is the area in hektars of a circular field 784^{m} in diameter?

12. Find the area of a circle 31^{cm} in diameter.

13. Find the area of a circle whose radius is 24^{m}.

14. If a circle has a radius of 7^{cm}, how many square centimeters does it contain?

15. In a rectangular sheet of zinc 1.76^{m} long and 89^{cm} wide are two circular openings, one of which has a radius of 10.5^{cm}, the other a radius of 9.2^{cm}. What is the area of the zinc left?

16. A piece of land in the form of a circle has a radius of 40^{m}; in the middle of it is a pond forming a circle of 15^{m} radius. What is the total surface? the surface of the pond? the surface of the land to cultivate?

17. How deep is a well, if the wheel whose diameter is 75^{cm} makes 26 revolutions in raising the bucket?

147. **A sphere** is a solid bounded by a curved surface, all points of which are equally distant from a point within called the *centre.*

148. A straight line drawn through the centre of a sphere, having its ends in the surface, is called a *diameter;* half a diameter is called a *radius.*

149. The area of the surface of a sphere is four times the area of a circle of the same diameter. Hence,

To Find the Area of the Surface of a Sphere,

Multiply the square of the diameter by 3.1416.

18. How many square centimeters of surface on a ball 7^{cm} in diameter?

19. How many square centimeters of surface on a ball 18^{cm} in diameter?

20. How many square meters of surface on a hemispherical dome 11.27^{m} in diameter?

NOTE. A hemisphere is half a sphere.

21. What is the interior surface of a hemispherical basin 12^{cm} in diameter?

22. What is the interior surface of a hemispherical vase 70^{cm} in diameter?

Carpeting Rooms.

150. Carpeting is made of various widths and is sold by the length.

In determining the number of meters required for a room, we first decide whether the strips shall run lengthwise or across the room, and then find the number of strips needed. The number of meters in a strip, including the waste in matching the pattern, multiplied by the number of strips will give the required number of meters.

23. How many meters of carpet 60^{cm} wide will be required for a room 6^m long and 5.4^m wide, the strips running lengthwise? how many meters would be required if the carpet were 80^{cm} wide?

Since the room is 540^{cm} wide, it will take $\frac{540}{60}$, or 9 widths of carpet 60^{cm} wide; that is, 9×6^m, or 54^m, will be required. If the carpet were 80^{cm} wide, it would take $\frac{540}{80}$, or 7 widths. Six widths would leave a surface 60^{cm} wide to be covered. This surface would require another strip, of which a width of 20^{cm} would be *turned under.*

24. How many meters of carpet 56^{cm} wide will be required for a room 8.32^m long and 6.6^m wide, strips running lengthwise?

25. How many meters of carpet 70^{cm} wide will be required for a room 7^m long and 5.4^m wide, strips running across the room?

26. How many meters of carpet 80^{cm} wide will be required for a room 6^m long and 5.47^m wide, strips running across the room?

27. How many meters of carpet 90^{cm} wide will be required for a room 5^m long and 4.5^m wide, strips running lengthwise? How much will it cost, at \$1.875 a meter?

28. How many meters of carpet 75^{cm} wide will be required for a room 5.25^m long and 4.75^m wide, strips running across the room? Find the cost, at \$2.125 a meter.

29. How many meters of carpet 75^{cm} wide will be required for a room 5.6^{m} square? How wide a strip will have to be turned under? How much will the carpet cost, at $1.25 a meter?

Papering and Plastering.

151. The area of the *four walls* of a room is equal to that of a rectangle whose length is the perimeter of the room, and whose breadth is the height of the room.

Perimeter = *twice the length* + *twice the breadth.*
Area = *height* × *perimeter.*

30. Find the area of the walls of a room whose length is 6.12^{m}, breadth 5.05^{m}, and height 3.5^{m}.

$$\text{Perimeter} = 2 \times (6.12^{m} + 5.05^{m}) = 22.34^{m}.$$
$$\text{Area} = (3.5 \times 22.34)^{qm} = 78.19^{qm}.$$

31. How many rolls of paper 45^{cm} wide and 8^{m} long, allowing 11.19^{qm} for doors and windows, will be required to paper the room of Ex. 30?

32. Find the cost of papering a room 8^{m} long, 5.5^{m} wide, and 4.5^{m} high, with paper 50^{cm} wide and 7.5^{m} in a roll, at $1.25 a roll, put on; if there is a baseboard 25^{cm} wide running round the room, and an allowance of 11^{qm} is made for doors and windows.

33. Find the cost of plastering the room of Ex. 32, at $0.50 a square meter.

34. Find the cost of papering a room 5.5^{m} long, 4.8^{m} wide, and 3.2^{m} high, with paper 45^{cm} wide, 7.5^{m} in a roll, at $0.875 a roll, put on, allowing 12^{qm} for baseboard, doors, etc.

35. Find the cost of plastering the room of Ex. 34, at $0.45 a square meter.

36. Find the cost of papering a room 6^{m} square and 3.5^{m} high, with paper 45^{cm} wide and 7.5^{m} in a roll, at

\$0.75 a roll, put on; and of putting on a border, at 5 cents per running meter.

37. Find the cost of plastering the room of Ex. 36, at \$0.36 a square meter.

38. Find the cost of papering a room 13^m long, 12^m wide, and 7^m high, with paper 45^{cm} wide and 7.5^m in a roll, at \$1.50 a roll, put on; and of putting on a border, at \$0.30 a running meter, allowing 115^{qm} for baseboard, doors, etc.

39. Find the cost of plastering the room of Ex. 38, at \$0.60 a square meter.

Board Measure.

152. Boards 25^{mm} or less in thickness are sold by the square meter.

Boards more than 25^{mm} in thickness and squared lumber are sold by the number of square meters of boards 25^{mm} in thickness to which they are equivalent.

Thus, a board 4^m long, 25^{cm} wide, and 25^{mm} thick contains 1^m *board measure;* if less than 25^{mm} thick it still contains 1^m; but if 75^{mm} thick, it contains 3^m board measure, for it is equivalent to *three* boards 4^m long, 25^{cm} wide, and 25^{mm} thick. Hence,

153. To Find the Board Measure of Boards more than 25^{mm} thick and of Squared Lumber,

Express the length and width in meters, and the thickness in millimeters; divide the product of these three numbers by 25 for the number of meters board measure.

If a board tapers regularly, its average width is found by taking one-half the sum of its end widths.

How many meters, board measure:

40. In a board 8^m long, 20^{cm} wide, and 20^{mm} thick?

41. In a joist 5^m long, 25^{cm} wide, and 75^{mm} thick?

42. In a stick of timber 15^m long and 40^{cm} square?

43. In 2 joists 5^m long, 27.5^{cm} wide, and 50^{mm} thick?

44. How many meters, board measure, in 10 planks, each 4^{m} long, 45^{cm} wide, and 10^{cm} thick? What is the value of these planks, at $25 a hundred meters?

45. How many meters, board measure, in 25 box boards, each 4^{m} long, 42^{cm} wide, and 20^{mm} thick? What is their value, at $14 a hundred meters?

Find the cost of:

46. Ten joists 4.5^{m} long, 10^{cm} wide, and 7.5^{cm} thick, at $11 a hundred meters.

47. Thirty-six planks, each 4^{m} long, 27.8^{cm} wide, and 75^{mm} thick, at $16 a hundred meters.

48. Three sticks of timber, each 8^{m} long, 22.5^{cm} wide, and 20^{cm} thick, at $17.50 a hundred meters.

49. A board 8.25^{m} long, 28^{cm} wide at one end and 35^{cm} at the other, and 31.25^{mm} thick, at $0.30 a meter?

50. A stick of timber 10^{m} long, 25^{cm} thick, 30^{cm} wide at one end and 25^{cm} wide at the other, at $14 a hundred meters.

51. The floor boards, 32^{mm} thick, for a two-story building 16^{m} by 10.5^{m}, at $30 a hundred meters.

52. The floor timbers, 25^{cm} by 50^{mm}, for the building of Ex. 51, if the timbers run lengthwise and are placed on edge 30^{cm} apart, and are worth $11.50 a hundred meters.

53. The fencing to enclose a field 150^{m} long and 75^{m} wide; the posts are set 2.5^{m} apart, and cost $0.25 apiece; the fence is 5 boards high; the bottom board is 30^{cm}, the top board 25^{cm}, and the other three each 22.5^{cm} wide, and the boards cost $13.25 a hundred meters.

154. Round logs are sold by board measure after 0.25 is deducted for slabs.

Large and heavy timber is sold by the ton. 0.20 is deducted for slabs, and the amount is reckoned in cubic measure instead of board measure.

Volumes.

155. A **solid** has three **dimensions,** *length, breadth,* and *thickness* (*height* or *depth*).

156. The **unit of volume** is a **cube,** each dimension of which is a unit of length.

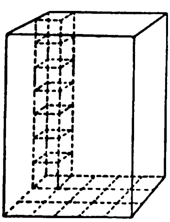
157. The **volume of a solid** is the **number of cubic units** it contains.

158. The boundary of a solid is called its **surface.** If the surface consists of planes, they are called the **faces** of the solid.

The faces of a solid meet in lines called the **edges** of the solid.

159. A **rectangular solid** is a solid bounded by six rectangles.

160. Find the volume of a rectangular solid whose length is 5^{cm}, breadth 3^{cm}, and height 7^{cm}.



The face on which the solid rests may be divided into square centimeters; there will be three rows of 5^{qcm} each; in all 15^{qcm}. Upon each square centimeter may be placed a pile of 7^{ccm}, so that the solid will contain 15×7^{ccm}; that is, $3 \times 5 \times 7^{ccm}$.

161. To Find the Volume of a Rectangular Solid,

Express the length, breadth, and height of the solid in the same linear unit; take the product of these numbers for its volume in cubic units of the same name as the linear unit.

If the number of cubic units in the volume is divided by the product of the numbers of linear units in any two dimensions, the quotient is the number of linear units in the third dimension.

Exercise 33.

1. How many cubic centimeters in a block 9^{cm} long, 7^{cm} wide, and 6^{cm} deep?

2. If wood is cut into 120^{cm} lengths, and a pile is 43.7^{m} long and 1.4^{m} high, how many sters of wood are there in the pile?

3. How many hektoliters of grain will a bin hold, 11.2^{m} long, 4.34^{m} wide, and 2.83^{m} deep?

4. If a liter of grain weighs 0.81 of the weight of a liter of water, find the weight of the grain in the bin of Ex. 3.

5. A bin 16^{m} by 9.7^{m}, and 2.8^{m} deep, is full of oats, worth $0.98 a hektoliter. What is the whole worth?

6. How many liters does a vat 197^{cm} long, 87^{cm} wide, and 63^{cm} deep hold? What weight of water will be required to fill it?

7. Add 1341^{ccm}, 231^{l}, and 2.13^{hl}, and give the sum in terms of each of the three units.

8. If a spring delivers 467.8^{l} each minute, how many hektoliters will it deliver in 60 minutes? in 37 minutes? in 78 minutes?

9. If 67.3^{l} of oil in a vat with perpendicular sides fills it to a depth of 173^{mm}, how deep will 13.7 times that quantity fill it? How many hektoliters will there be?

10. One cask contains 171.4^{l} of oil; another, 209.3^{l}; a third, 73.8^{l}; while a square vat, 137^{cm} each way, is filled to a depth of 69^{cm}. Find in liters and in hektoliters the amount of oil in the four vessels together.

11. How many liters of air in a room 7.8^{m} long, 6.23^{m} wide, and 3^{m} high?

12. If a person's breathing spoils the air at the rate of 0.2175^{cbm} a minute, how long will it take three persons sitting in the closed room of Ex. 11 to spoil the air?

13. How long, at the same rate as in Ex. 11, will the air in a hall 22^m long, 16^m wide, and 7^m high last 280 persons?

162. To Find the Volume of a Sphere,

Multiply the cube of the diameter by 0.5236 ($\frac{1}{6}$ of 3.1416).

14. How many cubic centimeters in a ball 10^{cm} in diameter?

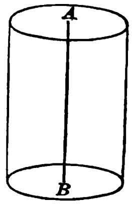
15. Into a cubical box 20^{cm} on an edge, and full of water, an iron ball 20^{cm} in diameter is gently lowered until it touches the bottom. Find in liters and in cubic centimeters the volume of the water left in the box.

16. If cast iron weighs 7.207 times as much as water, what is the weight of a cast iron ball 5^{cm} in diameter?

17. A rubber ball is 6.2^{cm} in diameter. What is the amount of rubber in the ball?

18. If the circumference of a cannon ball is 52^{cm}, find the volume of the ball.

163. A **cylinder** is a solid bounded by two equal and parallel circles called its *bases*, and a uniformly curved surface called its *lateral surface*.



Cylinder.

NOTE. Two circles are parallel if all points of one are equally distant from the other.

164. To Find the Volume of a Cylinder,

Multiply the number of square units in its base by the number of linear units in its height.

19. How many cubic centimeters of oil are there in a cylindrical cup 10^{cm} across when the oil is 38^{mm} deep?

20. What is the capacity of a cylindrical cup 95^{mm} across and 11.08^{cm} deep?

21. What is the capacity of a cylindrical vessel 16.24^{cm} across and 19.95^{cm} deep? 75.4^{mm} across and 87.9^{mm} deep?

22. How many cubic meters of wood in a round stick of equal size throughout, 37^{cm} in diameter and 8.4^{m} long?

23. A cylindrical stand-pipe whose diameter is 12^{m} and whose height is 22^{m} is filled with water. Find the weight of the water.

24. Find the number of liters of water in a well, if its diameter is 1.2^{m} and the depth of the water is 2^{m}.

25. A cylindrical cup 90^{mm} in diameter is partly filled with water. Into the cup is dropped a piece of iron, and the water rises 63^{mm}. What is the volume of the piece of iron?

Exercise 34.

1. What is the weight, in kilograms, of a hektoliter of water? of 73.8^{l} of water? of a cubic meter of water? of a cubic centimeter of water?

2. If a man buys half a ton of potatoes for $20, and retails them all, without waste, at 5 cents a kilogram, what profit does he make on the whole?

3. What is the weight of water required to fill a vat 98^{cm} long, 71^{cm} wide, and 38^{cm} deep?

4. If the vat of the last example is filled with brine weighing 1.04^{kg} to the liter, what is the weight of the brine?

5. If the vat of Ex. 3 is filled with wine weighing 0.981^{kg} to the liter, what is the weight of the wine?

6. What is the total weight of 13 men averaging 73.48^{kg} each?

7. How many kilograms, and how many tons, will 3.6175^{cbm} of brick weigh, at 2 tons to a cubic meter? at 2.34 tons?

8. From a barrel containing 67^{kg} of granulated sugar there are taken three parcels of 2.75^{kg} each, and four parcels of 7.50^{kg} each. How much is left in the barrel?

9. Into how many pills of 325^{mg} each can a mass of 7.8^{g} be divided?

10. A mass of 21.8^{g} is divided into 60 pills. What is the weight of each pill?

11. A bag, when empty, weighs 213^{g}; when full of silver five-franc pieces, 20^{kg} 5^{hg} 13^{g}. A five-franc piece weighs 25^{g}. How many five-franc pieces will the bag hold?

12. A vessel, when empty, weighs 2.7^{kg}; and when full of water 4235^{dkg}. What would it weigh if filled with milk, which is 1.03 times as heavy as water?

Specific Gravity.

165. The **specific gravity** of a substance is the *number* found by dividing the weight of the substance by the weight of an equal bulk of water.

Thus, if a sample of quicksilver has a specific gravity of 13.6, it is 13.6 times as heavy as water; a cubic centimeter of it would weigh 13.6^{g}; a liter of it would weigh 13.6^{kg}; and a cubic meter of it would weigh 13.6^{t}.

Again, if the specific gravity of a certain alcohol is 0.827, that is, if the alcohol weighs 0.827 as much as an equal bulk of water, then a cubic centimeter of it would weigh 0.827^{g}; a liter, 0.827^{kg}; and a cubic meter, 0.827^{t}.

166. The specific gravity of a substance, therefore, is the **number** that expresses the weight of a *cubic centimeter* of it in *grams;* of a *liter* in *kilograms;* of a *cubic meter* in *tons.*

167. *If a substance is in water, the water buoys it up by just the weight of the water displaced by it.*

168. Examples. 1. A lump of coal weighed in air is found to weigh 1017^{g}; weighed in water it is found to weigh 321^{g}. What is the specific gravity of the coal?

SOLUTION. $1017^g - 321^g = 696^g$. Since the lump of coal weighs 696^g less in water than in air, 696^g is the weight of the water displaced by the coal, and the lump contains 696^{ccm} of coal.

Therefore, the specific gravity of coal is $1017^g \div 696^g$, or 1.461; that is, the number found by dividing the weight of the lump of coal by the weight of an equal bulk of water.

2. A stone, weighing 1.1^{kg} in air and 0.6^{kg} in water, is tied to a block of wood; the two together weigh 1.28^{kg} in air and 0.54^{kg} in water. What is the specific gravity of the wood?

SOLUTION. The weight of the wood in air $= 1.28^{kg} - 1.1^{kg} = 0.18^{kg}$.

The weight of water displaced by stone and wood $= 1.28^{kg} - 0.54^{kg} = 0.74^{kg}$.

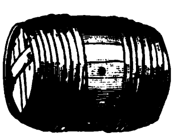
The weight of water displaced by stone alone $= 1.1^{kg} - 0.6^{kg} = 0.5^{kg}$.

The weight of water displaced by wood, therefore, $= 0.74^{kg} - 0.5^{kg} = 0.24^{kg}$.

Hence, the specific gravity of the wood $= 0.18^{kg} \div 0.24^{kg} = 0.75$.

EXERCISE 35.

1. If a stone weighs 1.3^{kg} in air and 0.68^{kg} in water, and the stone and a block of wood together weigh 1.55^{kg} in air and 0.63^{kg} in water, what is the specific gravity of the block of wood?



Hektoliter.

2. What is the weight of 8.17^{hl} of alcohol, specific gravity 0.83?

3. What will 97^{l} of alcohol weigh, of specific gravity 0.817? of specific gravity 0.819? of specific gravity 0.823? 0.838? 0.847?

4. A bar of aluminum 113^{mm} long, 17^{mm} wide, and 13^{mm} thick is said to be of specific gravity 2.57. What does it weigh? If it really is of specific gravity 2.67, what does it weigh?

5. What would be the specific gravity of the aluminum in Ex. 4 if the bar weighed 65.137^{g}?

6. What is the weight of a bar of aluminum 371^{mm} by 63^{mm} by 84^{mm}, specific gravity being 2.63?

7. An irregular mass of copper, gently lowered into a pail brimful of water, caused 1.374^{l} to run over. What did it weigh if of specific gravity 8.91? if 8.89?

8. What would be the specific gravity of the copper in Ex. 7 if the mass weighed 12.3016^{kg}?

169. To Find the Specific Gravity of a Substance,

Divide the weight in grams by the bulk in cubic centimeters, the weight in kilograms by the bulk in liters, or the weight in tons by the bulk in cubic meters.

9. A plate of iron 137^{cm} long, 64.3^{cm} wide, and 4.31^{cm} thick weighs 277.54^{kg}. What is its specific gravity? What would the same mass weigh at specific gravity 7.47? at 7.79?

10. What is the specific gravity of sea water when a hektoliter weighs 102.58^{kg}? when 3^{l} weighs 3077^{g}?

11. What is the specific gravity of a substance of which 7.3^{ccm} weighs 31.5^{g}?

12. If a cubic meter of sand weighs 1723^{kg}, what is its specific gravity? If 3.4^{cbm} of gravel weighs 7.134 tons, what is its specific gravity?

13. If a cubic centimeter of metal weighs 7.3^{g}, what is its specific gravity?

14. What is the specific gravity of a fluid weighing 2.317^{kg} to a liter?

15. If a body weighs 3.71^{kg} in air and 2.38^{kg} in water, what is its specific gravity?

16. A piece of ore weighing 3.77^{kg} weighs in water only 2.53^{kg}. What is its specific gravity?

17. How many cubic centimeters in a stone which loses 17.8^{g} of its weight when weighed in water? What is its specific gravity if it weighs 33.7^{g} in air?

18. In a wrought-iron bottle I find 2.63^{l} of quicksilver, weighing 35.81^{kg}; in another, 2.59^{l}, weighing 35.193^{kg}; in a third, 2.617^{l}, weighing 35.571^{kg}. What is the specific gravity of each? What would be the specific gravity of the mixture if the three were emptied into one vessel?

19. A plate of iron 89^{cm} by 17^{cm} by 7^{cm} weighs 79.43^{kg}. What is its specific gravity?

20. What is the specific gravity of a rectangular block of wood 1.6^{m} long, 0.3^{m} wide, and 0.15^{m} thick, if, floating in water on its face 0.3^{m} wide, it sinks to a depth of 0.12^{m}?

Exercise 36.

1. If 3 men eat 8^{kg} of bread a week, how much will 1 man eat at the same rate? How much will 7 men? How much will the 3 men eat in 1 day? How much will 1 man eat in 1 day? How much will 7 men eat in 1 day? in 1 week? in 5 weeks?

2. At the same rate as in Ex. 1, how much will 17 men eat in 3 weeks and 4 days?

3. If 1^{hl} of oats is enough for 5 horses 1 week, how much is enough for 1 horse 1 week? for 1 horse 7 weeks? for 11 horses 17 weeks?

4. If 2^{hl} of grain is enough for 3 horses 5 days, how much is enough for 3 horses 1 day? for 1 horse 1 day? for 7 horses 6 days?

5. Mix 17^{l} of vinegar, costing 6 cents a liter, with 39^{l} at 5 cents, 21^{l} at 7 cents, and 13^{l} of water costing nothing. Find the number of liters and the cost.

6. For how much a liter must I sell the mixture of of Ex. 5 to gain 96 cents? to gain $1.41?

7. A grocer sold 421 kegs of butter for \$4995.25; 56 kegs brought \$12.50 a keg, 91 brought \$11.75 a keg, and 100 kegs brought \$12.25 a keg. For how much a keg were the other kegs sold?

8. If 3 tons of coal cost \$15.75, how many tons will \$36.75 buy?

9. If 5^m of cloth cost \$18.75, what will 7^m cost?

10. If a tap running 3.5^l a minute fills a tub in 16 minutes, how long will a tap delivering 5^l a minute be in filling the same tub?

11. If both taps of the last example are opened at once, how soon will they fill the tub?

12. If 3 men can dig 378^m of ditch in 2 days, how long will it take 5 men, at the same rate, to dig 787^m?

13. Into a tub that will hold 48^l one tap is delivering water at the rate of 3.7^l a minute; while out of it, by another tap, the water is running at 2.5^l a minute. How long will it take to fill the tub, beginning with it empty?

14. A tap discharges into a tub 4.2^l a minute; from the tub water is also running, by a second tap; the water in the tub gains 30^l in 18 minutes. How fast is the second tap discharging?

15. If a wheel is 1.2^m across, how many times will it turn in going one kilometer?

16. How many times in a minute does the wheel of the last example turn, when the carriage is driven at the rate of 14^{km} an hour?

17. What is the weight of the water in a tank if it takes 1 hour and 38 minutes at the rate of 8.7^l a minute to empty the tank?

18. If we replace the water of Ex. 17 with oil worth \$18.75 a hektoliter, what will the contents of the tank be worth?

EXERCISE 37.

1. A train leaves Paris at 11 o'clock A.M. and reaches Lyons at 10 o'clock P.M. How many meters does it travel in an hour, the distance from Paris to Lyons being 512.7^{km}?

2. A railroad has a single track 11.450^{km} long. How many rails 4.569^{m} in length did it require to lay the track?

3. A book is 2.1^{cm} in thickness; each leaf is 0.05^{mm} thick. Find the number of pages in the book.

4. If the cost of opening a canal amounts to $25,400 a kilometer, how much will a canal cost which is 113.253^{km} in length?

5. The expense of laying out a paved road is $12,500 a kilometer. How much will a road cost which is 72.053^{km} long?

6. The cost of building a railroad is about $78,000 a kilometer in France, and only $25,000 in the United States. How much will it cost in each country to make a road 295.671^{km} long?

7. If you must go up 211 steps to reach the top of a tower, and each step is 195^{mm} high, what is the height of the tower?

8. A house has 5 stories, each story has 19 stairs, each stair is 16^{cm} in height. Find the height of the floor of the fifth story from the ground.

9. A ream of paper contains 20 quires, each quire has 24 sheets, the ream is 13.5^{cm} in thickness. Find the thickness of each sheet.

10. The equator on a terrestrial globe measures 0.80^{m} in circumference. By the aid of a tape measure we find that the distance between two cities on this globe is 0.046^{m}. What is really the distance in kilometers between the two cities? (The earth's equator is $40,075.45^{km}$.)

11. Upon a military map we find that the distance from Paris to St. Denis is 78^{mm}. What is the distance in kilometers from Paris to St. Denis? The map is made on the scale of 1 to 80,000; that is, 1^{m} on the map represents $80,000^{m}$ of actual measurement upon the ground.

12. Find the number of revolutions made by the wheels of a carriage in traveling 82^{km}. The wheels are 1354^{mm} in diameter.

13. How many hektars in a square kilometer? how many ars? how many square meters?

14. France has about $542,000^{qkm}$. How many hektars does it measure?

15. A piece of land 1224.5^{m} square is sold at $140 a hektar. How much does the land bring?

16. The total surface measurement of the glass in the windows of a house is 182^{qm}. How many panes of 53^{cm} by 48^{cm} will it take to supply the windows?

17. How many square slabs of marble 150^{qcm} on the surface will it require to pave a court whose area is 25.35^{qm}?

18. A speculator bought 31.0728^{ha} of land for $1296 a hektar. For how much a square meter must he sell it to realize a profit of $1937?

19. A man is offered $6000 for 2.5^{a} of land. He declines to sell; and soon after the town gives him $25.20 a square meter. How much did he make by refusing the first offer?

20. A man surveys a piece of land and finds that it measures 14.0715^{ha}. He afterwards discovers that his chain was too short by 0.03^{m}. How can he calculate the real superficial measurement of the land without surveying it again? (A surveyor's chain is 10^{m} long.)

21. A pile of wood is 4.25^{m} long, 1.33^{m} thick, and 2.60^{m} high. How many sters are there in it?

22. The railroad from Paris to Orleans has a double track; each rail is 4^m long, and the distance from Paris to Orleans is 121^{km}. What is the number of rails used in laying the track? If the width of the road is 15^m, how many hektars of land does the road include?

23. Find the number of ars in a surface which a ream of paper (480 sheets) will cover. The sheets are 30.3^{cm} long and 195^{mm} wide.

24. A beam is 7.070^m long; its two other dimensions are 0.258^m and 87^{mm}. Find its volume.

25. A bar of iron 3^m long measures 45^{mm} square on the end where it has been evenly cut. The bar is heated and drawn out to a greater length by being passed through an orifice 24^{mm} square. What is the length of the bar after the operation?

26. A reservoir is 1.50^m wide, 2.80^m long, and 1.25^m deep. Find how many liters it contains when full, and to what height it would be necessary to raise it that it might contain 10^{cbm}.

27. Suppose a box to be 3.75^m long, 3.50^m wide, and 0.50^m high. How much lime would it take to fill it with mortar, reckoning that 1^{cbm} of lime after being slaked becomes 1.80^{cbm} of mortar?

28. A chest has the following dimensions: 1.17^m, 0.90^m, 1.04^m. If 0.12 of the volume of the chest is deducted for packing, how many cakes of soap 13^{cm} square on the bottom and 29^{cm} thick could be put in it?

29. A cubic meter of dry plaster makes 1.18^{cbm} when tempered; tempered plaster increases 1 in every 100 twenty-four hours after it is mixed. What volume of tempered plaster would be obtained from 55 sacks of 25^l each of dry plaster?

30. A reservoir is 2.80^m long, 1.50^m wide, and 1.25^m deep. How many liters will be required to fill 0.80 of it?

31. A man buys 1415^{hl} of wheat for \$3.50 a hektoliter; but the measure used proves too small, the mistake amounting to 3^{l} in every hektoliter. What was the quantity of wheat delivered to the purchaser, the cost, and the reduction which ought to be made to him on account of the error?

32. The dimensions of a tile are as follows: length 22^{cm}, width 11^{cm}, thickness 55^{mm}. Find the volume of the tile, and the number of tiles in a pile of 25^{cbm}.

33. The measurement of a pile of wood shows that a ster could be filled from it 25.68 times. Find the volume of the pile in cubic meters, reckoning the length of the logs to be 1.15^{m}.

NOTE. A ster is a frame, as represented on page 64, one meter high and one meter between the upright posts. The ster may be filled with wood of any length, and the volume will be as many cubic meters as the sticks of wood are meters long.

34. A liter of air weighs 1.273^{g}. How much does a cubic meter of air weigh? How many times as heavy as air is water?

35. A package of candles that weighs 465^{g} is sold for 28 cents. At the same rate what is the price of a kilogram of candles?

36. How many times will 3.243^{t} of water fill a liter measure?

37. Express in kilograms the weight of 43.4578^{ccm} of pure water.

38. The volume of the axle of an engine is 0.245^{cbm}. Find its weight, if the specific gravity of the iron is 7.8.

39. Find the volume of a gram of the following substances: proof spirit, specific gravity 0.865; tin, specific gravity 7.291; lead, specific gravity 11.35; copper, specific gravity 8.85; silver, specific gravity 10.47; cork, specific gravity 0.240.

40. Olive oil costs 60 cents a kilogram. What is the price of a liter? The specific gravity of olive oil is 0.914.

41. Pure alcohol costs $1.87 a kilogram. What is the price of a liter? The specific gravity of alcohol is 0.792.

42. A man wishes to build a shed large enough to hold 135^{st} of wood; if the shed is to be 3^{m} high and 5^{m} wide, how long must it be?

43. In a country where firewood is cut 1.16^{m} long, what must be the height of the sides of the ster that it may hold a cubic meter?

44. If a ster of cork costs $20.00, how much would 100^{kg} cost, the cork weighing 0.25 as much as water?

45. A liter of powder weighs 825^{g}. What will be the volume in cubic centimeters of a charge for a gun if the charge weighs 5^{g}?

46. Out of gold which weighs 19.362 times as much as water sheets of gold foil are made which are 0.010^{mm} in thickness. What surface will 3^{g} of gold cover?

47. Find the weight of an oak board 3.25^{m} long, 0.31^{m} wide, and 0.04^{m} thick, if the specific gravity of the oak is 0.808.

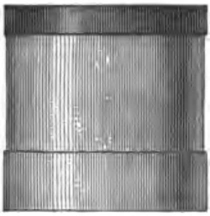
48. Find the weight of a bar of iron having the following dimensions: length 3.6^{m}, width 6^{cm}, thickness 2^{cm}, if the specific gravity of the iron is 7.8.

49. How many lead balls, each weighing 27^{g}, can be obtained by melting a cubic mass of lead 0.356^{m} on an edge, if the specific gravity of the lead is 11.35?

50. Marble costs $30.95 a cubic meter, and the specific gravity of marble is 2.73. If a block of marble weighs 1260^{kg}, what is its volume and what is it worth?

51. Sea water contains 28 parts, by weight, of salt in 1000. A liter of sea water weighs 1.025^{kg}. How many kilograms of salt can be obtained from 126.276842^{cbm} of sea water?

52. An empty cask weighs 17.06^{kg}; when filled with water it weighs 275.8^{kg}. How many liters does it hold? How many casks of this size will it take for the wine from a vat containing 3.008^{cbm}?


Hektoliter.

53. If it takes 2.048^{hl} of wheat to sow a hektar, how many cubic meters will it take to sow a square kilometer?

54. A piece of road 1^{km} long and 7^{m} wide is to be macadamized to the depth of 33^{cm}. What will the work cost at 43 cents a cubic meter?

55. A gasometer holds 28,000^{cbm} of gas. How many jets will this gasometer feed for an evening, when each jet burns 125^{l} an hour, and is used 4 hours?

56. The city of Venice is situated in the midst of a great lake of salt water, communicating with the sea, and all the rain water is caught for the cisterns. Ordinary years the fall of rain in Venice is 82^{cm}; the surface of the city, after the canals have been deducted, is 520^{ha}. Reckoning the population at 115,530, how many liters a day of rain water can each inhabitant have?

57. Find the weight of a bar of iron 5.35^{m} long, 4.56^{cm} thick, and 3.54^{cm} wide. Find, also, the width of an oak beam 4.30^{m} long, 9.12^{cm} thick, which has the same weight. The specific gravity of the oak to be reckoned at 1.026, that of the iron 7.788.

58. Find the specific gravity and volume of a body weighing 35^{kg} in air and 30^{kg} in water.

59. A ster of piled oak wood weighs 425^{kg}; the specific gravity of the wood is 0.74. What is the volume occupied by the spaces between the logs? For how much must 100^{kg} of separate sticks be sold to bring the same amount as when sold at $2.20 a ster?

60. Wrought iron sells for \$7.00 per 100^{kg}. A bar of iron 4.5^{cm} wide, 3.3^{cm} thick costs \$5.08; what is its length, reckoning the specific gravity of the iron at 7.4?

61. Experiment shows that water weighs 770 times as much as air; and the specific gravity of mercury is 13.6. How many liters of air will it take to weigh as much as a liter of mercury?

62. A mass of lead weighing 753^{kg} is made into sheets 0.1^{mm} thick. Find, in square meters, the surface which can be covered by the sheets thus obtained. The specific gravity of the lead is 11.3.

63. A rectangular sheet of tin of uniform thickness is 85^{cm} wide, 1.35^{m} long, and weighs 268^{g}. What is its thickness, if the specific gravity of tin is 7.3?

64. The fine coal which collects about the shafts of the mines and in the coalyards was for a long time wasted, because it could not be burned in stoves and grates. Now this dust is mixed with tar in the proportion of 92^{kg} of dust and 8^{kg} of tar; the mixture is heated, and afterwards pressed in rectangular moulds 14.75^{cm} by 18.5^{cm} by 29^{cm}; each one of these blocks weighs 10^{kg}. They are sold at \$3.00 a ton, and make excellent fuel for heating steam boilers. Find the specific gravity of this fuel; also the sum which would be realized in thus utilizing $800,000^{t}$ of coal dust, the cost of tar, mixing, etc., being \$0.50 a ton.

65. A bar of iron a millimeter square on the end will break under a tension of 30^{kg}. Find the length at which a suspended bar of iron will break from its own weight, if the specific gravity of the iron is 7.8.

66. Fifty-three kilograms of starch are obtained from 100^{kg} of wheat. A hektar of land produces 1363^{l} of wheat; a hektoliter of wheat weighs 78^{kg}. If the wheat harvested from a field measuring 2^{ha} and 33^{qm} is taken to a starch factory, how much starch will be made from it?

67. A gardener wishes to provide glass for his hotbeds. The beds cover 2.65^{a}; the panes will cover 0.75 of the whole surface, the rest being taken up by the frames and alleys. First find how many panes measuring 45^{cm} by 37^{cm} it will take to cover the beds; then find the price of the glass, at a cost of 95 cents a square meter.

68. A jar full of water weighs 1.325^{kg}; filled with mercury it weighs 12.540^{kg}. Find the capacity and the weight of the jar, if the specific gravity of the mercury is 13.59.

69. A hektoliter of rape seed weighs 63^{kg}, and 32^{l} of oil can be extracted from it. How many kilograms of the seed will it take to make a hektoliter of oil?

70. Common burning gas is 0.97 of the weight of air, and a liter of air weighs 1.293^{g}. In a shop there are 65 jets, each one of which burns 123^{l} an hour, and is used 5 hours in the winter evenings. Find the weight of the gas used in a month of 26 days, and the expense of lighting the shop, when gas costs 6 cents a cubic meter.

71. A merchant buys one kind of wine at 30 cents a liter, another kind at 21 cents a liter; he mixes the two kinds by putting 5^{l} of the first with 8^{l} of the second. For how much a liter must he sell the mixture in order to gain $3.75 a hektoliter?

72. If it requires 360 tiles to drain an ar of land, what will it cost to drain 17.784^{ha}, when the tiles cost $20 a thousand, and the expense of laying is the same as the cost of the tiles?

73. Hewn stone of medium durability ought not to support, as a permanent weight, more than 0.07 of the weight that is required to crush it. A certain kind of stone used for building will be crushed under a weight of 250^{kg} a square centimeter. What is the greatest height to which a wall constructed of this material can be safely carried, if the specific gravity of the stone is 2.1?

74. Several different kinds of wine are mixed as follows: 245^{l} at 20 cents a liter, 547^{l} at 15 cents a liter, 344^{l} at 25 cents a liter. How much does the mixture cost a liter?

75. A farmer wishes to drain a field of 8.75^{ha}. Each hektar requires 750^{m} of ditches. The opening of these ditches costs 10 cents a running meter; the tiles are 30^{cm} long and cost $15 a thousand. He pays 2 cents a meter for laying the tiles, and 4 cents a meter for filling the ditches. What is the cost of draining the field?

76. A silver five-franc piece weighs 25^{g}, and is composed of 9 parts of pure silver and 1 part of pure copper. A silver two-franc piece weighs 10^{g}, and is composed of 835 parts of pure silver and 165 parts of pure copper. A silver twenty-centime piece weighs 1^{g}, and has the same composition as the two-franc piece. Find the total weight of pure silver and of pure copper contained in 272 five-franc pieces, 145 two-franc pieces, and 179 twenty-centime pieces.

77. The dimensions of the bottom of a rectangular box are 70^{cm} by 50^{cm}. If the box contains exactly an hektoliter of wheat when full, what is the height of the box?

78. If a stick of oak timber 54 centimeters wide and 65 centimeters thick costs $25 at $16 a cubic meter, what is the length of the stick?

79. A rectangular box whose bottom is a square 28^{cm} on a side, and whose height is 19.2^{cm}, is exactly filled with gold twenty-franc pieces, in piles touching each other. If a twenty-franc piece is 35^{mm} in diameter, and 1.28^{mm} thick, what is the value of the gold in the box?

80. If 1^{hl} of coal yields 1854^{cbm} of gas, and one burner consumes 140^{l} of gas in an hour, how many hektoliters of coal are required to supply 2800 burners for 144 hours?

81. How many liters of water in a cylindrical well 1.96^{m} in diameter, if the water is 2.84^{m} deep?

CHAPTER VI.

MEASURES AND MULTIPLES OF NUMBERS.

170. Factors of a Number. The *factors* of a number are the numbers whose product is that number.

171. Prime Numbers. A *prime number* is a number that has *no integral factors,* except itself and one.

Thus, 2, 3, 5, 7, 11, 13, 17, 19 are prime numbers.

172. Composite Numbers. A *composite number* is a number that is the product of two or more integral factors.

Thus, 10, 21, 143 are composite numbers, for 10 is 2 × 5; 21 is 3 × 7; 143 is 11 × 13.

NOTE. In speaking of the integral factors of a number we exclude the number itself and one.

173. Prime Factors. A *prime factor* is a factor that is a prime number.

174. *A composite number can have but one set of prime factors.*

Thus, 12 cannot be expressed as the product of any *set of prime factors* except 2 × 2 × 3. It is the product of 2 × 6, and of 3 × 4, but one of the factors of 2 × 6 and one of 3 × 4 is *composite.*

175. A number that can be divided by another *without a remainder* is said to be *exactly divisible* by that number; and the divisor is called an *exact divisor.*

176. Even Numbers. An *even number* is a number that is exactly divisible by 2.

177. Odd Numbers. An *odd number* is a number that is *not* exactly divisible by 2.

178. To Determine Prime Numbers. Write the series of integral numbers in order, beginning with the smallest; then cancel the even numbers, and place a dot over each multiple of 3: we have

$1,\ \cancel{2},\ \dot{3},\ \cancel{4},\ 5,\ \dot{\cancel{6}},\ 7,\ \cancel{8},\ \dot{9},\ \cancel{10},\ 11,\ \dot{\cancel{12}},\ 13,\ \cancel{14},\ \dot{15},\ \cancel{16},\ 17,\ \dot{\cancel{18}},$
$19,\ \cancel{20},\ \dot{21},\ \cancel{22},\ 23,\ \dot{\cancel{24}},\ 25,\ \cancel{26},\ \dot{27},\ \cancel{28},\ 29,\ \dot{\cancel{30}},\ 31,$ etc.

Each multiple of 6 is cancelled, and also has the dot over it, and the only numbers without the cancelling line or the dot come just before or just after a multiple of 6. Therefore,

The only numbers greater than 6 that can be prime numbers are one less than or one greater than a multiple of 6.

179. Examples. 1. Find the prime factors of 144.

$$\begin{array}{r|r}
2 & 144 \\ \hline
2 & 72 \\ \hline
2 & 36 \\ \hline
2 & 18 \\ \hline
3 & 9 \\ \hline
 & 3
\end{array}$$

That is, $144 = 2 \times 2 \times 2 \times 2 \times 3 \times 3 = 2^4 \times 3^2$.

NOTE. Divide *as many times as possible* by the *smallest* prime number that will exactly divide the given number before taking the next larger prime number for a divisor.

2. Find the prime factors of 233.

By actual trial we find that the prime numbers 2, 3, 5, 7, 11, 13, and 17 are not factors of 233. We need not try any higher prime number than 17, as the quotient when 17 is tried is *less* than 17. Therefore, no prime number greater than 17 can be a factor; and we have found by trial that no prime number less than 17 is a factor. Therefore, 233 is a prime number.

Notice that 233 is one less than 234, or 39×6.

180. From these two examples we have the following

RULE. *Divide the given number by any prime number that exactly divides it; then the quotient by any prime number that exactly divides it; and so on until the quotient is itself a prime number. The several divisors and the last quotient are the prime factors.*

If no prime factor is found before the quotient becomes equal to or less than the divisor, the number is a prime number.

181. The following tests are very useful for determining without actual division whether a number contains certain factors:

1. A number is divisible by 2 if its *last digit is even.*
2. A number is divisible by 4 (2^2) if the number denoted by the *last two digits* is divisible by 4.
3. A number is divisible by 8 (2^3) if the number denoted by the *last three digits* is divisible by 8.
4. A number is divisible by 3 if the *sum of its digits* is divisible by 3.
5. A number is divisible by 6 if its *last digit is even* and *the sum of its digits* is divisible by 3.
6. A number is divisible by 9 (3^2) if the *sum of its digits* is divisible by 9.
7. A number is divisible by 5 if its *last digit* is either 5 or 0.
8. A number is divisible by 25 (5^2) if the number denoted by the *last two digits* is divisible by 25.
9. A number is divisible by 125 (5^3) if the number denoted by the *last three digits* is divisible by 125.
10. A number is divisible by 11 if the *difference between the sum of the digits in the even places and the sum of the digits in the odd places* is either 0 or a multiple of 11.

182. Other prime factors, 7, 13, 17, 19, etc., sometimes betray their presence to one familiar with the subject; but, practically, the best way to detect them is by actual division.

183. If we divide any number less than 121 (11^2) by 11, or by a number greater than 11, it is plain that the quotient is less than 11.

If we divide any number between 121 and 143 (11×13) by 11, the quotient will evidently lie between 11 and 13; and, since there are no prime numbers between 11 and 13, the quotient, if a whole number, must be composite, and contain factors smaller than 11.

What is true of 11 and 13 is evidently true of any two *adjacent* prime numbers; namely, that, excepting the *second power* of the smaller prime, *every composite number less than the product of two adjacent prime numbers contains prime factors less than the smaller of these two numbers.*

Thus, every composite number less than 4087 (61×67), except 3721 (61^2), contains prime factors less than 61.

184. The value of the following table, in discovering the prime factors of a given number, will be apparent.

Primes . .	7	11	13	17	19	23	29	31	37
Powers . .	49	121	169	289	361	529	841	961	1369
Products .	77	143	221	323	437	667	899	1147	1517
Primes . .	41	43	47	53	59	61	67	71	73
Powers . .	1681	1849	2209	2809	3481	3721	4489	5041	5329
Products .	1763	2021	2491	3127	3599	4087	4757	5183	5767
Primes . .	79	83	89	97	101	103	107	109	113
Powers . .	6241	6889	7921	9409	10201	10609	11449	11881	12769
Products .	6557	7387	8633	9797	10403	11021	11663	12317	14351

Opposite "Powers" are placed the squares of the primes from 7 to 113; and opposite "Products" are placed the products of the successive pairs of adjacent primes from 7 to 127.

185. Example. Find the prime factors of 610,764.

As 64 is divisible by 4, but 764 is not divisible by 8, 2^2 is the highest power of 2 contained in 610,764.

2^2	610,764
3	152,691
7	50,897
11	7,271
	661

As the sum of the digits 152,691 is divisible by 3, but not by 9, 3^1 is the highest power of 3 contained in 152,691.

The next quotient, 50,897, does not contain 5; but divided by 7 gives 7271. 7271 does not contain 7; but, since $7 + 7 - (2 + 1) = 11$, it is divisible by 11.

The quotient 661 when divided by 6 gives a remainder of 1, which shows that it *may be* a prime number. It cannot be divided by 11, 13, 17, or 19, and is seen by the table to be less than 667 (23×29), and not equal to 529 (23^2); therefore it is a prime number.

Therefore, $610,764 = 2^2 \times 3 \times 7 \times 11 \times 661$.

Exercise 38.

Find the prime factors of:

1. 148.
2. 264.
3. 178.
4. 183.
5. 173.
6. 187.
7. 346.
8. 343.
9. 210.
10. 353.
11. 5280.
12. 231.
13. 31,416.
14. 1369.
15. 1368.
16. 247.
17. 327.
18. 179.
19. 83.
20. 2125.
21. 2353.
22. 2333.
23. 165.
24. 168.
25. 2148.
26. 16,662.
27. 321.
28. 1551.
29. 38.
30. 82.
31. 129.
32. 72.
33. 66.
34. 68.
35. 65.
36. 76.
37. 86.
38. 88.
39. 142.
40. 326.
41. 368.
42. 464.
43. 292.
44. 362.
45. 365.
46. 730.
47. 42.
48. 868.
49. 999.
50. 822.
51. 1346.

52. 7641.	**59.** 128.	**66.** 78,309.
53. 6234.	**60.** 8192.	**67.** 25,179.
54. 234.	**61.** 8190.	**68.** 61,600.
55. 579.	**62.** 8197.	**69.** 48,048.
56. 577.	**63.** 3125.	**70.** 401,478.
57. 212.	**64.** 2401.	**71.** 278,208.
58. 126.	**65.** 1331.	**72.** 493,185.

186. A number is not only divisible by each of its prime factors, but by every possible combination of them.

Thus, 120 is $2^3 \times 3 \times 5$, and is divisible by 2, 4, 8, 6, 12, 24, 30, 60, 10, 20, 40, or 15.

187. The number 14.21 may be put in the form of 1421×0.01; and thus be resolved into $7^2 \times 29 \times 0.01$. But 0.01 is not properly a factor, it is a divisor; it is the reciprocal of $2^2 \times 5^2$. Nevertheless, it is frequently of great practical advantage to separate mixed decimals in this way, by first taking out the apparent factors, 0.1, 0.01, etc.

Thus, the factors of 142.1 may be said to be 7, 7, 29, and 0.1; of 1.421, 7, 7, 29, and 0.001.

EXERCISE 39.

Find the prime factors of:

1. 8.4.	**9.** 2.61.	**17.** 5.04.
2. 7.6.	**10.** 21.2.	**18.** 1.485.
3. 1.08.	**11.** 78.54.	**19.** 0.216.
4. 0.144.	**12.** 0.5236.	**20.** 34.87.
5. 0.036.	**13.** 0.00052.	**21.** 32.4.
6. 0.037.	**14.** 8.67.	**22.** 5.115.
7. 21.45.	**15.** 48.3.	**23.** 71.2.
8. 14.6.	**16.** 99.99.	**24.** 2.993.

Greatest Common Measure.

188. Measures of a Number. The *measures* of a number are the *exact divisors* of the number.

Thus, a man with five-dollar bills can make up a sum of $20, but not of $18. A wood chopper with the two-foot mark on his axe-handle can measure off lengths of 2, 4, 6, or 8 feet, but not of 3, 5, or 7 feet. A man with scales and a four-ounce weight can weigh out 16 ounces of tea, but not 22 ounces. A man with a four-quart measure can measure out 4, 8, or 12 quarts of molasses, but not 5, 6, or 7 quarts.

189. Common Measures. A common measure of two or more numbers is a number that exactly divides *each* of them.

Thus, $5 is a common measure of $35 and $40, being contained exactly 7 times in $35 and 8 times in $40. 3 feet is a common measure of 21 feet, 15 feet, and 12 feet. 1 yard is a common measure of 2 yards, 3 yards, and 5 yards. 4 is a common measure of 12, 16, and 20.

190. Greatest Common Measure. The greatest common measure of two or more numbers is *the greatest number* that exactly divides each of them.

Thus, the measures of 84 are 1, 2, 3, 4, 6, 7, 12, 14, 21, 28, 42, and the measures of 36 are 1, 2, 3, 4, 6, 9, 12, 18.

We see from the two series of measures that 1, 2, 3, 4, 6, 12 are the only measures of both 84 and 36, and that 12 is the greatest; therefore, 12 is the greatest common measure of 84 and 36.

191. The letters G. C. M. stand for the words Greatest Common Measure.

NOTE. Greatest Common Divisor is sometimes used instead of Greatest Common Measure, and then G. C. D. is used instead of G. C. M.

192. If two integral numbers have no common measure except 1, they are said to be *prime to each other*.

Thus, 27 and 32 are prime to each other, though both are composite numbers.

193. Examples. 1. Find the G. C. M. of 84, 126, and 210.

Resolve each of the numbers into its prime factors.

$$\begin{array}{r|r} 2 & 84 \\ \hline 2 & 42 \\ \hline 3 & 21 \\ \hline & 7 \end{array} \qquad \begin{array}{r|r} 2 & 126 \\ \hline 3 & 63 \\ \hline 3 & 21 \\ \hline & 7 \end{array} \qquad \begin{array}{r|r} 2 & 210 \\ \hline 3 & 105 \\ \hline 5 & 35 \\ \hline & 7 \end{array}$$

$84 = 2^2 \times 3 \times 7.$ $\quad 126 = 2 \times 3^2 \times 7.$ $\quad 210 = 2 \times 3 \times 5 \times 7.$

The factor 2 occurs once in all the numbers.
The factor 3 occurs once in all the numbers.
The factor 7 occurs once in all the numbers.
No other factor occurs in *all* the numbers.
Therefore, the G. C. M. is $2 \times 3 \times 7 = 42$.

2. Find the G. C. M. of 40 and 72.

$$40 = 2^3 \times 5. \qquad 72 = 2^3 \times 3^2.$$

Hence, 2 occurs *three* times as a factor in 40, and *three* times as a factor in 72. No other factor is common to 40 and 72. Therefore, 2^3, or 8, is the G. C. M. Hence,

194. To Find the G. C. M. of Two or More Numbers,

Separate the numbers into their prime factors. Select the lowest power of each factor that is common to the given numbers, and find the product of these powers.

195. The common factors of two or more numbers may be taken out at the same time as follows:

$$\begin{array}{r|rrr} 2 & 84 & 126 & 210 \\ \hline 3 & 42 & 63 & 105 \\ \hline 7 & 14 & 21 & 35 \\ \hline & 2 & 3 & 5 \end{array}$$

As all the numbers are *even*, 2 is a common factor. As 3 is an exact divisor of 42, 63, and 105, 3 is a common factor.

As 7 is an exact divisor of 14, 21, and 35, 7 is a common factor.

The quotients 2, 3, and 5 have no common factor. Therefore, the only *common* factors are 2, 3, and 7; and the G. C. M. is $2 \times 3 \times 7 = 42$.

Exercise 40.

Find the G. C. M. of :

1. 27 and 33.
2. 13 and 39.
3. 8 and 28.
4. 27 and 45.
5. 81 and 108.
6. 4, 10, 12.
7. 4, 6, 10.
8. 9, 12, 21.
9. 10, 15, 25.
10. 14, 98, 42.
11. 30, 18, 54.
12. 14, 56, 42.
13. 96, 36, 48.
14. 84, 105, 63.
15. 24, 60, 84, 128.
16. 45, 81, 27, 90.
17. 78, 18, 54, 42.
18. 98, 28, 70, 42.
19. 96, 112, 80, 32.
20. 24, 96, 48, 120.
21. 84, 252, 168, 210.
22. 33, 88, 77, 55.
23. 252, 315, 420, 504.
24. 128, 192, 320, 368, 432.
25. 136, 204, 357, 459.
26. 909, 1414, 2323, 4242.

196. Example. Find the G. C. M. of 69 and 184.

```
69)184(2
   138
    46)69(1
       46
       23)46(2
          46
```

Solution. We divide the greater number by the smaller, and the last divisor by the last remainder, and so on until there is no remainder. The final divisor, 23, is the greatest common measure.

This method of finding the G. C. M. can be employed when the numbers cannot readily be separated into their prime factors.

This method depends upon two principles :

1. *Every factor of a number is also a factor of every multiple of that number.*

2. *Every common factor of two numbers is also a factor of their sum and of their difference.*

Thus 4, which is a factor of 12, is also a factor of 24, 36, etc.; and 6, which is a common factor of 24 and 36, is also a factor of 60 and 12.

Let us apply these principles to this example:

Since 23 is a factor of itself and of 46, it is, by (2), a factor of 69.

Since 23 is a factor of 69, it is, by (1), a factor of 2 × 69, or 138; and therefore, by (2), it is a factor of 138 + 46, or 184.

Hence, 23 is a common factor of 69 and 184.

Again, every common factor of 69 and 184 is, by (1), a factor of 2×69, or 138; and, by (2), a factor of $184 - 138$, or 46.

Every such factor, being now a common factor of 69 and 46, is, by (2), a factor of $69 - 46$, or 23.

Therefore, the greatest common factor of 69 and 184 is contained in 23, and cannot be greater than 23. And 23, which we have shown to be a common factor of 69 and 184, must be their G. C. M.

197. In the course of this operation every remainder contains, as a factor of itself, the G. C. M. sought; and this G. C. M. is the greatest factor common to that remainder and the preceding divisor. Therefore,

If the remainder from any division is found to contain a factor that is not a factor of the preceding divisor, the remainder may be divided by that factor, and the quotient used as the next divisor.

If a factor common to any remainder and the preceding divisor is found, both remainder and divisor may be divided by the common factor, and that factor must be reserved as a factor of the G. C. M. sought.

198. We will illustrate by two examples.

1. Find the G. C. M. of 4627 and 8593.

```
4627)8593(1
     4627
   6|3966
     661)4627(7
         4627
```

The factor 6 is thrown out of the first remainder 3966, for it is prime to 4627, and therefore is not a factor of the G. C. M. sought.

Therefore, the G. C. M. is 661.

2. Find the G. C. M. of 72,471 and 134,589.

```
3|72471      134589
  24157     ) 44863(1
              24157
            6|20706
              3451)24157(7
                   24157
```

The common factor 3 is first taken out of both numbers. From the remainder 20,706 the factor 6, which is prime to 24,157, is ejected.

The G. C. M. is, therefore, 3×3451, or 10,353.

Exercise 41.

Find the G. C. M. of:

1. 2479 and 3589.
2. 3045 and 6195.
3. 568 and 712.
4. 11,023 and 6493.
5. 1485 and 2160.
6. 7040 and 7392.
7. 2760 and 4485.
8. 1177 and 2675.
9. 78,473 and 94,653.
10. 35,143 and 10,283.
11. 44,323 and 61,087.
12. 232,353 and 39,699.
13. 33,853 and 35,017.
14. 5115 and 7254.
15. 2268 and 3348.
16. 1003 and 2419.
17. 419 and 52,301.
18. 30,072 and 133,784.
19. 4257 and 10,836.
20. 17,104 and 27,794.

199. To find the G. C. M. of several large numbers, we find the G. C. M. of two of the numbers; then of that result and a third number; then of that result and a fourth; and so on. The last G. C. M. is the one required.

The work can often be very much shortened by removing from each of the numbers all factors less than 13.

Find the G. C. M. of 3555, 4977, and 6636.

3	3555		4977		6636
5	1185	3	1659	4	2212
3	237	7	553	7	553
	79		79		79

Hence, the G. C. M. required is 3 × 79, or 237.

Exercise 42.

Find the G. C. M. of:

1. 855, 1197, 1596.
2. 3864, 3404, 3657.
3. 15,561, 11,115, 13,585.
4. 2943, 2616, 4578.
5. 1177, 1391, 1819.
6. 4939, 1347, 3143.
7. 740, 333, 296.
8. 833, 1785, 1309.
9. 4994, 7491, 9988, 12,485, 16,571.

Least Common Multiple.

200. Multiples. If a number is multiplied by an integer, the product is called a *multiple* of the number.

Thus, \$20 is a multiple of \$5, since 4 times \$5 is \$20.

201. A series of multiples of a number is found by multiplying the number by the integers, 1, 2, 3, 4, 5, etc.

Since a composite number is the product of *only one set* of prime numbers (§ 174), every multiple of a number *contains all the prime factors of the number.*

202. Common Multiple. A multiple of two or more numbers is called a *common multiple* of the numbers.

Thus, $6 \times \$2 = \$12$; $4 \times \$3 = \$12$; $3 \times \$4 = \$12$; $2 \times \$6 = \$12$. Therefore, \$12 is a common multiple of \$2, \$3, \$4, and \$6.

203. Least Common Multiple. The *smallest* common multiple of two or more numbers is called their *least common multiple*; and it is the smallest number that is exactly divisible by each of them.

Thus, the multiples of 3 are 3, 6, 9, 12, 15, 18, 21, 24, etc.; and the multiples of 4 are 4, 8, 12, 16, 20, 24, etc. The common multiples of 3 and 4 are 12, 24, etc.; and the smallest of these is 12. Therefore, the least common multiple of 3 and 4 is 12.

204. The letters L. C. M. stand for the words Least Common Multiple.

205. The L. C. M. of two or more numbers is a number that contains *all the prime factors* of each of these numbers. Every prime factor, therefore, must occur in the L. C. M. the *greatest number of times* it occurs as a factor *in any one* of them.

Thus, $\$20 = 2 \times 2 \times 5 \times \$1$, and $\$30 = 2 \times 3 \times 5 \times \$1$.
The L. C. M. of \$20 and \$30 is, therefore, $2 \times 2 \times 3 \times 5 \times \$1$, or \$60.

206. Find the L. C. M. of 84, 168, 252, and 420.

Solution. Resolve each of the numbers into its prime factors.

$$\begin{array}{r|r} 2 & 84 \\ 2 & 42 \\ 3 & 21 \\ & 7 \end{array} \qquad \begin{array}{r|r} 2 & 168 \\ 2 & 84 \\ 2 & 42 \\ 3 & 21 \\ & 7 \end{array} \qquad \begin{array}{r|r} 2 & 252 \\ 2 & 126 \\ 3 & 63 \\ 3 & 21 \\ & 7 \end{array} \qquad \begin{array}{r|r} 2 & 420 \\ 2 & 210 \\ 3 & 105 \\ 5 & 35 \\ & 7 \end{array}$$

The factor 2 occurs *three* times in 168; the factor 3 occurs *twice* in 252; the factor 5 occurs *once* in 420; and the factor 7 occurs *once* in all the given numbers.

Therefore, the L. C. M. is $2^3 \times 3^2 \times 5 \times 7 = 2520$. Hence,

207. To Find the L. C. M. of Two or More Numbers,

Separate each number into its prime factors. Find the product of these factors, taking each factor the greatest number of times it occurs in any one of the given numbers.

208. Examples. 1. Find the L. C. M. of 18, 24, 27, 45.

Arrange the numbers in line, and divide by the smallest prime number that will divide *two* or more of the numbers.

$$\begin{array}{r|rrrr} 2 & 18 & 24 & 27 & 45 \\ 3 & \cancel{9} & 12 & 27 & 45 \\ 3 & & 4 & 9 & 15 \\ & & 4 & 3 & 5 \end{array}$$

We first divide by 2, and write the quotients and undivided numbers in a line below. In the first line of quotients we cancel 9, as it is an exact divisor of 27, and, therefore, 27 contains all the factors of 9. We next divide by 3, and the quotients by 3, and obtain the numbers 4, 3, 5 in the last line. No *two* of the numbers 4, 3, and 5 have a common factor. Hence, the L. C. M. is $2 \times 3 \times 3 \times 4 \times 3 \times 5 = 1080$.

2. Find the L. C. M. of 3, 9, 27, 54.

$$\cancel{3},\quad \cancel{9},\quad \cancel{27},\quad 54.$$

We cancel the 3, which is contained in 9; then the 9, which is contained in 27; then the 27, which is contained in 54, and have 54 for the L. C. M. of the numbers.

3. **Find** the L. C. M. of 13, 15, 26, 39:

$$\begin{array}{r|rrrr} 3) & \cancel{13} & 15 & 26 & 39 \\ \hline & & 5 & 26 & \cancel{13} \end{array}$$

We cancel the 13 of the first line and divide by 3, getting 5, 26, 13. **We cancel** the 13 of this line. The L. C. M. is $3 \times 5 \times 26 = 390$.

EXERCISE 43.

Find the L. C. M. of:

1. 6, 14, 21.
2. 8, 12, 3, 24.
3. 6, 10, 15.
4. 9, 12, 18, 4.
5. 15, 21, 35.
6. 12, 20, 24.
7. 14, 24, 28.
8. 12, 15, 20.
9. 16, 24, 32.
10. 21, 33, 77.
11. 27, 33, 99.
12. 7, 11, 13.
13. 77, 55, 35.
14. 16, 18, 27, 72.
15. 10, 12, 22, 33, 60.
16. 15, 16, 18, 20, 22, 24.
17. 56, 64, 70, 84, 112.
18. 48, 54, 81, 144, 162.
19. 75, 100, 120, 150, 180.
20. 112, 168, 196, 224.
21. 7, 14, 15, 21, 45.
22. 16, 25, 81.
23. 26, 39, 52, 65.
24. 80, 72, 225, 48.
25. 10, 20, 30, 40, 50, 60.
26. 30, 42, 105, 70.
27. 36, 24, 35, 20.
28. 7, 11, 14, 15.
29. 12, 18, 27, 63, 28.
30. 34, 26, 65, 85, 51, 39.
31. 12, 18, 96, 144.
32. 84, 156, 63, 99.
33. 17, 51, 119, 210.
34. 16, 30, 48, 56, 72.
35. 27, 33, 54, 69, 132.
36. 15, 26, 39, 65, 180.
37. 44, 126, 198, 280, 330.
38. 50, 338, 675, 975.
39. 552, 575, 920.
40. 228, 304, 342.
41. 1080 and 1260.
42. 600 and 480.
43. 1564 and 1932.
44. 2530 and 1760.
45. 936 and 2925.
46. 3432 and 4032.
47. 1875 and 2425.
48. 1632 and 2976.
49. 1001 and 2233.
50. 539 and 1463.

209. If the given numbers are large and contain no prime factors that can readily be detected, it is best to obtain the common factors by the process for finding the G. C. M. under like circumstances.

Example. Find the L. C. M. of 1247 and 1769.

```
1247)1769(1
     1247
   2| 522
   9| 261
      29)1247(43
          116
           87
           87
```

Hence, the G. C. M. of 1247 and 1769 is 29;
and $1247 = 29 \times 43$,
and $1769 = 29 \times 61$.

Therefore, the L. C. M. of 1247 and 1769 is $29 \times 43 \times 61 = 1247 \times 61$, or 1769×43, that is, 76,067.

210. From this process it will be seen that:

The L. C. M. of two numbers may be found by dividing either of the numbers by their G. C. M. and multiplying the quotient by the other number.

211. The L. C. M. of two prime numbers, or of two numbers prime to each other, is their product.

Exercise 44.

Find the L. C. M. of:

1. 424 and 583.
2. 319 and 407.
3. 1679 and 1932.
4. 1003 and 2419.
5. 1003 and 1357.
6. 899 and 961.
7. 407, 703, 444.
8. 411, 959, 2055.
9. 221 and 351.
10. 1426 and 989.
11. 3864, 3404, 3657.
12. 539 and 253.
13. 2943, 2616, 4578.
14. 2863 and 1151.
15. 1177, 1391, 1819.
16. 5317 and 2863.
17. 12,703 and 12,879.
18. 23,309 and 10,753.
19. 4939 and 3143.
20. 4199 and 6137.

CHAPTER VII.

COMMON FRACTIONS.

212. What is the name of one of the parts when a unit is divided into:

1. Two equal parts?
2. Three equal parts?
3. Four equal parts?
4. Five equal parts?
5. Six equal parts?
6. Eight equal parts?
7. Ten equal parts?
8. Twelve equal parts?
9. Twenty equal parts?
10. One hundred equal parts?

213. A unit contains how many:

1. Halves?
2. Thirds?
3. Fourths?
4. Fifths?
5. Sixths?
6. Eighths?
7. Ninths?
8. Sevenths?
9. Tenths?
10. Twelfths?
11. Elevenths?
12. Fifteenths?
13. Twentieths?
14. Twenty-fourths?
15. Thirtieths?
16. Fortieths?
17. Fiftieths?
18. Hundredths?

214. When a unit is divided into twelve equal parts, what is the name of:

1. One part?
2. Three parts?
3. Six parts?
4. Two parts?
5. Five parts?
6. Four parts?
7. Eight parts?
8. Ten parts?
9. Twelve parts?

215. Equal parts of a unit are called *fractional parts* of the unit.

216. In three quarters of a yard, the *unit* counted is a *quarter of a yard.*

217. A unit that is a fractional part of a whole unit is called a *fractional unit.*

218. Fractions. Numbers that count fractional units are called *fractions.*

219. Common Fractions. A fraction expressed by two numbers one under the other with a line between them is called a *common fraction.*

220. Simple Fractions. If the two numbers are whole numbers, the fraction is called a *simple fraction.*

Thus, three fifths, written $\frac{3}{5}$, is a simple fraction.

221. The lower number is called the *denominator* (name-giver), the upper number is called the *numerator* (number-giver); and the numerator and denominator together are called the *terms* of the fraction.

222. The fraction $\frac{3}{8}$ means 3 times $\frac{1}{8}$, where $\frac{1}{8}$ is the *fractional unit* and 3 is the *number* of them.

223. *The fractional unit is expressed by 1 divided by the denominator,* and the *number* of fractional units taken is expressed by the *numerator.*

Name the fractional unit and the integral unit of:

1. $\frac{3}{4}$ of an inch.
2. $\frac{4}{5}$ of a dollar.
3. $\frac{3}{8}$ of a bushel.
4. $\frac{2}{3}$ of a yard.
5. $\frac{5}{7}$ of a week.
6. $\frac{7}{12}$ of a foot.
7. $\frac{9}{16}$ of a pound.
8. $\frac{3}{4}$ of an acre.

224. To Read a Common Fraction,

Read the numerator and then the denominator.

Thus, $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{2}{5}$, $\frac{1}{6}$, $\frac{8}{11}$ are read one half, two thirds, three fourths or three quarters, two fifths, one sixth, eight elevenths.

Read: $\frac{3}{5}$; $\frac{7}{8}$; $\frac{4}{3}$; $\frac{11}{2}$; $\frac{3}{10}$; $\frac{12}{13}$; $\frac{5}{12}$; $\frac{1}{20}$; $\frac{17}{50}$; $\frac{85}{100}$.

Express in figures:

1. Two thirds.
2. Five sevenths.
3. Seven ninths.
4. Eight tenths.
5. Eleven sixteenths.
6. Seventeen twentieths.
7. One twenty-fifth.
8. Thirty hundredths.

225. A **proper fraction** is a fraction whose numerator is less than its denominator; as $\frac{7}{8}$.

226. An **improper fraction** is a fraction whose numerator is not less than its denominator; as $\frac{8}{8}$, $\frac{17}{4}$.

NOTE. When the numerator is greater than the denominator, more than one unit must be regarded as divided into equal parts.

Thus, $\frac{9}{4}$ means that three units have been divided each into four equal parts, and that all the parts of two units and one part of the third unit are taken; or, $\frac{9}{4}$ means that nine units, considered as one quantity, have been divided into four equal parts, and that one of these parts has been taken.

227. How many integral units must be divided into equal parts that we may have $\frac{9}{4}$ of the unit? $\frac{3}{2}$? $\frac{15}{8}$? $\frac{7}{9}$? $\frac{20}{3}$? $\frac{28}{7}$? $\frac{47}{10}$? $\frac{25}{8}$?

228. A **mixed number** is a whole number and a fraction; as $4\frac{3}{7}$, 5.35. These are read four *and* three sevenths, five *and* thirty-five hundredths.

NOTE. Every mixed number means that some entire units are taken and the fraction of another unit.

229. Name the proper fractions, the improper fractions, and the mixed numbers of the following expressions:

$3\frac{1}{7}$; $\frac{2}{5}$; $4\frac{2}{15}$; $\frac{8}{8}$; $\frac{3}{17}$; $\frac{9}{23}$; $11\frac{2}{9}$; $3\frac{15}{17}$; $\frac{19}{100}$; $\frac{19}{3}$; $\frac{13}{13}$; $\frac{107}{100}$; $18\frac{13}{37}$; $\frac{50}{41}$; $27\frac{11}{100}$; $\frac{23}{80}$; $\frac{11}{7}$; $\frac{19}{20}$; $3\frac{4}{5}$; $\frac{3}{19}$; $\frac{11}{5}$.

Reduction of an Improper Fraction to a Whole or a Mixed Number.

230. An improper fraction represents a quantity which can also be represented by a whole number, or else by a mixed number.

Thus, $\frac{17}{7} = 2\frac{3}{7}$; for, if we suppose several units to be divided each into seven equal parts, and we take 17 of these parts, 14 (that is, 2×7) will make two units, and the three parts remaining will be three sevenths of another unit. Hence,

231. To Reduce an Improper Fraction to a Whole or a Mixed Number,

Divide the numerator by the denominator.

The quotient will be the whole number, and the remainder, if any, will be the numerator of the fractional part, of which the denominator will be the denominator of the improper fraction.

Exercise 45.

Reduce to a whole or a mixed number :

1. $\frac{13}{7}$.	8. $\frac{44}{5}$.	15. $\frac{62}{5}$.	22. $\frac{481}{13}$.
2. $\frac{21}{8}$.	9. $\frac{9}{2}$.	16. $\frac{55}{6}$.	23. $\frac{292}{23}$.
3. $\frac{25}{4}$.	10. $\frac{19}{3}$.	17. $\frac{44}{4}$.	24. $\frac{522}{29}$.
4. $\frac{107}{11}$.	11. $\frac{50}{10}$.	18. $\frac{112}{19}$.	25. $\frac{787}{42}$.
5. $\frac{91}{9}$.	12. $\frac{51}{16}$.	19. $\frac{213}{16}$.	26. $\frac{8975}{25}$.
6. $\frac{63}{7}$.	13. $\frac{45}{15}$.	20. $\frac{242}{22}$.	27. $\frac{3823}{75}$.
7. $\frac{72}{8}$.	14. $\frac{45}{13}$.	21. $\frac{374}{25}$.	28. $\frac{4684}{177}$.

Reduction of a Whole or a Mixed Number to an Improper Fraction.

232. A whole number may be expressed as a fraction with any given denominator.

Thus, $7 = \frac{63}{9}$; for, as each unit contains 9 *ninths*, 7 units contain 7×9 *ninths*, that is, 63 *ninths*. Hence,

233. To Reduce a Whole Number to a Fraction,

Multiply the whole number by the denominator of the required fraction, and under this product write the denominator.

EXERCISE 46.

Reduce to an improper fraction :

1. $4 = \frac{\ }{2}$.	5. $11 = \frac{\ }{3}$.	9. $9 = \frac{\ }{14}$.
2. $5 = \frac{\ }{1}$.	6. $7 = \frac{\ }{7}$.	10. $18 = \frac{\ }{11}$.
3. $6 = \frac{\ }{5}$.	7. $3 = \frac{\ }{9}$.	11. $12 = \frac{\ }{12}$.
4. $8 = \frac{\ }{6}$.	8. $14 = \frac{\ }{12}$.	12. $16 = \frac{\ }{16}$.

234. A mixed number represents a quantity that can also be represented by an improper fraction.

Thus, $5\frac{7}{13} = \frac{72}{13}$; for each unit contains 13 *thirteenths;* therefore 5 units contain 5×13 *thirteenths*, or 65 *thirteenths;* which, together with the 7 *thirteenths*, make 72 *thirteenths*. Hence,

235. To Reduce a Mixed Number to a Fraction,

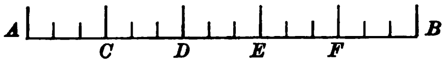
Multiply the whole number by the denominator of the fraction, and to the product add the numerator; under this sum write the denominator.

EXERCISE 47.

Reduce to an improper fraction :

1. $3\frac{4}{5}$.	10. $44\frac{6}{7}$.	19. $14\frac{17}{19}$.	28. $36\frac{83}{85}$.
2. $5\frac{9}{10}$.	11. $2\frac{108}{109}$.	20. $21\frac{17}{100}$.	29. $11\frac{109}{1000}$.
3. $12\frac{4}{11}$.	12. $3\frac{44}{45}$.	21. $6\frac{41}{50}$.	30. $3\frac{71}{75}$.
4. $8\frac{3}{7}$.	13. $10\frac{7}{19}$.	22. $16\frac{8}{25}$.	31. $16\frac{11}{12}$.
5. $25\frac{2}{5}$.	14. $12\frac{13}{15}$.	23. $11\frac{19}{24}$.	32. $15\frac{1}{42}$.
6. $17\frac{2}{3}$.	15. $84\frac{17}{20}$.	24. $8\frac{9}{16}$.	33. $108\frac{6}{25}$.
7. $8\frac{5}{12}$.	16. $16\frac{18}{23}$.	25. $12\frac{3}{4}$.	34. $51\frac{3}{17}$.
8. $9\frac{9}{14}$.	17. $17\frac{7}{12}$.	26. $27\frac{1}{2}$.	35. $40\frac{16}{19}$.
9. $162\frac{5}{11}$.	18. $19\frac{5}{9}$.	27. $111\frac{2}{3}$.	36. $864\frac{13}{57}$.

Reduction of a Fraction to Lower Terms.



236. If AB is divided into 5 equal parts, how many of these parts are there in AC? What part of AB is AC?

If AB is divided into 15 equal parts, how many of these parts are there in AC? What part of AB is AC?

Which is the greater fraction, $\frac{12}{15}$ or $\frac{4}{5}$?

What must be done to $\frac{12}{15}$ to make $\frac{4}{5}$?

Which is greater, $\frac{1}{2}$ or $\frac{2}{4}$? $\frac{6}{10}$ or $\frac{3}{5}$? $\frac{15}{21}$ or $\frac{5}{7}$? Hence,

237. To Reduce a Fraction to Lower Terms,

Divide both numerator and denominator by any common factor.

238. A fraction is expressed in its *lowest terms* when the numerator and denominator are prime to each other.

239. Examples. 1. Reduce $\frac{324}{756}$ to its lowest terms.

$$\frac{324}{756} = \frac{36}{84} = \frac{9}{21} = \frac{3}{7};$$

the common divisors used being 9, 4, and 3.

2. Reduce $\frac{259}{333}$ to its lowest terms.

We first find the prime factors of 333 to be 3, 3, and 37.

Since the factor 3 of 333 will not exactly divide 259 we try 37, and find it is contained 7 times in 259.

Dividing 259 and 333 each by 37, we have $\frac{259}{333} = \frac{7}{9}$.

3. Reduce $\frac{1261}{1649}$ to its lowest terms.

Since no common factor can be readily detected, we find the G. C. M. of 1261 and 1649 to be 97.

Dividing 1261 and 1649 each by 97, we have $\frac{1261}{1649} = \frac{13}{17}$. Hence,

240. To Reduce a Fraction to its Lowest Terms,

Cancel all the factors common to the numerator and denominator; or *divide both terms by their G. C. M.*

Exercise 48.

Reduce to lowest terms :

1. $\frac{120}{192}$.	8. $\frac{3960}{12672}$.	15. $\frac{5760}{7000}$.	22. $\frac{17526}{26145}$.
2. $\frac{105}{135}$.	9. $\frac{1848}{3003}$.	16. $\frac{875}{10000}$.	23. $\frac{44323}{61087}$.
3. $\frac{928}{1320}$.	10. $\frac{924}{1092}$.	17. $\frac{2208}{4140}$.	24. $\frac{339}{1243}$.
4. $\frac{1728}{2448}$.	11. $\frac{2640}{2970}$.	18. $\frac{1015}{1566}$.	25. $\frac{1177}{2675}$.
5. $\frac{1296}{6561}$.	12. $\frac{324}{1092}$.	19. $\frac{516}{2107}$.	26. $\frac{11445}{15369}$.
6. $\frac{2310}{3080}$.	13. $\frac{6732}{9108}$.	20. $\frac{3872}{92807}$.	27. $\frac{14141}{16289}$.
7. $\frac{1848}{2352}$.	14. $\frac{6840}{27360}$.	21. $\frac{78473}{94653}$.	28. $\frac{428571}{999999}$.

241. In practice, in the answers to all examples, unless the problem expressly demands the contrary, every fraction is left in its lowest terms, and every improper fraction is reduced to a whole or a mixed number.

Reduction of a Fraction to Higher Terms.

242. How many quarters are there in $\frac{1}{2}$? Which is greater, $\frac{1}{2}$ of an apple or $\frac{2}{4}$ of an apple?

How many sixths of an orange can be cut from $\frac{1}{2}$ of an orange? $\frac{1}{3}$ of an orange? $\frac{2}{3}$ of an orange?

Which is greater, $\frac{4}{5}$ or $\frac{8}{10}$? $\frac{3}{7}$ or $\frac{6}{14}$? $\frac{5}{8}$ or $\frac{15}{24}$? $\frac{7}{9}$ or $\frac{28}{36}$?

By what must we multiply both terms to change $\frac{5}{11}$ to $\frac{10}{22}$? $\frac{3}{5}$ to $\frac{15}{25}$? $\frac{5}{9}$ to $\frac{10}{18}$?

243. To Reduce a Fraction to Higher Terms,

Divide the required denominator by the denominator of the given fraction, and multiply both terms of the fraction by the quotient.

Exercise 49.

Reduce :

1. $\frac{3}{4}$ to 20ths.	4. $\frac{7}{13}$ to 39ths.	7. $\frac{3}{16}$ to 144ths.
2. $\frac{2}{3}$ to 24ths.	5. $\frac{5}{18}$ to 90ths.	8. $\frac{7}{18}$ to 144ths.
3. $\frac{3}{5}$ to 50ths.	6. $\frac{2}{9}$ to 108ths.	9. $\frac{7}{12}$ to 156ths.

Multiplication of Fractions.

244. Compound Fractions. A fraction *of* a whole number, of a mixed number, or of a fraction, is called a *ccmpound fraction.*

245. The expression $\frac{2}{3} \times \frac{1}{2}$ means the same as $\frac{2}{3}$ of $\frac{1}{2}$, and the sign $\times$ should be read *of* or *multiplied by* when it follows a fraction.

246. 7 times 3 horses are how many horses?
7 times three fifths ($\frac{3}{5}$) are how many fifths?
$\frac{3}{4}$ of 12 men are how many men?
$12 \times \$\frac{3}{4}$ are how many dollars?

247. To Find the Product of a Whole Number and a Fraction,

Divide the product of the numerator and the whole number by the denominator.

NOTE. A factor common to the whole number and the denominator should be cancelled. Thus,

$$\frac{3}{\cancel{5}} \text{ of } \overset{8}{\cancel{40}} = 24.$$

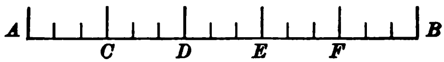
EXERCISE 50.

Find the product of:

1. $\frac{3}{4} \times 2$.
2. $\frac{3}{4} \times 9$.
3. $10 \times \frac{2}{5}$.
4. $15 \times \frac{3}{3}$.
5. $\frac{9}{21} \times 7$.
6. $16 \times \frac{5}{8}$.
7. $\frac{5}{8} \times 2$.
8. $\frac{2}{15} \times 5$.
9. $27 \times \frac{5}{9}$.
10. $\frac{13}{20} \times 2$.
11. $\frac{13}{20} \times 3$.
12. $\frac{13}{20} \times 4$.
13. $5 \times \frac{13}{20}$.
14. $6 \times \frac{13}{20}$.
15. $7 \times \frac{13}{20}$.
16. $8 \times \frac{13}{20}$.
17. $\frac{13}{20} \times 10$.
18. $\frac{13}{20} \times 12$.
19. $\frac{13}{20} \times 15$.
20. $\frac{13}{20} \times 20$.
21. $\frac{5}{36}$ of 324.
22. $\frac{7}{13}$ of 273.
23. $\frac{10}{11}$ of 242.
24. $340 \times \frac{8}{17}$.
25. $450 \times \frac{7}{10}$.
26. $\frac{6}{100} \times 1000$.
27. $\frac{9}{50} \times 210$.
28. $\frac{12}{25} \times 90$.
29. $\frac{5}{7}$ of 434.
30. $468 \times \frac{11}{9}$.
31. $30 \times \frac{12}{11}$.
32. $100 \times \frac{16}{15}$.
33. $\frac{25}{12} \times 54$.
34. $\frac{21}{32} \times 48$.
35. $72 \times \frac{19}{18}$.
36. $\frac{15}{32}$ of 128.

248. To Multiply a Fraction by a Fraction.

Multiply $\frac{4}{5}$ by $\frac{2}{3}$.



To multiply $\frac{4}{5}$ by $\frac{2}{3}$ means to find $\frac{2}{3}$ of $\frac{4}{5}$.

If the line AB is divided into 5 equal parts at the points C, D, E, and F, AF will be 4 of these parts, or $\frac{4}{5}$ of AB.

If each part is subdivided into three equal parts, there will be 15 of these smaller parts in the whole line, and each part will be $\frac{1}{15}$ of the line. Therefore, $\frac{1}{3}$ of $\frac{1}{5}$ is $\frac{1}{15}$ of the whole line.

Now $\frac{1}{3}$ of $\frac{4}{5}$ is 4 times as much as $\frac{1}{3}$ of $\frac{1}{5}$, or $\frac{4}{15}$ of the whole line.

And $\frac{2}{3}$ of $\frac{4}{5}$ is twice as much as $\frac{1}{3}$ of $\frac{4}{5}$, or $\frac{8}{15}$ of the whole line.

249. Therefore, we have the following

Rule. *Find the product of the numerators for the required numerator, and the product of the denominators for the required denominator.*

Note. Mixed numbers and whole numbers are brought under this rule by first reducing them to improper fractions.

Any factor common to a numerator and a denominator should be cancelled before multiplying.

Example. Find the product of $\frac{13}{15} \times 2\frac{8}{13} \times \frac{10}{17}$.

$$\frac{13}{15} \times 2\frac{8}{13} \times \frac{10}{17} = \frac{\cancel{13}}{\underset{3}{\cancel{15}}} \times \frac{\overset{2}{\cancel{34}}}{\cancel{13}} \times \frac{\overset{2}{\cancel{10}}}{\cancel{17}} = \frac{4}{3} = 1\frac{1}{3}.$$

We first reduce the mixed number $2\frac{8}{13}$ to an improper fraction and obtain $\frac{34}{13}$.

We cancel the 13 of the numerators and the 13 of the denominators. We cancel the common factor 17 of the 34 and the 17, and the common factor 5 of the 10 and the 15.

There remains in the numerators 2×2, and in the denominators 3, from which we obtain the improper fraction $\frac{4}{3}$.

Exercise 51.

Find the product of :

1. $\frac{3}{5}$ of $\frac{7}{11}$.
2. $\frac{3}{7}$ of $2\frac{1}{10}$.
3. $\frac{3}{7}$ of $\frac{5}{9}$.
4. $2\frac{2}{5} \times 2\frac{1}{2}$.
5. $4\frac{4}{5} \times 2\frac{1}{7}$.
6. $4\frac{5}{6} \times 9\frac{1}{3}$.
7. $\frac{1}{3}$ of $\frac{3}{5}$ of 10.
8. $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{2}{3}$.
9. $\frac{4}{5} \times \frac{5}{6} \times \frac{3}{7} \times 4\frac{1}{5}$.
10. $\frac{5}{6} \times 4\frac{1}{2}$.
11. $\frac{8}{9}$ of $\frac{9}{10}$ of $\frac{5}{7}$ of $\frac{3}{4}$ of $\frac{1}{5}$ of $15\frac{3}{4}$.
12. $5\frac{3}{4} \times 8\frac{2}{5}$.
13. $\frac{2}{3} \times \frac{4}{7} \times \frac{7}{15} \times 7\frac{1}{2}$.
14. $\frac{3}{5}$ of $\frac{10}{27}$ of $\frac{9}{20}$ of $8\frac{1}{3}$.
15. $\frac{8}{11} \times \frac{20}{21} \times \frac{35}{48} \times 2\frac{10}{13}$.
16. $\frac{42}{43} \times \frac{13}{105} \times 1\frac{7}{308}$.
17. $\frac{5}{8} \times \frac{120}{121} \times \frac{66}{85} \times 17$.
18. $\frac{38}{39} \times \frac{52}{57} \times \frac{62}{86} \times 1\frac{20}{23}$.
19. $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{5}{6}$ of $\frac{6}{7}$ of $\frac{7}{8}$ of $\frac{8}{9}$ of $\frac{9}{10}$ of 10.
20. $\frac{7}{25}$ of $\frac{8}{11}$ of 30.
21. $\frac{113}{355} \times \frac{85}{226} \times \frac{12}{35} \times 1\frac{3}{4}$.
22. $\frac{7}{8} \times \frac{3}{4} \times \frac{8}{11} \times \frac{4}{5}$ of $\frac{5}{6}$ of $\frac{3}{4}$ of 8.
23. $\frac{2}{13}$ of $\frac{39}{40}$ of $\frac{52}{117}$.
24. $\frac{9}{11} \times \frac{7}{12} \times \frac{22}{63} \times 48$.
25. $\frac{39}{40}$ of $\frac{7}{65}$ of $\frac{25}{42}$ of 12.
26. $1\frac{2}{3} \times 4\frac{1}{2} \times \frac{3}{8}$.
27. $2\frac{4}{9} \times 1\frac{3}{7} \times 1\frac{13}{15} \times 8$.
28. $3\frac{5}{7} \times 2\frac{1}{2}$ of $1\frac{8}{13} \times 1\frac{4}{11}$.
29. $\frac{11}{12} \times 5\frac{1}{3} \times 4\frac{1}{8} \times \frac{7}{22} \times 5$.
30. $\frac{2}{5}$ of $\frac{7}{15} \times 8\frac{2}{7} \times \frac{6}{29}$ of $1\frac{17}{18}$.
31. $\frac{17}{32} \times \frac{27}{35} \times \frac{123}{153}$.
32. $\frac{292}{465} \times \frac{573}{657} \times \frac{248}{1719}$.
33. $\frac{1205}{2763}$ of $\frac{712}{2169}$ of $\frac{1535}{1068}$.
34. $\frac{4}{25} \times 7\frac{1}{7} \times 6\frac{2}{3} \times \frac{21}{80}$.
35. $12\frac{1}{2} \times \frac{8}{15} \times 16\frac{2}{3} \times \frac{9}{50}$.
36. $37\frac{1}{2} \times \frac{12}{25} \times \frac{10}{11} \times \frac{11}{40}$.
37. $\frac{5}{16} \times \frac{9}{20} \times \frac{4}{21} \times 2\frac{1}{3}$.
38. $8\frac{1}{2} \times \frac{5}{12} \times 1\frac{1}{17} \times \frac{4}{5}$.
39. $62\frac{1}{2} \times \frac{9}{50} \times \frac{2}{3} \times 15$.
40. $\frac{8}{75} \times 87\frac{1}{2} \times \frac{3}{10} \times \frac{1}{7}$.
41. $1\frac{1}{9} \times 1\frac{1}{10} \times 3\frac{3}{11} \times \frac{1}{18}$.
42. $6\frac{3}{5} \times \frac{15}{22} \times \frac{8}{9} \times \frac{3}{8}$.
43. $\frac{7}{18}$ of $\frac{9}{14}$ of $\frac{35}{36}$ of $10\frac{2}{7}$.
44. $\frac{11}{68} \times 2\frac{19}{33} \times 1\frac{5}{9} \times 27$.
45. $2\frac{1}{28} \times 1\frac{32}{63} \times \frac{7}{180} \times 2\frac{4}{19}$.
46. $\frac{27}{44} \times 1\frac{29}{120} \times \frac{13}{60} \times 12\frac{2}{9}$.
47. $\frac{112}{119} \times 1\frac{3}{25} \times \frac{85}{98} \times \frac{7}{32}$.
48. $3\frac{1}{9} \times 2\frac{5}{36} \times 1\frac{5}{22} \times \frac{18}{35}$.
49. $1\frac{11}{25} \times \frac{22}{63} \times \frac{7}{18} \times 1\frac{1}{44}$.
50. $15\frac{3}{7} \times \frac{7}{36} \times \frac{19}{63} \times \frac{42}{57}$.
51. $\frac{162}{637} \times 9\frac{1}{9} \times \frac{1}{25} \times \frac{35}{54}$.
52. $1\frac{9}{247} \times 1\frac{28}{53} \times \frac{323}{432} \times \frac{117}{272}$.
53. $\frac{4}{9} \times \frac{3}{11} \times 6\frac{2}{7} \times 9\frac{3}{5} \times 2\frac{1}{2} \times 63 \times \frac{13}{144}$.
54. $6\frac{1}{2} \times 11\frac{3}{7} \times 16\frac{4}{11} \times \frac{2}{13}$ of $\frac{7}{80}$ of $\frac{1}{90}$.
55. $2\frac{3}{4} \times 7\frac{7}{11} \times 2 \times 1\frac{1}{3} \times \frac{3}{56} \times \frac{7}{27} \times \frac{27}{49}$.

250. To Find the Product of a Mixed Number and a Whole Number,

Find the product of the whole number and the fractional part of the mixed number, then the product of the whole number and the integral part, and add these products.

1. Multiply $7\frac{1}{8}$ by 9.

$$\begin{array}{r} 7\frac{1}{8} \\ 9 \\ \hline 64\frac{1}{8} \end{array}$$

2. Multiply 8 by $2\frac{2}{3}$.

$$\begin{array}{r} 8 \\ 2\frac{2}{3} \\ \hline 21\frac{1}{3} \end{array}$$

In Ex. 1, 9 times $\frac{1}{8}$ equals $\frac{9}{8}$, or $1\frac{1}{8}$. We write the $\frac{1}{8}$, and add the 1 to the product of 9×7, making 64.

In Ex. 2, $\frac{2}{3}$ of 8 equals $\frac{16}{3}$, or $5\frac{1}{3}$. We write the $\frac{1}{3}$, and add the 5 to the product of 2×8, making 21.

Exercise 52.

Find the product of :

1. $9 \times 6\frac{5}{8}$.
2. $8 \times 17\frac{1}{5}$.
3. $19 \times 5\frac{1}{4}$.
4. $7 \times 12\frac{1}{3}$.
5. $10 \times 15\frac{1}{2}$.
6. $6 \times 1\frac{7}{8}$.
7. $12 \times 2\frac{3}{4}$.
8. $17 \times 6\frac{1}{8}$.
9. $19 \times 1\frac{1}{19}$.
10. $24 \times 16\frac{5}{8}$.
11. $32 \times 22\frac{3}{8}$.
12. $40 \times 8\frac{1}{5}$.
13. $41 \times 9\frac{1}{6}$.
14. $18 \times 7\frac{4}{5}$.
15. $19 \times 6\frac{4}{13}$.
16. $20 \times 5\frac{1}{8}$.
17. $15 \times \$9\frac{1}{2}$.
18. $6 \times 8\frac{3}{5}$.
19. $11 \times 8\frac{4}{5}$.
20. $100 \times 6\frac{2}{3}$.
21. $5 \times 3\frac{1}{8}$.
22. $6 \times 17\frac{1}{7}$.
23. $32 \times 6\frac{2}{5}$.
24. $13 \times 3\frac{3}{7}$.
25. $12 \times 6\frac{3}{5}$.
26. $8\frac{2}{11} \times 12$.
27. $20\frac{1}{4} \times 5$.
28. $6\frac{2}{3} \times 18$.
29. $11 \times 11\frac{1}{9}$.
30. $18 \times 12\frac{4}{5}$.
31. $36 \times 4\frac{1}{6}$.
32. $12 \times 20\frac{2}{7}$.
33. $12 \times 48\frac{24}{25}$.
34. $11 \times 24\frac{4}{9}$.
35. $7 \times 19\frac{3}{5}$.
36. $8 \times 16\frac{1}{4}$.
37. $5 \times 29\frac{1}{6}$.
38. $16 \times 3\frac{3}{7}$.
39. $19 \times 12\frac{1}{17}$.
40. $23 \times 42\frac{5}{6}$.
41. $18 \times 12\frac{7}{8}$.
42. $22 \times 22\frac{1}{22}$.
43. $12 \times 161\frac{17}{19}$.
44. $9 \times 144\frac{1}{6}$.
45. $10 \times 112\frac{7}{9}$.
46. $14 \times 42\frac{5}{9}$.
47. $161 \times 4\frac{2}{3}$.
48. $140 \times 5\frac{7}{12}$.

Division of Fractions.

251. The *reciprocal* of a fraction is the fraction with its terms interchanged.

Thus, the reciprocal of $\frac{3}{7}$ is $\frac{7}{3}$, for $\frac{7}{3} \times \frac{3}{7} = 1$.

To write the reciprocal of a whole or a mixed number, we reduce the number to an improper fraction, and write the improper fraction with its terms interchanged.

Thus, the reciprocal of 4 is $\frac{1}{4}$; the reciprocal of $2\frac{2}{3}$ is $\frac{3}{8}$.

Multiplying by the reciprocal of a number gives the same result as dividing by the number (§101). Hence,

252. To Divide by a Whole Number or a Fraction,

Multiply the dividend by the reciprocal of the divisor.

EXERCISE 53.

Divide:

1. $\frac{24}{35}$ by 6.
2. $\frac{10}{11}$ by 5.
3. $\frac{3}{7}$ by 8.
4. $18\frac{2}{3}$ by 7.
5. $\frac{5}{8}$ by $\frac{3}{4}$.
6. $\frac{12}{16}$ by $\frac{3}{8}$.
7. $1\frac{3}{4}$ by $3\frac{1}{8}$.
8. $5\frac{1}{5}$ by $4\frac{2}{3}$.
9. $8\frac{2}{9}$ by $4\frac{1}{9}$.
10. $7\frac{1}{5}$ by $4\frac{2}{7}$.
11. $6\frac{3}{4}$ by $9\frac{1}{2}$.
12. $8\frac{2}{3}$ by $4\frac{2}{3}$.
13. $3\frac{8}{9}$ by $\frac{14}{27}$.
14. $4\frac{3}{7}$ by $6\frac{8}{9}$.
15. 5 by $4\frac{2}{7}$.
16. $4\frac{2}{3}$ by $\frac{7}{8}$.
17. $8\frac{3}{5}$ by $6\frac{1}{7}$.
18. $8\frac{4}{5}$ by $1\frac{1}{10}$.
19. 100 by $6\frac{2}{3}$.
20. $\frac{14}{15}$ by $\frac{12}{25}$.
21. $3\frac{1}{8}$ by 5.
22. 100 by $33\frac{1}{3}$.
23. 100 by $37\frac{1}{2}$.
24. $7\frac{1}{7}$ by $6\frac{1}{4}$.
25. $\frac{1}{9}$ by $\frac{1}{11}$.
26. $6\frac{2}{5}$ by 32.
27. $3\frac{1}{7}$ by $3\frac{3}{7}$.
28. $1\frac{7}{15}$ by $\frac{11}{25}$.
29. $11\frac{1}{5}$ by $\frac{8}{9}$.
30. 100 by $83\frac{1}{3}$.
31. 50 by $16\frac{2}{3}$.
32. $\frac{15}{22}$ by $1\frac{1}{2}$.
33. $1\frac{13}{38}$ by $1\frac{2}{19}$.
34. $20\frac{1}{4}$ by 5.
35. $16\frac{2}{3}$ by $\frac{4}{9}$.
36. $22\frac{2}{9}$ by $16\frac{2}{3}$.
37. $20\frac{5}{9}$ by $1\frac{10}{27}$.
38. $16\frac{2}{3}$ by $11\frac{1}{9}$.
39. $33\frac{1}{3}$ by $28\frac{4}{7}$.
40. $47\frac{7}{9}$ by $17\frac{1}{5}$.
41. $18\frac{4}{7}$ by $1\frac{5}{21}$.
42. $37\frac{4}{5}$ by $1\frac{4}{17}$.
43. $3\frac{4}{5}$ of $2\frac{1}{2}$ by $1\frac{1}{2}$ of $2\frac{1}{9}$.
44. $2\frac{2}{7}$ by $3\frac{1}{8}$ of $1\frac{1}{15}$.
45. $2\frac{9}{11}$ of $5\frac{1}{2}$ by $7\frac{3}{4}$.
46. $5\frac{3}{5}$ of $8\frac{1}{8}$ of $1\frac{4}{7}$ by $2\frac{1}{10}$ of $5\frac{5}{9}$.

A Short Method of Dividing a Mixed Number by a Whole Number.

253. 1. Divide $16\frac{4}{5}$ by 4. 2. Divide $16\frac{2}{3}$ by 7.

$$4)\underline{16\frac{4}{5}} \qquad\qquad 7)\underline{16\frac{2}{3}}$$
$$\quad 4\frac{1}{5} \qquad\qquad\quad 2\frac{8}{21}$$

In the first problem we simply divide the whole number 16 by 4, and then the fraction $\frac{4}{5}$ by 4, and obtain the result at once, $4\frac{1}{5}$.

In the second problem we divide the 16 by 7, and obtain the quotient 2 and a remainder 2. The remainder 2 is joined with the $\frac{2}{3}$, making $2\frac{2}{3}$, or $\frac{8}{3}$, and $\frac{8}{3} \div 7 = \frac{8}{21}$.

This method is the shortest method of dividing a mixed number by a whole number.

Exercise 54.

Find the quotient of :

1. $31\frac{7}{8} \div 5$.	5. $42\frac{6}{7} \div 6$.	9. $48\frac{24}{25} \div 12$.
2. $16\frac{7}{9} \div 6$.	6. $49\frac{4}{5} \div 7$.	10. $24\frac{4}{9} \div 11$.
3. $14\frac{2}{7} \div 2$.	7. $52\frac{4}{7} \div 8$.	11. $19\frac{3}{5} \div 7$.
4. $33\frac{1}{3} \div 7$.	8. $44\frac{4}{11} \div 12$.	12. $29\frac{1}{6} \div 8$.

254. Examples. 1. Find the value of $(2\frac{1}{2} \div \frac{5}{6}) \times \frac{3}{8}$.

$$(2\frac{1}{2} \div \frac{5}{6}) \times \frac{3}{8} = \frac{\cancel{5}}{\cancel{2}} \times \frac{\overset{3}{\cancel{6}}}{\cancel{5}} \times \frac{3}{8} = \frac{9}{8} = 1\frac{1}{8}.$$

We have to divide $2\frac{1}{2}$ by $\frac{5}{6}$, and to multiply the result by $\frac{3}{8}$. But $2\frac{1}{2}$ is divided by $\frac{5}{6}$ by multiplying $2\frac{1}{2}$ by $\frac{6}{5}$. Hence, we invert the divisor $\frac{5}{6}$ and find the product of the three fractions.

2. Find the value of $2\frac{1}{2} \div \frac{5}{6}$ of $\frac{3}{8}$.

$$2\frac{1}{2} \div \frac{5}{6} \text{ of } \frac{3}{8} = \frac{\cancel{5}}{\cancel{2}} \times \frac{\overset{\cancel{3}}{\cancel{6}}}{\cancel{5}} \times \frac{8}{\cancel{3}} = 8.$$

We have to divide $2\frac{1}{2}$ by $\frac{5}{6}$ of $\frac{3}{8}$; hence we may find the product of $\frac{5}{6}$ and $\frac{3}{8}$, and invert this product, or we may invert both factors of this product and multiply by the inverted factors $\frac{6}{5}$ and $\frac{8}{3}$.

Exercise 55.

Find the value of :

1. $2\frac{1}{5}$ of $2\frac{1}{2} \div \frac{3}{14}$ of $3\frac{2}{3}$.
2. $\frac{5}{8}$ of $6\frac{2}{3}$ of $\frac{6}{25} \div 5\frac{1}{2}$.
3. $\frac{3}{10} \div \frac{2}{5}$ of $2\frac{1}{4}$ of $1\frac{5}{7}$.
4. $\frac{3}{10} \div (\frac{2}{5} \times 2\frac{1}{4} \times 1\frac{5}{7})$.
5. $\frac{7}{5}$ of $\frac{15}{16} \div 1\frac{1}{27}$ of $1\frac{1}{35}$.
6. $\frac{3}{4}$ of $\frac{5}{6} \div (\frac{5}{8} \times \frac{4}{11})$.
7. $\frac{3}{7}$ of $\frac{14}{27} \div \frac{11}{13}$ of $\frac{26}{27}$.
8. $\frac{3}{8}$ of $\frac{32}{33} \div \frac{18}{19}$ of $\frac{76}{81}$.
9. $\frac{2}{7}$ of $1\frac{7}{18} \div \frac{27}{43}$ of $\frac{63}{81}$.
10. $\frac{4}{7}$ of $\frac{23}{30} \div \frac{8}{35}$ of 4.
11. $\frac{9}{10}$ of $\frac{110}{111} \div \frac{3}{4}$ of $1\frac{1}{11}$.
12. $\frac{2}{5}$ of $\frac{26}{27}$ of $\frac{5}{13} \div (\frac{1}{2} \times \frac{3}{4}$ of $\frac{4}{5})$.
13. $\frac{2}{5}$ of $\frac{5}{6}$ of $\frac{16}{25} \div \frac{4}{3}$ of $\frac{1}{15}$ of $1\frac{7}{11}$.
14. $(\frac{7}{9} \div 1\frac{1}{8}) \div (5\frac{6}{17} \div 4\frac{33}{34})$.
15. $(14\frac{2}{3} \div 4\frac{8}{9}) \div (3\frac{1}{12} \div 9\frac{2}{5})$.
16. $\frac{3}{5}$ of $\frac{10}{33}$ of $8\frac{1}{4} \div 3\frac{1}{11}$ of $\frac{1}{17}$ of $5\frac{1}{2}$.

255. Example. If $\frac{3}{4}$ of a barrel of flour costs \$3, what is the cost of $\frac{1}{4}$ of a barrel? What is the cost of a barrel?

Solution. Since $\frac{3}{4}$ of a barrel of flour costs \$3, $\frac{1}{4}$ of a barrel will cost $\frac{1}{3}$ of \$3, or \$1. If $\frac{1}{4}$ of a barrel costs \$1, a barrel will cost $4 \times$ \$1, or \$4. Hence,

256. To Find the Whole when a Fractional Part is Given,

Divide the given part by the numerator of the fraction and multiply the quotient by the denominator.

Exercise 56.

1. If $\frac{5}{8}$ of a ton of hay costs \$15, what is the cost of one ton?
2. 15 is $\frac{5}{8}$ of what number?
3. If $\frac{5}{7}$ of a roll of carpeting is worth \$75, what is the whole roll worth?
4. A man sold $6\frac{2}{3}$ yards of cloth, which was $\frac{4}{15}$ of the whole piece. How many yards were there in the piece?
5. A farmer sold $\frac{3}{7}$ of his hay for \$195.60. What was the value of his entire crop of hay?

6. $21\frac{2}{3}$ is $\frac{13}{17}$ of what number?

7. $6\frac{2}{3}$ is $\frac{14}{27}$ of what number?

8. $2\frac{13}{22}$ is $\frac{10}{11}$ of what number?

9. If $\frac{3}{7}$ of an acre of land is worth $32, what is the value of an acre?

10. If $\frac{4}{5}$ of a bushel of wheat is worth 48 cents, what is the value of $2\frac{7}{12}$ bushels of wheat?

11. If $\frac{5}{7}$ of a ton of hay is worth $15, what is the value of $7\frac{1}{3}$ tons of hay?

12. If $\frac{5}{8}$ of a cord of wood is worth $4, find the value of 7 cords of wood?

13. If $\frac{4}{11}$ of a barrel of apples is worth 44 cents, what is the value of 12 barrels of apples?

14. $125 is $\frac{1}{4}$ more than (that is, $\frac{5}{4}$ of) what sum of money?

15. $132 is $\frac{1}{4}$ less than what sum of money?

16. 495 is $\frac{1}{8}$ more than what number?

17. 217 is $\frac{1}{8}$ less than what number?

18. 495 is $\frac{2}{13}$ less than what number?

19. 495 is $\frac{2}{13}$ more than what number?

20. If $\frac{4}{5}$ of a yard of silk is worth $1, find the value of 4 yards of silk.

21. If $\frac{2}{3}$ of a yard of linen is worth 60 cents, what is the value of $2\frac{1}{3}$ yards of linen?

22. If a man who owned $\frac{1}{3}$ of a schooner sold $\frac{3}{4}$ of his share for $1200, what was the value of the schooner?

23. One fourth of one third of three sevenths of a number is 60. What is the number?

24. Three fourths of two ninths of six sevenths of a number is $12\frac{5}{7}$. What is the number?

25. If $\frac{5}{16}$ of the goods in a store were sold for $1000, what was the value of the whole stock of goods?

26. If $\frac{5}{32}$ of a farm is worth $1200, what is the value of the whole farm?

Least Common Denominator.

257. Similar Fractions. Fractions that have a common denominator are called *similar fractions.*

258. Least Common Denominator. The L. C. M. of the denominators of a series of fractions is called their *least common denominator.* The letters L. C. D. stand for the words Least Common Denominator.

259. Example. Change $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$ to similar fractions.

The L. C. D. of the fractions is 12.

If both terms of $\frac{2}{3}$ are multiplied by 4, the value of the fraction will not be altered, but the form will be changed to $\frac{8}{12}$.

If both terms of $\frac{3}{4}$ are multiplied by 3, the equivalent fraction will be $\frac{9}{12}$.

And if both terms of $\frac{5}{6}$ are multiplied by 2, the equivalent fraction will be $\frac{10}{12}$.

The multipliers, 4, 3, and 2, are obtained by dividing 12, the L. C. D. of the fractions, by the respective denominators of the given fractions. Therefore,

260. To Change Fractions to Similar Fractions,

Divide the least common denominator of the fractions by the denominator of the first fraction, and multiply both terms of this fraction by the quotient. Proceed in the same way with each of the other given fractions.

Change $\frac{3}{4}$, $\frac{7}{8}$, $\frac{11}{12}$ to similar fractions.

$$4 = 2^2;\ 8 = 2^3;\ 12 = 2^2 \times 3.$$

Hence, the L. C. D. is $2^3 \times 3$, or 24.

$$\frac{3}{4} = \frac{18}{24},$$
$$\frac{7}{8} = \frac{21}{24},$$
$$\text{and } \frac{11}{12} = \frac{22}{24}.$$

The result may be written as follows:

$$\frac{18 \quad 21 \quad 22}{24}$$

By this operation the parts represented by the given fractions have been subdivided into smaller parts all of *one size*, and the numerators of the resulting fractions show the *number* of these smaller parts contained in the given fractions. Thus, the quantities denoted by $\frac{3}{4}$, $\frac{7}{8}$, and $\frac{11}{12}$ are each subdivided into 24ths of the unit, and contain respectively 18, 21, and 22 of these subdivisions.

261. *Fractions may be compared by first reducing them to equivalent fractions having a common denominator.*

Determine the greater of the fractions $\frac{5}{7}$ and $\frac{11}{16}$.

In this case the least common denominator is 112.

Hence, $\frac{5}{7} = \frac{80}{112}$, and $\frac{11}{16} = \frac{77}{112}$.

Therefore, $\frac{5}{7}$ is greater than $\frac{11}{16}$.

Exercise 57.

Change to similar fractions :

1. $\frac{1}{2}, \frac{2}{3}, \frac{5}{6}$.
2. $\frac{2}{3}, \frac{5}{9}, \frac{7}{8}, \frac{9}{10}$.
3. $\frac{5}{6}, \frac{1}{8}, \frac{5}{21}, \frac{19}{35}$.
4. $\frac{2}{15}, \frac{7}{20}, \frac{3}{25}, \frac{8}{45}$.
5. $\frac{12}{25}, \frac{17}{40}, \frac{13}{60}, \frac{19}{75}$.
6. $\frac{3}{8}, \frac{7}{30}, \frac{4}{35}, \frac{3}{28}, \frac{19}{24}$.
7. $\frac{11}{16}, \frac{7}{18}, \frac{13}{20}, \frac{23}{30}, \frac{17}{54}$.
8. $\frac{4}{5}, \frac{8}{9}, \frac{11}{12}, \frac{13}{15}$.
9. $\frac{5}{6}, \frac{5}{18}, \frac{13}{24}, \frac{19}{30}$.
10. $\frac{7}{8}, \frac{17}{24}, \frac{19}{32}, \frac{11}{48}$.
11. $\frac{2}{3}, \frac{5}{6}, \frac{7}{12}, \frac{15}{16}$.
12. $\frac{2}{7}, \frac{3}{14}, \frac{5}{18}, \frac{7}{9}, \frac{2}{21}$.
13. $\frac{3}{8}, \frac{3}{4}, \frac{3}{16}, \frac{3}{64}, \frac{3}{256}$.
14. $\frac{3}{5}, \frac{7}{15}, \frac{2}{9}, \frac{11}{24}, \frac{7}{8}, \frac{17}{45}$.
15. $\frac{2}{3}, \frac{3}{4}, \frac{5}{7}, \frac{7}{12}, \frac{13}{18}, \frac{4}{27}$.
16. $\frac{11}{12}, \frac{9}{10}, \frac{14}{15}, \frac{5}{6}, \frac{17}{20}, \frac{29}{30}$.
17. $\frac{1}{2}, \frac{3}{4}, \frac{5}{8}, \frac{6}{11}, \frac{7}{44}, \frac{9}{22}$.
18. $\frac{9}{14}, \frac{7}{10}, \frac{13}{28}, \frac{17}{70}, \frac{3}{4}, \frac{31}{56}$.
19. Which is the greater, $\frac{13}{20}$ or $\frac{17}{25}$? $\frac{5}{6}$ or $\frac{7}{9}$? $\frac{3}{5}$ or $\frac{7}{12}$?
20. Arrange the fractions $\frac{7}{12}, \frac{11}{18}, \frac{13}{24}$ in order of magnitude.
21. Arrange the fractions $\frac{5}{12}, \frac{8}{15}, \frac{4}{11}, \frac{7}{18}$ in order of magnitude.
22. Arrange the fractions $\frac{3}{7}, \frac{4}{9}, \frac{9}{19}, \frac{10}{23}$ in order of magnitude.

Addition of Fractions.

262. Examples. 1. Find the sum of $\frac{5}{12}$, $\frac{1}{12}$, $\frac{7}{12}$.

$$\tfrac{5}{12} + \tfrac{1}{12} + \tfrac{7}{12} = \frac{5+1+7}{12} = \tfrac{13}{12} = 1\tfrac{1}{12}.$$

2. Find the sum of $\frac{3}{4}$, $\frac{5}{6}$, $\frac{7}{9}$, $\frac{5}{12}$.

$$\text{Denominators} \ldots \begin{cases} 4 = 2^2, \\ 6 = 2 \times 3, \\ 9 = 3^2, \\ 12 = 2^2 \times 3. \end{cases}$$

Hence, the L. C. D. $= 2^2 \times 3^2$, or 36.

$$\begin{aligned} \tfrac{3}{4} + \tfrac{5}{6} + \tfrac{7}{9} + \tfrac{5}{12} &= \tfrac{27+30+28+15}{36}, \\ &= \tfrac{100}{36}, \\ &= 2\tfrac{28}{36}, \\ &= 2\tfrac{7}{9}. \end{aligned}$$

3. Find the sum of $4\frac{11}{20}$, $2\frac{7}{15}$, $5\frac{5}{12}$.

$$\text{Denominators} \ldots \begin{cases} 20 = 2^2 \times 5, \\ 15 = 3 \times 5, \\ 12 = 2^2 \times 3. \end{cases}$$

Hence, the L. C. D. $= 2^2 \times 3 \times 5 = 60$.

$$\begin{aligned} 4\tfrac{11}{20} + 2\tfrac{7}{15} + 5\tfrac{5}{12} &= 11\tfrac{33+28+25}{60}, \\ &= 11\tfrac{86}{60}, \\ &= 12\tfrac{26}{60}, \\ &= 12\tfrac{13}{30}. \quad \text{Hence,} \end{aligned}$$

263. To Add Fractions,

Change the fractions to similar fractions, if they are not similar, and write the sum of their numerators over their common denominator.

If any of the expressions to be added are whole or mixed numbers, add together separately the fractions and the integers, and find the sum of the results.

Exercise 58.

Find the sum of :

1. $\frac{1}{2} + \frac{3}{2}$.
2. $\frac{1}{3} + \frac{2}{3} + \frac{1}{3}$.
3. $\frac{1}{4} + \frac{1}{4} + \frac{3}{4}$.
4. $1\frac{1}{2} + 2\frac{1}{2}$.
5. $1\frac{1}{3} + 2\frac{2}{3}$.
6. $3\frac{1}{4} + \frac{3}{4}$.
7. $2\frac{3}{5} + 3\frac{4}{5}$.
8. $1\frac{7}{8} + \frac{3}{8}$.
9. $\frac{9}{17} + \frac{3}{17} + \frac{14}{17} + \frac{11}{17}$.
10. $8\frac{9}{17} + 6\frac{3}{17} + 5\frac{14}{17} + \frac{11}{17}$.
11. $\frac{4}{5} + \frac{5}{6}$.
12. $\frac{3}{4} + \frac{7}{8}$.
13. $\frac{1}{2} + \frac{1}{6}$.
14. $\frac{4}{15} + \frac{11}{20}$.
15. $\frac{5}{16} + \frac{11}{24}$.
16. $12\frac{5}{8} + 7\frac{3}{16}$.
17. $85\frac{7}{12} + 27\frac{11}{18}$.
18. $\frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5}$.
19. $\frac{1}{2} + \frac{2}{3} + \frac{3}{4} + \frac{4}{5}$.
20. $\frac{5}{6} + \frac{11}{12} + \frac{8}{15} + \frac{7}{20} + \frac{13}{30}$.
21. $5\frac{17}{20} + 11\frac{19}{30} + 24\frac{21}{40} + \frac{9}{50} + 17\frac{8}{15} + 14 + 11\frac{5}{12}$.
22. $9\frac{4}{7} + 15\frac{11}{28} + 163\frac{17}{63} + 1\frac{11}{42} + 10\frac{1}{4}$.
23. $3\frac{3}{5} + 4\frac{2}{3} + 1\frac{5}{6} + 2$.
24. $1\frac{3}{20} + 2\frac{2}{25} + 5\frac{7}{30} + \frac{4}{15}$.
25. $\frac{2}{7} + 1\frac{4}{9} + 2 + 3\frac{3}{8} + 4\frac{5}{12}$.
26. $4\frac{4}{9} + 3\frac{3}{8} + 2\frac{2}{7} + 1\frac{1}{6} + \frac{9}{14}$.
27. $\frac{11}{35} + \frac{7}{40} + 10 + \frac{23}{60}$.
28. $\frac{27}{50} + \frac{29}{60} + \frac{31}{80} + \frac{33}{100} + \frac{37}{240}$.
29. $2 + \frac{2}{3} + 1\frac{3}{4} + 4\frac{8}{9} + 5\frac{13}{24}$.
30. $3\frac{5}{8} + 6 + \frac{4}{11} + 2\frac{3}{10} + 5\frac{5}{16} + \frac{9}{20}$.
31. $\frac{8}{15} + \frac{7}{18} + 3\frac{17}{20} + 1\frac{19}{24} + 2\frac{19}{120}$.
32. $\frac{5}{14} + \frac{6}{11} + 9\frac{1}{2}$.
33. $20\frac{5}{12} + 11\frac{7}{20} + 5\frac{1}{8} + 305$.
34. $\frac{11}{38} + \frac{14}{57} + \frac{17}{76}$.
35. $\frac{5}{17} + \frac{11}{34} + \frac{14}{51} + \frac{19}{68}$.
36. $317\frac{2}{5} + 17\frac{3}{51} + 4\frac{9}{10} + \frac{7}{15} + 6\frac{2}{3} + \frac{5}{17}$.
37. $4\frac{7}{15} + 8\frac{5}{21} + 4\frac{7}{11} + 5\frac{2}{7} + 5\frac{4}{5} + \frac{2}{3}$.
38. $3\frac{2}{3} + 5\frac{3}{40} + 8\frac{7}{240} + \frac{39}{60} + 1\frac{29}{2880}$.
39. $4\frac{5}{13} + 7\frac{5}{39} + 5\frac{47}{78} + 275\frac{37}{156} + 2\frac{57}{91}$.
40. $\frac{17}{33} + 7\frac{5}{12} + 6\frac{2}{11} + 400\frac{3}{28} + 51\frac{25}{56}$.
41. $3\frac{2}{3} + 1\frac{1}{4} + 2\frac{1}{9} + 3\frac{5}{6} + 107\frac{5}{18} + 2\frac{7}{36}$.
42. $5\frac{1}{14} + 5\frac{3}{5} + 2\frac{1}{7} + 7\frac{8}{21} + 12\frac{4}{15}$.
43. $4\frac{3}{4} + 2\frac{1}{8} + 3\frac{3}{8} + 7\frac{1}{6} + 8\frac{15}{16}$.
44. $6\frac{1}{2} + 7\frac{2}{3} + 8\frac{3}{4} + 9\frac{5}{6} + 8\frac{11}{18}$.
45. $7\frac{5}{8} + 8\frac{3}{4} + 5\frac{15}{16} + 7\frac{11}{12} + 9\frac{1}{2}$.
46. $5\frac{1}{2} + 6\frac{2}{3} + 7\frac{11}{24} + 9\frac{17}{48} + 3\frac{11}{16} + 2\frac{1}{6}$.
47. $9\frac{3}{4} + 10\frac{6}{7} + 11\frac{2}{3} + 5\frac{17}{42} + 7\frac{8}{21} + 18\frac{5}{6}$.
48. $\frac{13}{21} + \frac{3}{49} + \frac{16}{35} + \frac{11}{63} + \frac{4}{7} + \frac{4}{15} + \frac{17}{45}$.

Subtraction of Fractions.

264. Example. From $\frac{17}{24}$ take $\frac{5}{18}$.

The L. C. D. of the fractions is 72.

$$\tfrac{17}{24} - \tfrac{5}{18} = \tfrac{51-20}{72} = \tfrac{31}{72}.$$ Hence,

265. To Subtract One Fraction from Another,

Change the fractions to similar fractions, if they are not similar.

Subtract the numerator of the subtrahend from the numerator of the minuend, and write the result over their common denominator.

266. Examples. 1. Subtract $3\frac{11}{24}$ from $4\frac{11}{15}$.

The L. C. D. of the fractions is 120.

$$4\tfrac{11}{15} - 3\tfrac{11}{24} = 1\tfrac{88-55}{120} = 1\tfrac{33}{120} = 1\tfrac{11}{40}.$$

If the terms are mixed numbers, subtract *separately* the integers and the fractions, and unite the results.

2. Subtract $2\frac{7}{8}$ from $5\frac{5}{12}$.

$$5\tfrac{5}{12} - 2\tfrac{7}{8} = 3\tfrac{10-21}{24} = 2\tfrac{34-21}{24} = 2\tfrac{13}{24}.$$

The difference between $5\frac{5}{12}$ and $2\frac{7}{8}$ is $3\frac{10-21}{24}$. Since we cannot subtract $\frac{21}{24}$ from $\frac{10}{24}$, we take 1, that is, $\frac{24}{24}$, from the 3, and add it to $\frac{10}{24}$, making $\frac{34}{24}$.

3. From 8 take $2\frac{31}{34}$.

$$8 = 7\tfrac{34}{34}, \text{ and } 7\tfrac{34}{34} - 2\tfrac{31}{34} = 5\tfrac{34-31}{34} = 5\tfrac{3}{34}.$$

4. Subtract $5\frac{7}{8}$ from $15\frac{3}{4}$.

$$5\tfrac{7}{8} + \tfrac{1}{8} = 6, \text{ and } 15\tfrac{3}{4} + \tfrac{1}{8} = 15\tfrac{7}{8}.$$

Then $15\frac{7}{8} - 6 = 9\frac{7}{8}$.

Since adding the same number to both the minuend and the subtrahend does not alter the difference between them, we may add to the subtrahend such a fraction as will make it a whole number, provided we add the same fraction to the minuend.

EXERCISE 59.

Find the value of:

1. $52\frac{1}{8} - 46$.
2. $\frac{8}{9} - \frac{3}{8}$.
3. $\frac{3}{4} - \frac{2}{3}$.
4. $\frac{8}{15} - \frac{5}{12}$.
5. $\frac{11}{18} - \frac{3}{14}$.
6. $4 - \frac{1}{2}$.
7. $7 - \frac{2}{3}$.
8. $3 - \frac{5}{6}$.
9. $8 - \frac{3}{7}$.
10. $5 - \frac{4}{5}$.
11. $5 - \frac{7}{9}$.
12. $6\frac{1}{3} - 5\frac{1}{6}$.
13. $4\frac{2}{5} - 3\frac{3}{7}$.
14. $7\frac{1}{3} - 2\frac{3}{10}$.
15. $7\frac{2}{5} - 4\frac{8}{9}$.
16. $6\frac{2}{3} - 2\frac{3}{4}$.
17. $9\frac{4}{5} - 4\frac{5}{6}$.
18. $4\frac{2}{3} - \frac{1}{2}$.
19. $6\frac{3}{4} - 4\frac{2}{3}$.
20. $7\frac{1}{2} - 2\frac{3}{4}$.
21. $8\frac{1}{5} - 4\frac{4}{7}$.
22. $85\frac{7}{22} - 27\frac{11}{18}$.
23. $8\frac{7}{10} - 2\frac{11}{16}$.
24. $10 - 3\frac{5}{8}$.
25. $120\frac{21}{32} - 110\frac{13}{24}$.
26. $5\frac{17}{28} - \frac{27}{35}$.
27. $13\frac{3}{40} - 2\frac{15}{44}$.
28. $2\frac{151}{240} - 1\frac{163}{192}$.
29. $4 - 1\frac{2317}{4000}$.
30. $1473 - 279\frac{11}{12}$.
31. $1473\frac{5}{13} - 279\frac{11}{12}$.
32. $1473\frac{7}{18} - 279\frac{11}{12}$.
33. $278\frac{15}{16} - 30\frac{5}{12}$.
34. $125\frac{5}{22} - 10\frac{17}{33}$.
35. $118\frac{5}{11} - 17\frac{3}{14}$.
36. $94\frac{5}{11} - 91\frac{13}{14}$.
37. $7\frac{5}{21} - 2\frac{11}{14}$.
38. $\frac{235}{357} - \frac{13}{51}$.
39. $\frac{17}{63} - \frac{29}{108}$.
40. $\frac{9}{38} - \frac{43}{209}$.
41. $\frac{146}{273} - \frac{268}{637}$.
42. $\frac{359}{360} - \frac{199}{200}$.

Plus and Minus Terms.

267. Example. Simplify $5\frac{4}{5} - 4\frac{3}{4} + 3\frac{2}{3} - 2\frac{7}{10}$.

$$5\frac{4}{5} + 3\frac{2}{3} = 8\frac{12+10}{15} = 8\frac{22}{15} = 9\frac{7}{15},$$

$$4\frac{3}{4} + 2\frac{7}{10} = 6\frac{15+14}{20} = 6\frac{29}{20} = 7\frac{9}{20},$$

$$\text{and } 9\frac{7}{15} - 7\frac{9}{20} = 2\frac{28-27}{60} = 2\frac{1}{60}.$$

We first add the two plus terms and obtain $9\frac{7}{15}$.
Then we add the two minus terms and obtain $7\frac{9}{20}$.
Then we subtract the sum of the minus terms from the sum of the plus terms and obtain $2\frac{1}{60}$. Hence,

268. To Simplify an Expression of Plus and Minus Terms,

Subtract the sum of the minus terms from the sum of the plus terms.

Exercise 60.

Simplify :

1. $3\frac{2}{5} - 2\frac{5}{8} + 4\frac{3}{10} + 1\frac{7}{9} - 5\frac{8}{15}$.
2. $1\frac{5}{11} - \frac{11}{12} + 7\frac{3}{8} - 2\frac{1}{3} - 1\frac{11}{16}$.
3. $12 - 3\frac{2}{7} - 1\frac{3}{10} - 4\frac{5}{28} + 2\frac{13}{20} - 4\frac{3}{5}$.
4. $43\frac{7}{15} - 1\frac{1}{3} - 1\frac{31}{48} - 1\frac{23}{24} - 2\frac{13}{48} - 2\frac{7}{12} - 2\frac{43}{48} - 3\frac{5}{12}$.
5. $\frac{1}{2} + \frac{4}{13} + 7\frac{9}{40} + 8\frac{14}{39} + 7\frac{1}{4} + 8\frac{3}{10} + 4\frac{1}{12} - 36\frac{1}{40}$.
6. $(8\frac{5}{18} + 1\frac{19}{27} + 17\frac{11}{36} + 40) - (30\frac{13}{40} + 11\frac{11}{20})$.
7. $(172\frac{19}{78} + 93\frac{14}{117}) + (172\frac{19}{78} - 93\frac{14}{117})$.
8. $(172\frac{19}{78} + 93\frac{14}{117}) - (172\frac{19}{78} - 93\frac{14}{117})$.
9. $(\frac{3}{13} - \frac{2}{39}) + (\frac{5}{78} + \frac{7}{156})$.
10. $\frac{4}{9} - \frac{3}{11} - 2\frac{3}{4} + 3\frac{2}{3} + 7\frac{7}{12} - 1\frac{3}{5} - \frac{3}{22}$.
11. $\frac{3}{10} - \frac{7}{100} - \frac{9}{1000} - \frac{5}{10000}$.
12. $9\frac{5}{8} - 7 - \frac{3}{4} - \frac{5}{6}$.
13. $5\frac{2}{3} + 8\frac{3}{4} - 1\frac{3}{5} - 4\frac{7}{9}$.
14. $6\frac{3}{4} - 5\frac{2}{3} + 4\frac{2}{5} - 4\frac{5}{12}$.
15. $14\frac{7}{18} + 9\frac{3}{5} - 6\frac{3}{4} - 12\frac{4}{5} - 3\frac{2}{5}$.
16. $20\frac{2}{3} - 2\frac{5}{8} - 9\frac{5}{9} + 10\frac{3}{10} - 14\frac{7}{12}$.
17. $95\frac{2}{3} - 9\frac{7}{20} - 8\frac{3}{7} - 14\frac{2}{5} + 74\frac{2}{9}$.
18. $12\frac{3}{4} + 23\frac{3}{8} - (4\frac{3}{10} + 12\frac{2}{5} + 7\frac{14}{15})$.
19. $16\frac{2}{15} + 18\frac{5}{24} - (5\frac{2}{5} + 9\frac{9}{40} + 14\frac{5}{24})$.
20. $97\frac{8}{9} - (20 + 9\frac{3}{4} + 18\frac{2}{25} + 24\frac{49}{50})$.
21. $2\frac{13}{16} + 3\frac{15}{36} - (1\frac{53}{70} + 1\frac{31}{32} + \frac{49}{56})$.
22. $\frac{143}{100} + \frac{2471}{1000} - \frac{82643}{100000}$.

Complex Fractions.

269. Complex Fractions. A fraction that has a fraction in one or both of its terms is called a *complex fraction.*

Thus, $\frac{\frac{3}{4}}{7}$, $\frac{\frac{2}{3}}{\frac{5}{6}}$, $\frac{3\frac{1}{2}}{4}$, $\frac{2 + 3\frac{5}{6}}{1 - \frac{7}{8}}$, $\frac{6 \div 1\frac{1}{6}}{4 \times \frac{7}{12}}$ are complex fractions.

The simplest meaning to give to a complex fraction is that of an *indicated* division. Hence,

270. To Simplify a Complex Fraction,

Multiply its numerator by the reciprocal of its denominator.

271. Examples. 1. $\dfrac{3\frac{4}{5}}{20}=\dfrac{19}{5}\div 20=\dfrac{19}{5}\times\dfrac{1}{20}=\dfrac{19}{100}.$

2. $\dfrac{2\frac{5}{6}}{4\frac{6}{7}}=\dfrac{17}{6}\div\dfrac{34}{7}=\dfrac{\cancel{17}}{6}\times\dfrac{7}{\underset{2}{\cancel{34}}}=\dfrac{7}{12}.$

3. $\dfrac{2\frac{2}{3}-1\frac{5}{9}}{1\frac{5}{6}-1\frac{1}{8}}.$

$$2\frac{2}{3}-1\frac{5}{9}=1\frac{6-5}{9}=1\frac{1}{9};\quad 1\frac{5}{6}-1\frac{1}{8}=\frac{20-3}{24}=\frac{17}{24};$$

$$\text{and } 1\frac{1}{9}\div\frac{17}{24}=\frac{10}{\underset{3}{\cancel{9}}}\times\frac{\overset{8}{\cancel{24}}}{17}=\frac{80}{51}=1\frac{29}{51}.$$

4. $\dfrac{4\frac{7}{9}-3\frac{1}{2}-2\frac{1}{3}+1\frac{7}{18}}{3\frac{5}{9}-2\frac{2}{3}+2\frac{1}{2}-\frac{7}{18}}=\dfrac{86-63-42+25}{64-48+45-7}=\dfrac{6}{54}=\dfrac{1}{9}.$

A complex fraction may be simplified by multiplying both its terms by the smallest number that will make them integral. This multiplier will always be the L. C. D. of the fractions contained in the terms of the given fraction.

Here each term of the numerator and denominator is made integral by multiplying it by 18, the L. C. D. of the fractions contained in the numerator and denominator of the given fraction.

5. $\dfrac{6\frac{3}{4}-1\frac{8}{9}}{\frac{7}{12}\text{ of }1\frac{2}{3}}=\dfrac{6\frac{3}{4}-1\frac{8}{9}}{\frac{35}{36}}=\dfrac{243-68}{35}=\dfrac{175}{35}=5.$

Here the compound term $\frac{7}{12}$ of $1\frac{2}{3}$ is first reduced to the simple term $\frac{35}{36}$, and then the numerator and denominator of the resulting fraction are multiplied by 36, the L. C. D. of the small fractions.

Compound terms must first be reduced to simple terms.

Exercise 61.

Simplify:

1. $\dfrac{2\frac{3}{11}}{3\frac{3}{4}}$.

2. $\dfrac{3}{7\frac{1}{8}}$.

3. $\dfrac{17\frac{1}{7}}{13\frac{1}{3}}$.

4. $\dfrac{\frac{5}{9}}{8\frac{1}{3}}$.

5. $\dfrac{5\frac{1}{9}}{8\frac{4}{11}}$.

6. $\dfrac{1\frac{4}{5} \text{ of } 3\frac{1}{7}}{4\frac{1}{8} \text{ of } \frac{9}{10}}$.

7. $\dfrac{2\frac{1}{2} - 1\frac{5}{9}}{1\frac{5}{6} - 1\frac{5}{8}}$.

8. $\dfrac{10\frac{2}{5} - 1\frac{5}{7}}{7\frac{1}{8} - 3\frac{3}{40}}$.

9. $\dfrac{\frac{3}{7} \text{ of } 2\frac{1}{17}}{1\frac{2}{3} \div 2\frac{3}{7}}$.

10. $\dfrac{6\frac{3}{4} - 1\frac{5}{14}}{2\frac{1}{6} + 1\frac{3}{7}}$.

11. $\dfrac{5\frac{4}{5} + 2\frac{3}{7}}{4\frac{2}{3} - 3\frac{11}{13}}$.

12. $\dfrac{8\frac{3}{4}}{14} - \dfrac{\frac{2}{3}}{1\frac{1}{7}}$.

13. $\dfrac{3\frac{3}{7}}{11\frac{1}{4}} \text{ of } \dfrac{3\frac{3}{8}}{2\frac{2}{5}}$.

14. $\dfrac{5\frac{8}{9} - 4\frac{11}{12}}{5\frac{3}{8} - 2\frac{17}{18}}$.

15. $\dfrac{2\frac{3}{4} + 2\frac{7}{8}}{4\frac{3}{4} - 3\frac{1}{7}}$.

16. $\dfrac{2\frac{3}{5} \times \frac{9}{11}}{3\frac{5}{7} \div 4\frac{1}{8}}$.

17. $\dfrac{\frac{17}{20} + \frac{11}{15} + \frac{7}{10} + \frac{4}{5}}{\frac{17}{20} - \frac{11}{15} + \frac{7}{10} - \frac{4}{5}}$.

18. $\dfrac{4\frac{1}{7} - 2\frac{1}{4}}{6\frac{1}{2} - 2\frac{1}{7}}$.

19. $\dfrac{2\frac{27}{40} - 4\frac{4}{7} + 3\frac{1}{8}}{5\frac{6}{7} - 4\frac{7}{8} + \frac{2}{5}}$.

20. $\dfrac{1\frac{1}{4} \times 1\frac{2}{7} + \frac{1}{3} \text{ of } 2\frac{1}{4} - \frac{13}{28} \times 2}{\frac{13}{28} \text{ of } 2 + \frac{1}{3} \text{ of } 2\frac{1}{4} - 1\frac{1}{4} \text{ of } 1\frac{2}{7}}$.

21. $2\frac{1}{4} \times \dfrac{10\frac{3}{4} - 4\frac{11}{12}}{6\frac{3}{16} + 7\frac{2}{3}} \times \dfrac{3\frac{5}{11}}{1\frac{2}{5} \times 9\frac{1}{11}}$.

22. $\dfrac{8\frac{7}{8} - 7\frac{6}{7} + 5\frac{5}{6} - 4\frac{4}{5}}{9\frac{9}{10} - 8\frac{13}{15} + 7\frac{7}{8} - 6\frac{6}{7}}$.

23. $\dfrac{\frac{1}{7}}{8} \times \dfrac{\frac{8}{7}}{9\frac{1}{2}} \times \dfrac{7\frac{1}{9}}{\frac{8}{9}} \times \dfrac{4\frac{3}{4}}{7\frac{3}{14}} \times \frac{3}{27} \times 1\frac{1}{8}$.

24. $\dfrac{27}{37\frac{4}{5}} \times \dfrac{87\frac{2}{9}}{98\frac{1}{8}} \times \dfrac{\frac{7}{8}}{2\frac{1}{2}} \times \dfrac{89\frac{5}{11}}{128}$.

25. $\dfrac{4\frac{1}{17}}{6\frac{1}{19}} \times \dfrac{170}{399} \div \dfrac{12\frac{2}{3}}{7\frac{7}{9}}$.

26. $\left(1 - \dfrac{426}{697} + \dfrac{2\frac{1}{2}}{8\frac{1}{2}}\right) \div \dfrac{3\frac{1}{2}}{5\frac{1}{8}}$.

27. $\dfrac{\frac{1}{6} \text{ of } 1\frac{13}{16} + 1\frac{1}{6} \text{ of } 6\frac{1}{4} - 1\frac{1}{3} \text{ of } 5\frac{4}{9}}{\frac{1}{6} \text{ of } 2\frac{5}{6} \text{ of } 5\frac{2}{3}.}$.

28. $\dfrac{\frac{37}{68} \times \frac{51}{111} \times \frac{152}{187} \times \frac{153}{171}}{5\frac{1}{2}}$.

29. $\dfrac{\frac{3}{11} \times 9\frac{3}{13} \times 3\frac{1}{7} \times 9\frac{1}{10}}{\frac{4}{17} \times 3\frac{9}{19} \times 12\frac{1}{7} \times 2\frac{10}{33} \times \frac{7}{20}}$.

30. $\dfrac{2\frac{3}{4} \times 7\frac{7}{11}}{\frac{1}{2} \times \frac{3}{4} \times 18\frac{2}{3}}$.

To Express One Number as the Fraction of Another.

272. Examples. 1. What fraction of 8 is 7?

Since 1 is $\frac{1}{8}$ of 8,
7 is 7 *times* $\frac{1}{8}$ of 8;
that is, 7 is $\frac{7}{8}$ of 8.

Here the number denoting the part is the numerator and the number denoting the whole is the denominator of the required fraction.

2. What fraction of $\frac{3}{2}$ is $\frac{3}{7}$?

Taking the number denoting the part for the numerator, and the number denoting the whole for the denominator, we have

$$\frac{\frac{3}{7}}{\frac{3}{2}};\text{ and this becomes } \frac{3}{7} \times \frac{2}{3} = \frac{2}{7}. \quad \text{Therefore,}$$

273. To Find the Fraction that One Number is of Another,

Take the number denoting the part for the numerator, and the number denoting the whole for the denominator.

Exercise 62.

What fraction of:

1. 8 is 3?
2. 3 is 8?
3. 9 is 7?
4. 7 is 9?
5. 8 is 12?
6. 12 is 8?
7. $2\frac{1}{5}$ is $\frac{3}{5}$?
8. $\frac{3}{5}$ is $2\frac{1}{5}$?
9. $2\frac{3}{4}$ is $1\frac{1}{4}$?
10. $1\frac{1}{4}$ is $2\frac{3}{4}$?
11. $2\frac{1}{3}$ is $7\frac{1}{8}$?
12. $7\frac{1}{3}$ is $2\frac{1}{8}$?
13. $3\frac{1}{3}$ is $8\frac{1}{7}$?
14. \$2 is $\$1\frac{1}{2}$?
15. $\$2\frac{1}{2}$ is \$5?
16. $\$\frac{3}{4}$ is $\$\frac{1}{4}$?
17. $\$\frac{5}{4}$ is $\$\frac{3}{8}$?
18. $\$2\frac{3}{4}$ is $\$\frac{1}{3}$?
19. $\$\frac{1}{2}$ is $\$\frac{1}{10}$?
20. \$1 is $\$\frac{7}{8}$?
21. \$10 is $\$\frac{2}{3}$?
22. \$100 is \$6?
23. \$100 is $\$4\frac{1}{2}$?
24. \$4 is \$25?
25. $100\frac{5}{9}$ is $8\frac{4}{9}$?
26. 21 is $\frac{15}{19}$ of $3\frac{4}{5}$?
27. $18\frac{117}{256}$ is $\frac{5}{8}$ of $33\frac{3}{4}$?
28. $3\frac{1}{3}$ is $\frac{2}{3} \times 1\frac{1}{3}$?
29. $3\frac{1}{11} \times 5\frac{1}{27}$ is 1720?
30. $3\frac{1}{2} \times \frac{8}{9}$ of $\frac{4}{7}$ is $1\frac{3}{5}$?

What part of:

31. $\frac{28}{39} \times \frac{52}{63}$ is $\frac{1}{6} \times 4 \times \frac{2}{3}$?
32. $13\frac{5}{8} \times \frac{2}{3} \times \frac{9}{65}$ is $\frac{2}{3}$ of $1\frac{44}{65}$ of $1\frac{1}{8}$?
33. $\frac{17}{20} + \frac{11}{15} + \frac{7}{10} + \frac{4}{5}$ is $\frac{17}{20} - \frac{11}{15} + \frac{7}{10} - \frac{4}{5}$?

34. $4\frac{1}{7} - 2\frac{1}{4}$ is $6\frac{1}{2} - 2\frac{1}{7}$?

35. $17\frac{2}{3} - 12\frac{6}{7}$ is $5 - \frac{1}{13} - \frac{4}{39} - \frac{1}{25}$?

36. $24 - 17\frac{4}{13}$ is $7 + \frac{2}{15} - \frac{5}{81} - \frac{11}{45}$?

37. $\frac{3}{7}$ of $2\frac{1}{17}$ is $1\frac{2}{3} \div 2\frac{3}{7}$?

38. $$\left(\frac{7}{4-\frac{5}{6}} - \frac{5}{6-\frac{3}{8}}\right) \div \left(\frac{4}{7-\frac{4}{7}} + \frac{2}{4-\frac{2}{5}}\right) \text{ is}$$

$$\left(14 - \frac{1}{\frac{1}{2}-\frac{6}{31}}\right) \div \left(\frac{1}{\frac{1}{2}-\frac{27}{59}} - 13\right) ?$$

Conversion of Fractions.

274. A decimal may be changed to a common fraction.

Examples. **1.** Reduce 0.527 to a common fraction.

0.527 means $\frac{5}{10} + \frac{2}{100} + \frac{7}{1000} = \frac{500+20+7}{1000} = \frac{527}{1000}$.

2. $0.525 = \frac{525}{1000} = \frac{21}{40}$.

3. $18.375 = 18\frac{375}{1000} = 18\frac{15}{40} = 18\frac{3}{8}$. Hence,

275. To Reduce a Decimal to a Common Fraction,

For the numerator, write the figures of the decimal; for the denominator, write 1 with as many zeros after it as there are figures in the given decimal.

Exercise 63.

Reduce to a common fraction or to a mixed number :

1. 0.125.
2. 0.625.
3. 0.675.
4. 10.864.
5. 50.84.
6. 3.00025.
7. 8.1075.
8. 35.01024.
9. 7.015625.
10. 20.100256.
11. 10.012575.
12. 104.235.
13. 50.0004.
14. 100.001.
15. 8.00725.
16. 20.018375.
17. 125.6048.
18. 0.128.
19. 0.73125
20. 1.1875.
21. 0.603125.
22. 6.03125.
23. 60.3125.
24. 7.0315.
25. 12.0625.
26. 4.7168.
27. 0.0425.
28. 6.46875.
29. 0.00256.
30. 0.000375.

276. A common fraction may be reduced to a decimal.

Example. Reduce $\frac{5}{8}$ to a decimal.

$$\begin{array}{r} 8\underline{)5.000} \\ 0.625 \end{array}$$

We annex zeros to the numerator of the fraction, inserting a decimal point before the zeros; and then divide the numerator by the denominator.

By this operation the *form* of the quotient is changed from $\frac{5}{8}$ to 0.625, but the *value* remains unchanged. Hence,

277. To Reduce a Common Fraction to a Decimal,

Divide the numerator by the denominator.

Exercise 64.

Reduce to a decimal:

1. $\frac{7}{8}$.
2. $\frac{15}{16}$.
3. $\frac{9}{32}$.
4. $\frac{9}{25}$.
5. $\frac{5}{64}$.
6. $4\frac{11}{800}$.
7. $5\frac{5}{32000}$.
8. $9\frac{123}{256000}$.
9. $11\frac{19}{4000}$.
10. $\frac{9}{125}$.
11. $\frac{17}{4000}$.
12. $\frac{119}{128}$.
13. $\frac{13}{625}$.
14. $\frac{11}{256}$.
15. $\frac{3}{160}$.
16. $\frac{124}{16}$.
17. $\frac{2}{3}$ of $1\frac{4}{5}$.
18. $\frac{3}{4}$ of $\frac{5}{8}$ of $\frac{7}{10}$.
19. $3\frac{3}{5}$ of $4\frac{1}{9}$.
20. $\frac{29}{32}$ of $\frac{49}{64}$.

Exercise 65.

Simplify by common fractions, then by reducing the common fractions to decimals, and show that the results in each example agree:

1. $7\frac{2}{5} + 4\frac{5}{8} + 9\frac{13}{20} + 11\frac{29}{32}$.
2. $84\frac{13}{20} + 19\frac{11}{21} + \frac{41}{50}$.
3. $4\frac{27}{64} + 13\frac{17}{20} + 42\frac{37}{50} + 2\frac{13}{16} + 1\frac{1}{2}$.
4. $5\frac{7}{8} + 13\frac{4}{5} + 19\frac{7}{16} + 7\frac{3}{20}$.
5. $5\frac{5}{10} + \frac{2}{3}$ of $1\frac{4}{5} + \frac{7}{8}$ of $2\frac{2}{7} + \frac{3}{4}$ of $\frac{5}{8}$.
6. $1\frac{5}{12}$ of $2\frac{5}{8}$.
7. $3\frac{5}{16} + 2\frac{19}{20}$.
8. $7\frac{2}{5} - 4\frac{5}{8}$.
9. $82\frac{1}{5} - 37\frac{11}{16}$.
10. $100 - 17\frac{113}{625}$.
11. $5\frac{1}{2} - 1\frac{1}{2}$ of $1\frac{13}{24}$.
12. $\frac{14}{25} - \frac{11}{64}$.
13. $8\frac{1}{5} - 1\frac{1}{2}$ of $\frac{3}{16}$.
14. $\frac{19}{64} \times 1000$.

Repeating Decimals.

278. If the denominator of a common fraction in its lowest terms contains no other factors than 2 and 5 (the prime factors of 10), the fraction can be expressed *exactly* by a decimal; otherwise it cannot.

Thus, if we take the fraction $\frac{3}{11}$ to express as a decimal, we have 0.27272727..... and the division will never end, however far it is carried.

279. A decimal that contains a *constantly recurring figure or series of figures* is called a **repeating decimal** or a **circulating decimal.**

Thus, the decimal 0.27272727..... is a repeating decimal, the series of figures constantly recurring being 27.

280. Repetend. The figure or series of figures that constantly recurs is called the *repetend.*

281. If the repetend begins at the *first* place in the decimal, the decimal is called a *pure* repeating decimal. If the repetend begins at *any place except the first,* it is called a *mixed* repeating decimal.

282. In writing a repeating decimal, we place dots over the first and last figures of the repetend.

Thus, we write 0.272727..... $0.\dot{2}\dot{7}$, and we write 0.333..... $0.\dot{3}$.

283. Examples. 1. Reduce $\frac{6}{7}$ to a decimal.

The denominator contains neither 2 nor 5; the *first figure* of the decimal *begins* the repetend; and we reach the *end* of the repetend *when this figure appears as a remainder.*

$$\begin{array}{r} 0.857142 \\ 7\overline{)6.000000} \end{array}$$

$0.\dot{8}5714\dot{2}$ *Ans.*

2. Reduce $12\frac{15}{28}$ to a decimal.

```
      0.53571428
28)15.0
   140
    100
     84
     160.....(1)
     140
      200
      196
        40
        28
       120
       112
         80
         56
        240
        224
         16 the same remainder as (1).
```

We reach the *end* of the repetend when *a previous remainder reappears.*

When 2 or 5 is a factor of the denominator, *the number of decimal places preceding the repetend is equal to the highest exponent which either of these factors has.* Thus, 28 is equal to $2^2 \times 7$, and the quotient contains *two* decimal places preceding the repetend.

The remainder at the *beginning* of the repetend is the same as the remainder at the *end* of the repetend, and when this remainder reappears we need carry the division no further.

Hence, $12\frac{15}{28} = 12.53\dot{5}7142\dot{8}$.

We may shorten the work by cancelling the factor 4 common to 28 and the second dividend 100, giving 25 divided by 7.

Exercise 66.

Reduce to a decimal :

1. $\frac{5}{9}$.	5. $3\frac{17}{48}$.	9. $9\frac{11}{108}$.	13. $\frac{13}{33}$.
2. $\frac{5}{11}$.	6. $2\frac{5}{37}$.	10. $11\frac{4}{35}$.	14. $\frac{37}{70}$.
3. $3\frac{5}{12}$.	7. $\frac{3}{3700}$.	11. $\frac{15}{56}$.	15. $2\frac{58}{255}$.
4. $\frac{11}{60}$.	8. $11\frac{11}{84}$.	12. $\frac{8}{21}$.	16. $5\frac{6}{26}$.

17. If $\dfrac{117}{5^7 \times 2^8}$ is expressed as a decimal, how many decimal places will the quotient contain?

18. If $\dfrac{119}{2^5 \times 13}$ is expressed as a decimal, how many decimal places will *precede* the repetend?

19. If $\dfrac{57}{5^2 \times 7}$ is reduced to a decimal, how many decimal places will precede the repetend?

To Reduce a Repeating Decimal to a Common Fraction.

284. Examples. 1. Reduce $0.\dot{2}\dot{7}$ to a common fraction.

From $100 \times 0.\dot{2}\dot{7}$, or $27.2727.....$
take $\quad 1 \times 0.\dot{2}\dot{7}$, or $\underline{0.2727.....}$
Then $\quad 99 \times 0.\dot{2}\dot{7} = 27.$
Therefore, $0.\dot{2}\dot{7} = \frac{27}{99} = \frac{3}{11}$.

2. Reduce $0.5\dot{2}4\dot{3}$ to a common fraction.

From $10{,}000 \times 0.5\dot{2}4\dot{3}$, or $5243.243243.....$
take $\quad \underline{10} \times 0.5\dot{2}4\dot{3}$, or $\underline{5.243243.....}$
Then $\quad 9990 \times 0.5\dot{2}4\dot{3} = 5243 - 5.$
Therefore, $0.5\dot{2}4\dot{3} = \frac{5243-5}{9990} = \frac{5238}{9990} = \frac{97}{185}$.

We multiply the decimal by such a number as will put the decimal point at the *end of the repetend*, then by such a number as will put the decimal point at the *beginning of the repetend;* and divide the difference of the products by the difference of the multipliers. Hence,

285. To Reduce a Repeating Decimal to a Fraction,

For the numerator, write the difference between two numbers, one expressed by the figures of the repetend, and the other by the figures that precede the repetend.

For the denominator, write a 9 for each figure of the repetend, and annex a 0 for each figure that precedes the repetend.

EXERCISE 67.

Reduce to a common fraction or to a mixed number:

1. $0.2\dot{4}\dot{5}$.	**9.** $1.41\dot{6}$.	**17.** $0.2\dot{3}6\dot{8}$.
2. $0.4\dot{2}\dot{5}$.	**10.** $0.5\dot{5}7\dot{5}$.	**18.** $1.1\dot{3}\dot{6}$.
3. $53.00\dot{2}4\dot{3}$.	**11.** $2.0\dot{8}\dot{1}$.	**19.** $1.\dot{5}3\dot{1}$.
4. $7.2\dot{0}1\dot{1}$.	**12.** $5.12\dot{2}9\dot{7}$.	**20.** $3.28\dot{9}6\dot{3}$.
5. $2.53\dot{0}\dot{6}$.	**13.** $0.35\dot{9}\dot{0}$.	**21.** $5.878\dot{3}$.
6. $0.004\dot{2}\dot{6}$.	**14.** $4.3\dot{1}6\dot{2}$.	**22.** $1.6\dot{9}40\dot{8}$.
7. $31.2\dot{0}\dot{3}$.	**15.** $0.7\dot{2}8\dot{3}$.	**23.** $0.48\dot{3}2\dot{4}$.
8. $0.\dot{3}5\dot{1}$.	**16.** $5.1\dot{4}2857\dot{7}$.	**24.** $0.00\dot{1}221\dot{3}$.

The Greatest Common Measure and the Least Common Multiple of Fractions.

286. If we divide $\frac{8}{5}$ by $\frac{2}{15}$, we obtain the quotient 12.

If we divide $\frac{8}{5}$ by $\frac{2}{7}$, we obtain the quotient $5\frac{3}{5}$.

If we divide $\frac{3}{7}$ by $\frac{8}{35}$, we obtain the quotient $1\frac{7}{8}$.

We see from these three examples that in dividing one fraction by another the quotient is integral only when the *numerator* of the divisor is a *measure* of the numerator of the dividend, and the *denominator* of the divisor is a *multiple* of the denominator of the dividend.

Therefore, that a fraction may be a common measure of a series of fractions, its numerator must be a measure of each numerator, and its denominator a multiple of each denominator; and that a fraction may be the *greatest common measure* of a series of fractions, its numerator must be the greatest common measure of the numerators of the fractions, and its denominator the least common multiple of the denominators of the fractions. Hence,

287. To Find the G. C. M. of a Series of Fractions,

Find the G. C. M. of the numerators for the numerator of the required measure, and the L. C. M. of the denominators for the denominator of the required measure.

288. Conversely, that a fraction may be a common multiple of a series of fractions, its numerator must be a multiple of each numerator, and its denominator a measure of each denominator; and, that a fraction may be the *least common multiple* of a series of fractions, its numerator must be the least common measure of the numerators of the fractions, and its denominator the greatest common measure of the denominators of the fractions. Hence,

289. To Find the L. C. M. of a Series of Fractions,

Find the L. C. M. of the numerators for the numerator of the required multiple, and the G. C. M. of the denominators for the denominator of the required multiple.

290. Examples. 1. Find the G. C. M. of $\frac{5}{36}$, $\frac{25}{9}$, $\frac{35}{99}$.

The G. C. M. of 5, 25, 35 = 5;
and the L. C. M. of 36, 9, 99 = 396.
Hence, $\frac{5}{396}$ is the G. C. M. required.

2. Find the L. C. M. of $\frac{5}{36}$, $\frac{25}{9}$, $\frac{35}{99}$.

The L. C. M. of 5, 25, 35 = 175;
and the G. C. M. of 36, 9, 99 = 9.
Hence, $\frac{175}{9}$ is the L. C. M. required.

EXERCISE 68.

Find the G. C. M. and L. C. M. of:

1. $\frac{7}{9}$, $\frac{14}{27}$, $\frac{18}{45}$.	**7.** $50\frac{1}{2}$, $67\frac{1}{3}$, $44\frac{8}{9}$, $84\frac{1}{6}$, 707.
2. $2\frac{2}{9}$, $2\frac{2}{5}$, $\frac{4}{40}$.	**8.** $\frac{4}{5}$, $\frac{5}{6}$, $\frac{6}{7}$, $\frac{7}{8}$, $\frac{8}{9}$, $\frac{9}{10}$.
3. $33\frac{3}{7}$, $50\frac{5}{8}$.	**9.** $1\frac{1}{14}$, $1\frac{19}{21}$, $4\frac{2}{7}$, $\frac{25}{42}$.
4. $\frac{7}{24}$, $\frac{35}{36}$, $\frac{49}{60}$.	**10.** $18\frac{2}{5}$, $57\frac{1}{2}$.
5. $5\frac{1}{2}$, $7\frac{1}{3}$, $8\frac{1}{4}$, $4\frac{8}{9}$, $9\frac{1}{6}$, $6\frac{5}{12}$.	**11.** $134\frac{3}{4}$, $128\frac{1}{3}$, $115\frac{1}{2}$.
6. $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, $\frac{1}{10}$, $\frac{1}{12}$.	**12.** $2\frac{22}{25}$, $1\frac{37}{75}$, $\frac{63}{100}$.

13. A, B, and C start together to walk in the same direction round a circular island. It takes A $2\frac{1}{3}$ days, B $2\frac{5}{6}$ days, C $2\frac{7}{8}$ days to walk round the island. They walk until they all meet at the point of starting. In how many days will they be together at the point of starting?

14. If the step of a man is $2\frac{1}{3}$ feet, and that of a horse is $2\frac{3}{4}$ feet, find the smallest number of feet which is an exact number of steps for a man and for a horse.

15. Find the largest number that is contained without remainder in $2\frac{5}{9}$, $6\frac{7}{18}$, $11\frac{1}{2}$, and $19\frac{1}{6}$.

EXERCISE 69. — REVIEW.

1. Simplify $\frac{2709}{6966}$, $\frac{43785}{56835}$, $\frac{2436}{567216}$, $\frac{4087}{5063}$.

2. Which is greater, and by how much, $\frac{7}{9}$ or $\frac{19}{24}$?

3. Find the sum of $3\frac{2}{5}$, $2\frac{4}{11}$, $5\frac{1}{2}$, $7\frac{7}{10}$, $1\frac{3}{22}$.

4. Simplify $5\frac{1}{2} - 3\frac{3}{7} + 2\frac{9}{10} - 1\frac{3}{5}$.

5. Simplify $1\frac{4}{5} + 3\frac{5}{6} - 2\frac{7}{12} + 4\frac{3}{20} - 3\frac{7}{15}$.

6. Simplify $\dfrac{3\frac{1}{2} + 3\frac{5}{6}}{4\frac{1}{3} - 2\frac{7}{12}}$.

7. Simplify $7 \div 2\frac{3}{4}$; $\dfrac{7}{1\frac{3}{8}}$; $\dfrac{95\frac{1}{2}}{8\frac{7}{11}}$; $15 \div \frac{2}{3}$; $\dfrac{16}{5\frac{1}{3}}$; $7\frac{4}{11} \div 9$; $43\frac{1}{4} \div 37\frac{1}{3}$; $\dfrac{6\frac{7}{11}}{18\frac{1}{4}}$; $5\frac{4}{5} \div 4\frac{5}{6}$; $\dfrac{\frac{3}{8} \text{ of } 4\frac{1}{2}}{\frac{7}{4} \times \frac{3}{2}}$; $106 \div 8\frac{5}{6}$; $\dfrac{17}{4\frac{7}{17}}$.

8. Simplify $7\frac{19}{32} \times 8$; $43\frac{13}{33} \times 6\frac{5}{8}$; $6\frac{5}{6} \div 8\frac{1}{5}$; $5\frac{1}{17} \times 51$; $\frac{11}{12}$ of $\frac{11}{13}$; $\frac{30}{49}$ of $\frac{7}{15}$ of $\frac{7}{8}$ of $\frac{2}{3}$ of $\frac{2}{5}$; $\frac{17}{19}$ of $\frac{228}{361}$; $\frac{1}{2} \times \frac{3}{4} \times \frac{7}{11} \times \frac{8}{9} \times \frac{3}{7}$.

9. By what must $\frac{1}{6}$ be multiplied to obtain $\frac{1}{2}$? $\frac{1}{6}$ to obtain $\frac{2}{3}$? $\frac{1}{2}$ to obtain $\frac{5}{6}$? $\frac{2}{3}$ to obtain $\frac{5}{6}$? $\frac{3}{5}$ to obtain $\frac{7}{8}$?

10. By what must $\frac{1}{6}$ be divided to obtain $\frac{1}{2}$? $\frac{2}{3}$ to obtain $\frac{1}{6}$? $\frac{7}{8}$ to obtain $\frac{4}{5}$? $\frac{3}{5}$ to obtain $\frac{7}{8}$? 8 to obtain $7\frac{19}{32}$?

11. What number exceeds $5\frac{2}{9}$ by $4\frac{7}{8}$?

12. From what must $6\frac{3}{5}$ be subtracted to leave $\frac{1}{2}$ of $3\frac{1}{3}$?

13. What fraction falls short of $\frac{7}{12}$ by $\frac{3}{20}$?

14. What fraction must be added to $\frac{5}{76}$ to make $\frac{11}{57}$?

15. Reduce to decimals: $\frac{1}{2}$; $\frac{1}{4}$; $\frac{1}{3}$; $\frac{3}{4}$; $\frac{1}{8}$; $\frac{3}{8}$; $\frac{5}{8}$; $\frac{7}{8}$; $\frac{1}{16}$; $\frac{3}{16}$; $\frac{5}{16}$; $\frac{7}{16}$; $\frac{9}{16}$; $\frac{11}{16}$; $\frac{13}{16}$; $\frac{15}{16}$; $\frac{1}{6}$; $\frac{5}{6}$; $\frac{3}{7}$; $\frac{5}{9}$; $\frac{3}{11}$; $\frac{7}{40}$.

16. Reduce to common fractions: 0.16; 0.016; 0.125; 0.13; 0.725; 0.625; 0.00625; 0.8125; 0.03125; 0.08; $0.\dot{5}\dot{4}$; $0.01\dot{6}$; $0.5\dot{4}3\dot{7}$; $0.0\dot{2}\dot{7}$; $0.27\dot{7}$; $0.68\dot{4}9\dot{4}$; $1.3\dot{4}\dot{5}$.

17. Simplify $\dfrac{2.8 \text{ of } 2.\dot{2}\dot{7}}{1.1\dot{3}\dot{6}}$.

18. Multiply $6.9\dot{5}\dot{4}$ by $5.\dot{3}0\dot{3}$, and express the result as a whole number and common fraction.

19. Simplify $1\frac{1}{3}$ of $2\frac{4}{5} + 6\frac{7}{8} \div 2\frac{3}{4}$ and reduce the result to a decimal.

20. From what number can $4\frac{17}{36}$ be taken 9 times and leave no remainder?

21. Of what fraction is $17\frac{1}{3}$ the 7th part?

22. Add $\frac{4}{5}$, 0.35, $\frac{5}{8}$, $\frac{3}{4}$, 0.112, 45.28.

23. Reduce to decimals: $\frac{13}{16}$; $\frac{3}{11}$; $\frac{4}{35}$; $\frac{17}{60}$; $\frac{14}{19}$; $\frac{5}{13}$.

24. What part of $\frac{15}{73}$ is $\frac{3}{1241}$?

25. Divide 0.0015 by 0.012, and express the result as a common fraction in its lowest terms.

26. Reduce to decimals: $\frac{3}{32}$; $\frac{3}{32000}$; $\frac{17}{74}$; $\frac{1}{7}$.

27. The product of two factors is $\frac{5}{8}$, and one factor is $1\frac{1}{4}$; find the other factor.

28. The dividend is $\frac{11}{12}$, the quotient $6\frac{1}{2}$; find the divisor.

29. The dividend is $12\frac{17}{72}$, quotient 3, remainder $1\frac{5}{12}$; find the divisor.

30. Find the G. C. M. and the L. C. M. of 833, 1127, 1421, 343.

31. Arrange in order of magnitude $\frac{9}{11}$, $\frac{32}{45}$, $\frac{19}{27}$, $\frac{17}{20}$, $\frac{29}{33}$.

32. Find the L. C. M. of $\frac{15}{17}$, $\frac{26}{51}$, $\frac{65}{102}$.

33. Find the G. C. M. of $\frac{65}{68}$, $\frac{39}{2}$, $\frac{91}{64}$, and $6\frac{1}{2}$.

34. Reduce to common fractions $7.2\dot{0}1\dot{1}$; $6.9\dot{5}\dot{4}$.

35. Simplify $\dfrac{3\frac{7}{9} \times 1\frac{1}{17} + 4\frac{1}{12} - 3\frac{9}{16}}{5\frac{1}{9} - 7\frac{7}{8} \div 28\frac{7}{20} + \frac{1}{3}}$.

36. Simplify $\dfrac{6\frac{3}{4} + 5\frac{1}{2} \times 3\frac{1}{7} - 7\frac{1}{4}}{3\frac{1}{5} + 2\frac{1}{2} - 4\frac{1}{10}}$.

37. Simplify $\dfrac{2\frac{4}{5} - 1\frac{1}{2} + 9\frac{1}{11}}{4\frac{1}{5} - 2\frac{1}{4} + 13\frac{7}{11}}$.

38. Simplify $\dfrac{(3.71 - 1.908) \times 7.03}{2.\dot{2} - \frac{74}{333}}$.

39. Simplify $\dfrac{5\frac{5}{8} \div \frac{2}{3}}{1\frac{1}{5} \text{ of } \frac{5}{9} \div 10\frac{1}{3}} \times \frac{2}{5}$ of $\dfrac{1\frac{1}{2} \text{ of } 4\frac{1}{9}}{13\frac{7}{8} \text{ of } 5\frac{1}{3}}$.

40. Simplify $1\frac{1}{2}$ of $2\frac{4}{5} + 6\frac{7}{8} \div 2\frac{3}{4} + \left(5\frac{1}{2} + \dfrac{0.24 + 0.5\dot{3}}{2.2 - 0.6\dot{4}}\right)$.

41. Simplify 0.9 of $\frac{5}{8}$ of $\frac{4}{7}$ of $15\frac{3}{4}$.

42. What part of $\frac{2}{3}$ is $\frac{1}{2}$?

43. What part of 0.390625 is 0.05?

44. What fraction of $0.20\dot{4}\dot{5}$ is $0.\dot{0}\dot{9}$?

45. Reduce to decimals: $\frac{42}{67}$; $\frac{14}{37}$; $\frac{62}{73}$.

46. The G. C. M. of three numbers is 15, and their L. C. M. is 450. What are the numbers?

47. A merchant, after selling $5\frac{1}{8}$ yards and $3\frac{1}{2}$ yards from a remnant of calico, found that he had $7\frac{3}{8}$ yards left. What was the entire length of the remnant?

48. If $3\frac{3}{4}$ yards of cloth are required for a coat, how many coats can be made from $56\frac{1}{4}$ yards of cloth?

49. A grocer bought a hogshead of sugar weighing 744 pounds at $4\frac{7}{8}$ cents per pound, and sold it at $5\frac{1}{4}$ cents per pound. How much did he gain?

50. A man, after selling $\frac{2}{9}$ of his field, sold $\frac{2}{7}$ of the remainder and then had $13\frac{1}{3}$ acres left. How many acres did he own at first?

51. A railroad train passed over $\frac{7}{12}$ of its route in $3\frac{1}{2}$ hours. In how many hours would it pass over the entire route? In how many hours over $\frac{2}{5}$ of the route? $\frac{7}{8}$? $\frac{9}{14}$?

52. A boy, being asked to find the value of $8\frac{4}{15} + 2\frac{7}{9} + 3\frac{3}{5} + 4\frac{2}{3}$, gave as his answer 20. How great was his error?

53. The meter is equal to $3\frac{7}{25}$ feet, very nearly. Express in centimeters the value of $4\frac{1}{10}$ feet.

54. For a piano cover a lady bought $2\frac{5}{8}$ yards of plush at $\$3\frac{1}{2}$ per yard, the same amount of lining flannel at $\$\frac{7}{8}$ per yard, $1\frac{1}{4}$ yards of satin at $\$1\frac{1}{2}$ per yard, and $1\frac{1}{8}$ yards of fringe at $\$1\frac{1}{4}$ per yard. If the making cost \$5, what was the cost of the piano cover?

55. A mason built $6\frac{1}{2}$ yards of wall on Monday, $4\frac{5}{8}$ yards on Tuesday, $4\frac{2}{5}$ yards on Wednesday, and $7\frac{2}{3}$ yards on Thursday. If he is paid \$0.80 per yard, how much has he earned in the four days together?

56. A coal dealer sold 100 tons of coal. If he shipped by six cars $14\frac{1}{2}$, $14\frac{1}{10}$, $14\frac{3}{14}$, $14\frac{8}{35}$, $14\frac{7}{15}$, $14\frac{1}{70}$ tons respectively, how many tons must he load on the seventh car to complete his shipment?

57. The moon's diameter is $\frac{3}{11}$ that of the earth, and the sun's diameter is 110 times that of the earth. What fraction of the sun's diameter is the moon's diameter?

58. If a silver rupee in Calcutta is worth $\$\frac{12}{25}$, what is the value in dollars and cents of a fan costing $4\frac{7}{8}$ rupees?

59. If a man can do $\frac{2}{11}$ of a piece of work in 25 days, what fraction of the work can he do in $62\frac{6}{7}$ days?

60. I paid a tailor $\$3\frac{1}{2}$ a yard for $5\frac{1}{4}$ yards of broadcloth. On measuring it, I found that there were only $4\frac{7}{8}$ yards. How much money ought the tailor to return?

61. From a tank full of water $\frac{3}{5}$ of the water was drawn off. Then 35 gallons were added, and the tank was just half full. What is the capacity of the tank?

62. What number exceeds the sum of its fourth, fifth, sixth, and seventh parts by 101?

63. A trader bought wheat at 75 cents a bushel, and sold it at 71 cents a bushel. How many cents did he lose on every dollar he paid?

64. How many bushels of potatoes at $\$\frac{2}{5}$ per bushel will pay for 16 bushels of wheat at $\$\frac{13}{20}$ per bushel?

65. From a piece of calico containing $35\frac{7}{8}$ yards, there have been sold at different times $12\frac{3}{4}$ yards, $2\frac{1}{2}$ yards, $2\frac{3}{16}$ yards, and $8\frac{5}{8}$ yards. How many yards remain?

66. If gun metal is composed of $90\frac{1}{2}$ parts of copper to $9\frac{1}{2}$ parts of tin by weight, how many ounces of tin are there in one pound (16 ounces) of gun metal? how many ounces of copper in one pound?

67. One man mows $\frac{1}{3}$ of a field, a second $\frac{2}{7}$ of it, and a third $\frac{5}{21}$ of it. What fraction of the field remains to be mowed?

68. Bell metal by weight consists of 4 parts of copper to 1 part of tin. What is the cost of a bell weighing 12,400 pounds, if the copper costs 19 cents per pound, the tin $22\frac{1}{2}$ cents per pound, and the cost of making is $500?

69. If an ore loses $\frac{17}{40}$ of its weight in roasting, and $\frac{8}{15}$ of the remainder in smelting, how many tons of ore must be mined to obtain 466 tons of pure metal?

70. The amount of starch in potatoes is $\frac{11}{50}$ of their weight, but the amount that can usually be extracted is only $\frac{2}{15}$. How many pounds of starch can be obtained from 100 pounds of potatoes, and how many pounds of starch will be left in the potatoes?

71. How many pairs of trousers, each pair requiring $2\frac{3}{8}$ yards, can be made from $33\frac{1}{4}$ yards of cloth?

72. If $3\frac{1}{2}$ yards of cloth are required for a shirt, how many shirts can be made from 12 pieces of cloth, each piece measuring $47\frac{1}{4}$ yards?

73. Green coffee when roasted loses $\frac{1}{5}$ of its weight. If a dealer buys green coffee at $22\frac{1}{2}$ cents a pound, and sells it roasted at 30 cents a pound, what will be his gain in selling 1000 pounds of roasted coffee, the cost of roasting the whole quantity being $2.25?

74. If an iron bar, when heated 1 degree, expands $\frac{1}{143480}$ of its length, what is the length at 212 degrees of a bar whose length at 32 degrees is $10\frac{5}{8}$ feet?

75. If a horse eats $\frac{7}{16}$ of a ton of hay in 30 days, how long will $4\frac{9}{10}$ tons of hay last 5 horses?

76. If 4 is added to both terms of the fraction $\frac{11}{16}$, by how much is the value of the fraction increased?

77. If 4 is subtracted from both terms of the fraction $\frac{11}{16}$, by how much is the value of the fraction decreased?

78. Find the least number of apples that arranged in groups of 8, 9, 10, or 12 will have just 6 over in each case.

79. The diameter of a bicycle wheel is $2\frac{1}{3}$ feet, and the circumference is $3\frac{1}{7}$ times the diameter. How many times does the wheel turn in going 1 mile (5280 feet)?

80. What is the least number of yards of carpet in a roll that can be cut into lengths of exactly $13\frac{1}{3}$ yards, 8 yards, or $11\frac{3}{7}$ yards?

81. What is the length of the longest pole that will exactly measure the sides of a field whose lengths are respectively $135\frac{1}{3}$ yards, $118\frac{2}{3}$ yards, 152 yards, and $202\frac{2}{3}$ yards?

82. Find the least multiplier of $\frac{7}{9}$, $\frac{14}{27}$, and $\frac{28}{45}$ that will make each product an integral number.

83. Find the least integral number that is exactly divisible by $5\frac{1}{4}$, $3\frac{1}{2}$, and 7.

84. Four bells commence tolling together, and toll at intervals of 1, $1\frac{1}{8}$, $1\frac{1}{12}$, and $1\frac{3}{10}$ seconds, respectively. In how many seconds will all four toll again at the same instant?

85. What number multiplied by $\frac{7}{11}$ of $\frac{9}{14}$ of $29\frac{1}{3}$ will give $102\frac{2}{3}$ for the product?

86. How many miles an hour must a man walk to go 28 miles in $7\frac{7}{15}$ hours?

87. If the rent of $5\frac{5}{12}$ acres of land is $\$21\frac{2}{3}$, what will be the rent of $19\frac{9}{16}$ acres at the same rate?

88. If the English acre is $\dfrac{31\frac{1}{4}}{49}$ of an Irish acre, how many English acres are there in $218\frac{3}{4}$ Irish acres?

89. Resolve the denominator of $\frac{23}{48}$ into its prime factors; from the result state the number of figures the equivalent decimal will have, and the number that will precede the repetend.

90. Find the greatest common measure of 9083, 9207, 8897.

CHAPTER VIII.

COMPOUND QUANTITIES.

291. A quantity expressed in units of *one* denomination is called a **simple quantity.** A quantity expressed in units of *two or more* denominations is called a **compound quantity,** or a **compound denominate number.**

Thus, $4\frac{1}{4}$ gallons is a simple quantity; but its equivalent, 4 gallons 1 quart, is a compound quantity.

292. Reduction. The process of changing the *denomination* in which a quantity is expressed, without changing the *value* of the quantity, is called *reduction.*

Measures of Capacity.

293. Liquid Measure. Liquid measure is used in measuring liquids, as water, milk, etc.

TABLE.

4 gills (gi.) = 1 pint (pt.).
2 pints = 1 quart (qt.).
4 quarts = 1 gallon (gal.).

1 gal. = 4 qt. = 8 pt. = 32 gi. 1 gal. contains 231 cubic inches.

$31\frac{1}{2}$ gal. = 1 barrel (bbl.).
63 gal. = 1 hogshead (hhd.).

NOTE. Casks holding from 28 gal. to 43 gal. are called barrels, and casks holding from 54 gal. to 63 gal. are called hogsheads.

Whenever barrels or hogsheads are used as *measures*, a barrel means $31\frac{1}{2}$ gallons, and a hogshead means 63 gallons.

294. Dry Measure. Dry measure is used in measuring dry articles, as grain, seeds, fruit, vegetables.

TABLE.

2 pints (pt.)	= 1 quart (qt.).
8 quarts	= 1 peck (pk.).
4 pecks	= 1 bushel (bu.).

1 bu. = 4 pk. = 32 qt. 1 bu. contains 2150.42 cubic inches.

NOTE. In measuring grain, seeds, and small fruits, the measure must be *even* full. In measuring apples, potatoes, and other large articles, the measure must be *heaping* full.

The quart of liquid measure contains $57\frac{3}{4}$ cubic inches, and the quart of dry measure $67\frac{1}{5}$ cubic inches.

In Great Britain, the quart of liquid measure is of the same size as the quart of dry measure, and contains 69.3185 cubic inches. Therefore, the imperial gallon of Great Britain contains 277.274 cubic inches, and the imperial bushel contains 2218.192 cubic inches.

The quarter contains 8 imperial bushels.

Reduction of Compound Quantities.

295. If the reduction is from a higher denomination to a lower, it is called *reduction descending*. If the reduction is from a lower denomination to a higher, it is called *reduction ascending*.

Thus, 2 pecks = 16 quarts is an example of reduction descending; and 24 quarts = 6 gallons is an example of reduction ascending.

296. Example. Reduce 12 gal. 2 qt. 1 pt. to pints.

gal.	qt.	pt.
12	2	1
4		
50		
2		
101		

SOLUTION. 12 gal. = 12 × 4 qt. = 48 qt., and 48 qt. with the 2 qt. added are 50 qt.

50 qt. = 50 × 2 pt. = 100 pt., and 100 pt. with the 1 pt. added are 101 pt.

The multiplicand and multiplier are interchanged in the operation. Hence,

297. In Reduction Descending,

Multiply the given number of units of the highest denomination by the number of units of the next lower denomination required to make one of this higher; and add to the product the given number of units of this lower denomination.

Proceed in this way with each successive result until the required denomination is reached.

298. Example. Reduce 221 pt. to higher units.

2	221 pt.	
4	110 qt. . . . 1 pt.	
	27 gal. . . 2 qt.	

SOLUTION. 221 pt. = 110 qt. and 1 pt. over. 110 qt. = 27 gal. and 2 qt. over. Therefore, 221 pt. = 27 gal. 2 qt. 1 pt. Hence,

299. In Reduction Ascending,

Divide by the number of units required to make one of the next higher denomination.

Divide this quotient and each successive quotient in like manner until the required denomination is reached.

The last quotient with the several remainders, arranged in order, is the answer sought.

EXERCISE 70.

Reduce:

1. 3 pk. 5 qt. 1 pt. to pints.
2. 4234 pt. (dry measure) to higher units.
3. 24 gal. 2 qt. 1 pt. 2 gi. to gills.
4. 3047 gills to higher units.
5. 1715½ bu. to pints.
6. 508 dry quarts to higher units.
7. 1016 liquid pints to higher units.
8. 44 gal. 3 qt. 1 pt. to pints.
9. 44 bu. 3 pk. 7 qt. 1 pt. to pints.
10. 272 liquid quarts to dry quarts.
11. 429 dry quarts to liquid quarts.

Addition and Subtraction of Compound Quantities.

300. Examples. 1. Add 3 gal. 2 qt. 1 pt.; 5 gal. 3 qt.; 13 gal. 1 qt. 1 pt.; 9 gal. 1 pt.

gal.	qt.	pt.
3	2	1
5	3	0
13	1	1
9	0	1
31	3	1

SOLUTION. We write units of the same name in the same column, and add the columns, beginning with the pints. 3 pt. = 1 qt. and 1 pt. over. We write the 1 pt. under the pints, and add the 1 qt. to the quarts. 7 qt. = 1 gal. and 3 qt. over. We write the 3 qt. under the quarts, and add the 1 gal. to the gallons. The required sum is, therefore, 31 gal. 3 qt. 1 pt.

2. From 5 bu. 2 pk. 3 qt. take 3 bu. 3 pk. 3 qt.

bu.	pk.	qt.
5	2	3
3	3	3
1	3	0

SOLUTION. 3 qt. − 3 qt. = 0 qt. We write 0 under the quarts. Since we cannot take 3 pk. from 2 pk., we take 1 bu. (4 pk.) from the 5 bu. and add it to the 2 pk. 4 pk. + 2 pk. = 6 pk., and 6 pk. − 3 pk. = 3 pk. We write 3 under the pecks. Then 4 bu. − 3 bu. = 1 bu. The required difference is, therefore, 1 bu. 3 pk.

EXERCISE 71.

1. Add 5 bu. 3 pk. 6 qt. 1 pt.; 6 bu. 2 pk. 7 qt.; 7 bu. 1 pk. 1 qt. 1 pt.; 1 pk. 7 qt.; 2 bu. 3 pk. 1 pt.

2. Add 50 gal. 3 qt. 1 pt. 3 gi.; 12 gal. 1 qt. 1 pt. 1 gi.; 5 gal. 2 qt. 1 pt. 2 gi.; 75 gal. 3 qt. 1 pt. 3 gi.; 80 gal. 3 qt. 1 gi.; 17 gal. 1 qt. 1 pt. 3 gi.

3. Add 4 gal. 3 qt. 1 pt.; 3 gal. 2 qt. $1\frac{1}{2}$ pt.; 12 gal. 3 qt.; 14 gal. $1\frac{1}{2}$ pt.; 5 gal. 2 qt. 1 pt.

4. Subtract 5 bu. 1 pk. 6 qt. 1 pt. from 5 bu. 3 pk. 3 qt.

5. Subtract 22 gal. 3 qt. 1 pt. from 30 gal. 2 qt.

6. Add 6 bu. 1 pk. 7 qt. 1 pt.; 2 bu. 2 pk. 5 qt. $\frac{1}{2}$ pt.; 19 bu. 3 pk. 0 qt. 1 pt.; 14 bu. 2 pk. 4 qt. $1\frac{1}{2}$ pt.; 10 bu. 1 pk. 3 qt. 1 pt.; 5 bu. 3 pk. 2 qt.

7. Find the difference between 2 bu. and 5 qt. 1 pt.

Multiplication and Division of Compound Quantities.

301. Examples. 1. Multiply 15 gal. 3 qt. 1 pt. by 5.

gal.	qt.	pt.
15	3	1
		5
79	1	1

SOLUTION. 5 × 1 pt. = 5 pt. = 2 qt. 1 pt. We write the 1 pt. under the pints, and reserve the 2 qt. to be added to 5 × 3 qt. 5 × 3 qt. = 15 qt., and this with the 2 qt. reserved = 17 qt. = 4 gal. 1 qt. We write the 1 qt under the quarts, and add the 4 gal. to 5 × 15 gal., having 79 gal. The required product is, therefore, 79 gal. 1 qt. 1 pt.

2. Divide 122 bu. 2 pk. 7 qt. 1 pt. by 5.

	bu.	pk.	qt.	pt.
5)	122	2	7	1
	24	2	1	1

SOLUTION. 122 bu. ÷ 5 = 24 bu. and 2 bu. over. We write 24 under the bushels. 2 bu. = 2 × 4 pk. = 8 pk., and 8 pk. + 2 pk. = 10 pk. 10 pk. ÷ 5 = 2 pk. We write 2 under the pecks. 7 qt. ÷ 5 = 1 qt. and 2 qt. over. We write 1 under the quarts. 2 qt. = 2 × 2 pt. = 4 pt., and 4 pt. + 1 pt. = 5 pt. 5 pt. ÷ 5 = 1 pt. We write 1 under the pints. The required quotient is, therefore, 24 bu. 2 pk. 1 qt. 1 pt.

EXERCISE 72.

1. Multiply 19 gal. 3 qt. 1 pt. by 70.
2. Multiply 43 bu. 2 pk. 6 qt. 1 pt. by 63.
3. Multiply 17 bu. 3 pk. 6 qt. by 8.
4. Multiply 26 gal. 2 qt. 1 pt. 3 gi. by 12.
5. Multiply 12 bu. 3 pk. 7 qt. 1 pt. by 25.
6. Divide 34 gal. 3 qt. 1 gi. by 7.
7. Divide 147 gal. 2 qt. 1 pt. 2 gi. by 17.
8. Divide 54 bu. 3 pk. 2 qt. 1 pt. by 11.
9. Divide 34 bu. 3 pk. 5 qt. 1 pt. by 15.

302. *All* compound quantities are reduced, added, subtracted, multiplied, divided, by the methods given for measures of capacity.

Measures of Weight.

303. Troy Weight. Troy weight is used in weighing gold, silver, and precious stones.

TABLE.

24 grains (gr.)	= 1 pennyweight (dwt.).
20 pennyweights	= 1 ounce (oz.).
12 ounces	= 1 pound (lb.).

The pound troy contains 5760 grains.

304. Avoirdupois Weight. Avoirdupois weight is used in weighing all articles except gold, silver, and precious stones.

TABLE.

16 ounces (oz.)	= 1 pound (lb.).
100 pounds	= 1 hundredweight (cwt.).
2000 pounds	= 1 ton (t.).

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112 pounds	= 1 long hundredweight.
2240 pounds	= 1 long ton.

The long ton is used in the United States Custom Houses, and in wholesale transactions in iron and coal.

The pound avoirdupois contains 7000 troy grains.

EXERCISE 73.

1. Reduce 27,587 gr. to higher troy units.
2. Reduce 34,652 pounds avoirdupois to long tons, etc.
3. Reduce 136,851 ounces avoirdupois to higher units.
4. Reduce 864,205 gr. to higher troy units.
5. Reduce 864,205 gr. to higher avoirdupois units.
6. Reduce 5 lb. 7 oz. 6 dwt. 12 gr. to grains.
7. Reduce 745 lb. avoirdupois to troy measures.
8. Reduce 745 lb. troy to avoirdupois measures.
9. Reduce 1,440,445 oz. avoirdupois to higher units.

10. Reduce 5,640,773 oz. avoirdupois to higher units.

11. Add 48 t. 13 cwt. 75 lb. 6 oz.; 25 t. 12 cwt. 27 lb. 8 oz.; 51 t. 10 cwt. 44 lb.; 80 t. 5 cwt. 6 oz.; 19 cwt. 27 lb.; 25 lb. 8 oz.; 5 t. 5 cwt. 5 lb.

12. Add 13 lb. 4 oz. 8 dwt. 6 gr.; 25 lb. 8 oz. 13 dwt. 20 gr.; 8 lb. 11 oz. 14 gr.; 20 lb. 16 dwt. 8 gr.; 15 lb. 9 oz. 12 dwt.; 4 oz. 3 dwt.

13. Subtract 23 lb. 8 oz. 19 dwt. 10 gr. from 58 lb. 6 oz. 17 dwt. 21 gr.

14. Subtract 17 t. 7 cwt. 17 lb. 6 oz. from 25 t. 13 cwt. 15 lb. 12 oz.

15. Multiply 3 lb. 4 oz. 8 dwt. 10 gr. by 10.

16. Multiply 5 t. 10 cwt. 67 lb. 4 oz. by 15.

17. Divide 17 t. 19 cwt. 79 lb. 8 oz. by 8.

18. Divide 60 lb. 6 oz. 10 dwt. 20 gr. by 7.

19. How many bags each holding 2 bu. 1 pk. 3 qt. are required to hold 234 bu. 1 pk. 4 qt. of corn?

NOTE. Reduce both quantities to quarts.

20. What is the value at $4\frac{1}{2}$ cents a pound of a calf weighing 184 lb. 6 oz.?

21. How many tablespoons each weighing 2 oz. 17 dwt. 12 gr. can be made from 155 oz. 5 dwt. of silver?

305. In compounding medicines, apothecaries make use of the following:

Apothecaries' Weight.

20 grains (gr.)	=	1 scruple (℈).
3 scruples	=	1 dram (ʒ).
8 drams	=	1 ounce (℥).
12 ounces	=	1 lb. troy.

Apothecaries' Measure.

60 minims (♏)	=	1 dram (♏ lx.).
8 drams	=	1 ounce (fl. drm. viij.).
16 ounces	=	1 pint (fl. oz. xvj.).

Measures of Length.

306. The unit of measure for lengths is the **yard.** From the yard are derived the units of surface and volume.

307. The **standard yard** of Great Britain, as defined by Act of Parliament, is the distance between the centres of two cylindrical holes in a certain bar of gun metal, when the metal has a temperature of 62 degrees Fahrenheit.

The standard yard of the United States conforms as nearly as possible to that of Great Britain.

308. Measures of length are used in measuring lines or distances.

TABLE.

12 inches (in.)	= 1 foot (ft.).
3 feet	= 1 yard (yd.).
$5\frac{1}{2}$ yards, or $16\frac{1}{2}$ feet	= 1 rod (rd.).
320 rods	= 1 mile (mi.).

1 mi. = 320 rd. = 1760 yd. = 5280 ft.

NOTE. A hand (used in measuring the height of horses) = 4 in.; a knot (used in navigation) = 6086 ft.; a league = 3 knots; a fathom = 6 ft.; a cable length = 120 fathoms; a line = $\frac{1}{12}$ in.; a barleycorn = $\frac{1}{3}$ in.; a palm = 3 in.; a span = 9 in.; a cubit = 18 in.; a military pace = 28 in.; a furlong = $\frac{1}{8}$ mi.

309. Examples. 1. Change 106,760 ft. to higher denominations.

SOLUTION. There are $16\frac{1}{2}$ ft., or 33 half-feet, in a rod; so we change the 106,760 ft. to half-feet, and *these* to rods, by dividing by 33. The remainder is 10 half-feet, or 5 ft. 6470 rd. = 20 mi. 70 rd. Therefore, 106,760 ft. = 20 mi. 70 rd. 5 ft.

$16\frac{1}{2}$)106760 ft.
2
33)213520 half-feet.
320)6470 rd. . . . 10 half-feet = 5 ft.
20 mi. . . . 70 rd.

2. Add 4 mi. 110 rd. 5 yd. 1 ft. 8 in. and 6 mi. 25 rd. 4 yd. 1 ft. 6 in.

mi.	rd.	yd.	ft.	in.
4	110	5	1	8
6	25	4	1	6
10	136	$3\frac{1}{2}$	0	2
			1	6
10	136	3	1	8

SOLUTION. We have for the sum 10 mi. 136 rd. $3\frac{1}{2}$ yd. 0 ft. 2 in. We reduce the $\frac{1}{2}$ yd. of this sum and add its value 1 ft. 6 in. to the 0 ft. 2 in. We have for the sum, therefore, 10 mi. 136 rd. 3 yd. 1 ft. 8 in.

EXERCISE 74.

1. Reduce 3 yd. 2 ft. to inches.
2. Reduce 4 mi. 124 rd. 3 yd. 2 ft. to feet.
3. Reduce 27 rd. 4 yd. 9 in. to inches.
4. Reduce 290 leagues to feet.
5. Reduce 82,976,432 in. to higher units.
6. Reduce 7 mi. 3 yd. 1 ft. 6 in. to inches.
7. Reduce 22 mi. 222 rd. 4 ft. 8 in. to inches.
8. Reduce 712 mi. to feet.
9. Reduce 540,451 ft. to higher units.
10. Reduce 271,256 in. to higher units.
11. Reduce 723,964 ft. to higher units.
12. Reduce 233,205 in. to higher units.
13. How many feet high is a horse 16 hands high?
14. Add 6 mi. 120 rd. 3 yd. 2 ft. 2 in.; 18 mi. 15 rd. 1 yd. 1 ft. 6 in.; 3 mi. 215 rd. 2 yd. 2 ft. 3 in.; 7 mi. 95 rd. 1 yd. 1 ft. 8 in.
15. Subtract 3 mi. 217 rd. 4 yd. 1 ft. 3 in. from 4 mi. 100 rd. 3 yd. 2 in.
16. Multiply 5 mi. 126 rd. 9 ft. 6 in. by 7125.
17. Divide 54 mi. 124 rd. 1 yd. 2 ft. 6 in. by 33.
18. If a man builds 1 rd. 1 yd. 1 ft. 6 in. of stone wall in one day, how much will he build in 26 days?
19. A man builds 25 rd. 2 yd. 1 ft. 6 in. of wall in 20 days. How much does he build per day?

Measures of Surface.

310. The *area* of a surface is the number of square units it contains.

TABLE.

144 square inches (sq. in.)	= 1 square foot (sq. ft.).
9 square feet	= 1 square yard (sq. yd.).
$30\frac{1}{4}$ square yards, or $272\frac{1}{4}$ square feet	= 1 square rod (sq. rd.).
160 square rods or 43,560 square feet	= 1 acre (A.).
640 acres	= 1 square mile (sq. mi.).

NOTE. In measures of surface the scale ascends and descends by the *squares* of the units of length; thus, $144 = 12^2$; $9 = 3^2$; $30\frac{1}{4} = (5\frac{1}{2})^2$; $272\frac{1}{4} = (16\frac{1}{2})^2$.

EXERCISE 75.

1. Reduce 92,638 sq. yd. to square inches.
2. Reduce 1,223,527 sq. in. to higher units.
3. Reduce 721 sq. mi. to square rods.
4. Reduce 34,729 sq. yd. to higher units.
5. Reduce to square inches 3 A. 107 sq. rd. 27 sq. yd. 7 sq. ft. 23 sq. in.
6. Reduce 99,894,712 sq. in. to higher units.
7. Reduce 15,376 sq. yd. to higher units.
8. Reduce 562,934 sq. in. to higher units.
9. Add 74 A. 21 sq. rd. 5 sq. yd. 4 sq. ft. 100 sq. in.; 123 A. 23 sq. rd. 13 sq. yd. 5 sq. ft. 83 sq. in.; 112 A. 106 sq. rd. 17 sq. yd. 8 sq. ft. 7 sq. in.; 541 A. 50 sq. rd. 23 sq. yd. 24 sq. in.
10. From 20 A. take 13 A. 150 sq. rd. 98 sq. ft. 10 sq. in.
11. Multiply 27 A. 76 sq. rd. 22 sq. yd. 5 sq. ft. by 90.
12. Divide 74,128 sq. mi. 517 A. 80 sq. rd. by 10,000.

Surveyors' Measure.

311. Surveyors use a chain, called Gunter's chain, which is 4 rods, or 66 feet, long. The chain has 100 links, and therefore links are written as *hundredths* of a chain.

SURVEYORS' TABLE OF

Measures of Length.	*Measures of Surface.*
7.92 in. = 1 link (l.).	16 sq. rd. = 1 sq. ch.
100 links = 1 chain (ch.)	10 sq. ch. = 1 A.
80 chains = 1 mile (mi.).	640 A. = 1 sq. mi.
	1 sq. mi. = 1 section (sec.).
	36 sec. = 1 township (tp.).

EXERCISE 76.

1. Reduce 10 ch. to inches.
2. Reduce 3168 in. to chains.
3. How many acres are there in a township?
4. Reduce 6400 sq. ch. to acres; to square miles.
5. Reduce 82,426 sq. ch. to higher units.
6. Add 4 sq. mi. 412 A. 6 sq. ch. 8 sq. rd.; 7 sq. mi. 88 A. 2 sq. ch. 11 sq. rd.; 3 sq. mi. 367 A. 7 sq. ch. 2 sq. rd.; 11 sq. mi. 344 A. 9 sq. ch. 15 sq. rd.
7. Subtract 1 mi. 75 ch. 85 l. from 4 mi. 44 ch. 38 l.
8. What is the area of a field if it can be divided into 12 lots each containing 2 sq. ch. 7 sq. rd.?
9. Multiply 3 sq. mi. 172 A. 5 sq. ch. 7 sq. rd. by 11.
10. Divide 6 sq. mi. 422 A. 2 sq. ch. 13 sq. rd. by 5.
11. A field is divided into 47 gardens each containing 1 sq. ch. 9 sq. rd. What is the area of the field?
12. A field containing 5 A. 4 sq. ch. 11 sq. rd. is divided into 25 equal lots. What is the area of each lot?
13. Find the rent of 8 sq. ch. 10 sq. rd. at $2 an acre.
14. If a field contains 3 A. 6 sq. ch. 12 sq. rd., what is it worth at 14 cents a square foot?

Measures of Volume.

312. The *volume* of a body is the number of cubic units it contains.

TABLE.

1728 cubic inches (cu. in.)	=	1 cubic foot (cu. ft.).
27 cubic feet	=	1 cubic yard (cu. yd.).

NOTE. In measures of volume, the scale ascends and descends by the *cubes* of the units of length; thus $1728 = 12^3$; $27 = 3^3$.

313. In measuring wood and small, irregular stones the following is the

TABLE.

16 cubic feet	=	1 cord foot (cd. ft.).
8 cord feet, or 128 cubic feet	=	1 cord (cd.).

NOTE. A cord is a pile 8 ft. long, 4 ft. wide, and 4 ft. high. One foot of the length of such a pile is called a *cord foot*.

EXERCISE 77.

1. Reduce 25 cu. yd. 5 cu. ft. 143 cu. in. to cubic inches.
2. Reduce 921,730 cu. in. to higher units.
3. Wood cut in lengths of 4 ft. is piled $3\frac{1}{2}$ ft. high. How long must the pile be to contain 2 cords?
4. How many cords in a pile of 4-ft. wood 43 ft. long and 6 ft. high?
5. Add 130 cu. yd. 5 cu. ft. 820 cu. in.; 56 cu. yd. 20 cu. ft. 304 cu. in.; 37 cu. yd. 4 cu. ft. 86 cu. in.; 8 cu. yd. 10 cu. ft. 129 cu. in.; 12 cu. yd. 19 cu. ft. 175 cu. in.
6. Subtract 32 cu. yd. 13 cu. ft. 1600 cu. in. from 39 cu. yd. 17 cu. ft. 1400 cu. in.
7. Multiply 12 cd. 4 cd. ft. by 14.
8. Divide 5 cu. yd. 10 cu. ft. 371 cu. in. by 6.

Measures of Value.

314. **Money** is the measure of value.

315. **Currency** is the medium of exchange employed in buying and selling.

316. **Coins** or **Specie** are stamped pieces of metal of fixed purity and weight issued by governments as money.

317. **Bullion** is uncoined gold or silver, of standard purity, usually in the shape of bars.

318. **Paper Money** is stamped paper containing the promise of the government or of a bank of issue to pay the holder on presentation a specified sum of standard coined money.

319. **United States Money.** The unit of value in the United States is the *dollar.*

TABLE.

10 mills (m.)	=	1 cent (ct.).
10 cents	=	1 dime (d.).
10 dimes or 100 cents	=	1 dollar ($).

Cents are made of bronze; half-dimes of nickel; dimes, quarter-dollars, half-dollars, and dollars of silver; pieces of two and a half dollars, five dollars, ten dollars, and twenty dollars of gold.

A 10-dollar gold coin is called an eagle, and a 20-dollar gold coin is called a double eagle.

NOTE. The standard gold dollar weighs 25.8 gr. and contains 23.22 gr. of pure gold.

320. In common use dimes and cents are read together as cents; figures to the right of mills are read as the decimal of a mill; mills are used only in computation.

Thus, $4.27765 is read four dollars and twenty-seven cents, seven and sixty-five hundredths mills; $4.273 is reckoned $4.27, and $4.277 is reckoned $4.28.

321. English Money. The unit of value in Great Britain is the *pound sterling*, which is equivalent in United States money to \$4.8665.

TABLE.

4 farthings	=	1 penny (d.).
12 pence	=	1 shilling (s.).
20 shillings	=	1 pound (£).

A guinea = 21s.; a sovereign = 20s.; a half-sovereign = 10s.; a crown = 5s.; a half-crown = 2s. 6d.; a florin = 2s.

NOTE 1. The penny and half-penny are made of copper; the three-penny piece, the six-penny piece, the shilling, the florin, the half-crown, and the crown are made of silver; the half-sovereign and the sovereign of gold.

NOTE 2. Farthings are generally written as the common fraction of a penny. Thus, 1, 2, and 3 farthings are written $\frac{1}{4}$d., $\frac{1}{2}$d., and $\frac{3}{4}$d., respectively.

EXERCISE 78.

1. Reduce £583 6s. 8d. to pence.
2. Reduce £79 18s. 11$\frac{1}{2}$d. to farthings.
3. Reduce 28,572d. to higher units.
4. Reduce 27,281 crowns to guineas.
5. Reduce 1,716,114 guineas to pounds.
6. Reduce 706,126d. to higher units.
7. Add £35 2s. 6$\frac{3}{4}$d.; £18 5s. 4d.; £27 3s. 10d.; £12 5d.; £6 7s. 8d.; £14 19s. 11d.; £29 16s. 2d.
8. Subtract £92 15s. 1$\frac{1}{4}$d. from £120 13s. 4d.
9. Multiply £31 2s. 6$\frac{1}{2}$d. by 8.
10. Divide £394 2s. 10$\frac{1}{2}$d. by £5 2s. 4$\frac{1}{2}$d.
11. Divide £108 15s. 4d. by 13.
12. Find the value in United States money of the money in a box containing 35 sovereigns, 27 half-sovereigns, 13 crowns, 41 half-crowns, and 85 shillings.

322. Values of Foreign Coins. Oct. 1, 1897.

Country.	Standard.	Monetary Unit.	Value in terms of U. S. gold dollar.
Argentine Republic	Gold and silver	Peso	$0.965
Austria-Hungary	Gold	Crown	0.203
Belgium	Gold and silver	Franc	0.193
Bolivia	Silver	Boliviano	0.412
Brazil	Gold	Milreis	0.546
British Possessions, N. A. (except Newfoundland).	Gold	Dollar	1.000
Central Amer. States —			
Costa Rica	Gold	Colon	0.465
British Honduras	Gold	Dollar	1.000
Guatemala Honduras Nicaragua Salvador	Silver	Peso	0.412
Chile	Gold	Peso	0.365
China	Silver	Tael { Canton Tael { Shanghai	0.664 0.608
Colombia	Silver	Peso	0.412
Cuba	Gold and silver	Peso	0.926
Denmark	Gold	Crown	0.268
Ecuador	Silver	Sucre	0.412
Egypt	Gold	Pound (100 piasters)	4.943
Finland	Gold	Mark	0.193
France	Gold and silver	Franc	0.193
German Empire	Gold	Mark	0.238
Great Britain	Gold	Pound sterling	4.866½
Greece	Gold and silver	Drachma	0.193
Haiti	Gold and silver	Gourde	0.965
India	Silver	Rupee	0.196
Italy	Gold and silver	Lira	0.193
Japan	Gold	Yen	0.498
Liberia	Gold	Dollar	1.000
Mexico	Silver	Dollar	0.446
Netherlands	Gold and silver	Florin	0.402
Newfoundland	Gold	Dollar	1.014
Norway	Gold	Crown	0.268
Persia	Silver	Kran	0.076
Peru	Silver	Sol	0.412
Portugal	Gold	Milreis	1.080
Russia	Gold	Ruble	0.772
Spain	Gold and silver	Peseta	0.193
Sweden	Gold	Crown	0.268
Switzerland	Gold and silver	Franc	0.193
Turkey	Gold	Piaster	0.044
Uruguay	Gold	Peso	1.034
Venezuela	Gold and silver	Bolivar	0.193

The "British dollar" has the same legal value as the Mexican dollar in Hongkong, the Straits Settlements, and Labuan.

By Imperial ukase, January 3/15, 1897, 1½ paper rubles = 1 gold ruble, giving paper ruble a value of 51$\frac{4}{10}$ cents U. S. money.

Measures of Time.

323. The unit of time is the *day*.

324. The interval of time, measured from the instant the sun is due south until it is due south the next day, is called a *solar day*. The length of the solar day varies slightly, and its average length, called the *mean solar day*, is the unit of time.

TABLE.

60 seconds (sec.)	= 1 minute (min.).
60 minutes	= 1 hour (hr.).
24 hours	= 1 day (dy.).
7 days	= 1 week (wk.).
365 days	= 1 common year (yr.).
366 days	= 1 leap year.
100 years	= 1 century.

325. A *year* is the time in which the earth performs one revolution round the sun, and consists of 365.242218 mean solar days.

NOTE. Before the time of Julius Cæsar the year was reckoned as 365 days. On the supposition that $365\frac{1}{4}$ days was the true length, he introduced a calendar in which every fourth year (*every year which will give an integral quotient when its number is divided by 4*) was to consist of 366 days. The year of 366 days is called a *leap year*.

The error of the Julian calendar, 365.25 − 365.242219, or 0.007781 of a day, would amount to 3.1124 days in four centuries. To correct this error Pope Gregory XIII, in 1582, introduced a calendar in which three leap years in every four centuries were reckoned as common years. Hence, the centuries are not leap years unless *the number of the century* divided by 4 gives an integral quotient. Thus, 1600 and 1884 were leap years; 1800 and 1885 were not; 1900 will not be a leap year; 2000 will be a leap year.

The present, or Gregorian, calendar leaves a slight error equal to one day in about 3300 years.

326. The year is divided into twelve *calendar months*.

The names of the months (mo.) and the number of days in each are:

	dy.		dy.
1. January (Jan.) . . .	31	7. July	31
2. February (Feb.)	28 or 29	8. August (Aug.) . . .	31
3. March (Mar.) . . .	31	9. September (Sept.) . .	30
4. April (Apr.)	30	10. October (Oct.) . . .	31
5. May	31	11. November (Nov.) . .	30
6. June	30	12. December (Dec.) . .	31

February has 28 days in common years and 29 days in leap years.

NOTE. The number of days in each month may be easily remembered by committing to memory the following lines:

"Thirty days hath September,
April, June, and November;
All the rest have thirty-one,
Except the second month alone,
Which has but twenty-eight, in fine,
Till leap year gives it twenty-nine."

A *lunar month* is the time between two new moons, and is a little more than 29 dy. 12 hr. 44 min.

EXERCISE 79.

1. Reduce 6 hr. 17 min. 25 sec. to seconds.
2. Reduce 1 yr. 13 dy. 8 hr. 4 min. to minutes.
3. Reduce 48,567 min. to higher units.
4. Reduce 7,423,922 sec. to higher units.
5. How many minutes are there from midnight of March 7 to midnight of June 20?
6. Find the number of seconds from eight o'clock Monday morning till six o'clock the next Saturday evening.
7. Which of the years 1600, 1656, 1700, 1734, 1800, 1818, 1880, 1900, 1924, 2000 are leap years?
8. Add 8 dy. 14 hr. 21 min. 37 sec.; 44 dy. 17 hr. 13 min. 32 sec.; 208 dy. 9 hr. 47 min. 43 sec.; 161 dy. 12 hr. 53 min. 54 sec.; 88 dy. 22 hr. 17 min. 50 sec.

9. Subtract 2 yr. 213 dy. 17 hr. 48 min. 48 sec. from 3 yr. 147 dy. 14 hr. 14 min. 32 sec.

10. Multiply 34 dy. 10 hr. 13 min. 12 sec. by 108.

11. Divide 16 yr. 357 dy. 17 hr. 20 min. 48 sec. by 18.

12. Divide 22 wk. 2 dy. by 11 hr. 31 min. 12 sec.

Difference between Two Dates.

327. Examples. 1. Find the difference in time between July 4, 1897, and December 25, 1848.

SOLUTION. In finding the period of time between *long* dates, 30 days are considered a month. As July is the seventh and December the twelfth month, we write 7 and 12 instead of the names of the months. The required difference is 48 yr. 6 mo. 9 dy.

yr.	mo.	dy.
1897	7	4
1848	12	25
48	6	9

2. Find the number of days from July 25 to September 5.

SOLUTION. The number of days in July = 6
The number of days in Aug. = 31
The number of days in Sept. = 5
Total number of days = 42

In finding the period of time between *short* dates the exact number of days is counted; and the last day named is included.

EXERCISE 80.

1. Napoleon was born Aug. 15, 1769, and died at the age of 51 yr. 8 mo. 20 dy. What was the date of his death?

2. Daniel Webster was born Jan. 18, 1782, and died Oct. 24, 1852. How old was he when he died?

3. A note dated July 14, 1897, has 63 days to run. When is the note due?

4. A note dated Feb. 11, 1896, has 93 days to run. When is the note due?

5. A note dated Feb. 11, 1897, has 63 days to run. When is the note due?

6. In the morning of July 5 a man went into the country for his vacation, and returned in the evening of Sept. 26. Express in weeks and days the length of his vacation.

7. Find the difference in time between Oct. 12, 1492, and July 4, 1776.

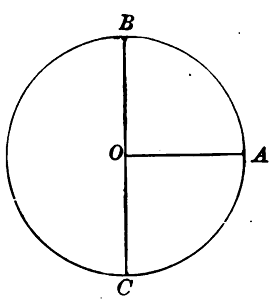
8. Jan. 1, 1859, fell on Saturday. What day of the week was Jan. 1, 1860? Jan. 1, 1861?

Circular and Angular Measures.

328. Any portion of the circumference of a circle is called an *arc.*

329. If a straight line fixed at one end is revolved in a plane, the other end describes the arc of a circle; and the straight line in moving from its original position to any other given position describes an *angle.*

Thus, if OA revolves on a fixed point O, the end A makes the circumference ABC. When OA has reached the position OB, the arc AB has been made by A, and the angle AOB between OA and OB has been made by OA. The angle AOB is such a part of the angular magnitude about O as AB is of the circumference.



The circumference of every circle is divided into 360 equal parts, called *degrees* (arc-degrees), and corresponding to every one of these equal parts is an angle at the centre of the circle. Hence, the whole angular magnitude about any point in a plane is divided into 360 equal parts called *degrees* (angle-degrees), and the number of degrees in the angle formed by two lines drawn from the centre of a circle is the same as the number of degrees in the arc which is intercepted between these two lines.

330. An angle described by a line making one fourth of a revolution contains 90 degrees, and is called a *right angle*, as AOB; and OA and OB are said to be *perpendicular* to each other. An angle less than a right angle is called an *acute angle;* an angle greater than a right angle and less than two right angles is called an *obtuse angle.*

TABLE.

60 seconds (″) = 1 minute (′).
60 minutes = 1 degree (°).
360 degrees = 1 revolution or circumference.

NOTE. A degree of the circumference of the earth at the equator contains 60 geographical miles, or 69.16 statute miles.

EXERCISE 81.

1. Reduce 2° 30′ 25″ to seconds.
2. Reduce 15° 3′ 22″ to seconds.
3. Reduce 56,760″ to higher units.
4. Reduce 212,221″ to higher units.
5. Add 60° 50′ 50″; 20° 41′ 52″; 30° 25′ 20″; 20° 32′ 43″.
6. Subtract 58° 33′ 36″ from 90° 11′ 21″.
7. Multiply 12° 14′ 32″ by 48.
8. Divide 321° 49′ 24″ by 22.
9. Divide 38° 37′ 42″ by 5° 31′ 6″.

Miscellaneous Tables.

331.

NUMBERS.

12 units = 1 dozen.
12 dozen = 1 gross.
12 gross = 1 great gross.
20 units = 1 score.

PAPER.

24 sheets = 1 quire.
20 quires = 1 ream.
2 reams = 1 bundle.
5 bundles = 1 bale.

WEIGHTS.

1 bu. of corn or rye = 56 lb.	1 bu. of barley = 48 lb.
1 bu. of corn meal, rye meal, or cracked corn = 50 lb.	1 bu. of timothy-seed = 45 lb.
1 bu. of wheat = 60 lb.	1 stone of iron or lead = 14 lb.
1 bu. of potatoes, beets, etc. = 60 lb.	1 pig of iron or lead = $21\frac{1}{2}$ stone.
1 bu. of beans or peas = 60 lb.	1 fother of iron or lead = 8 pigs.
1 bu. of oats = 32 lb.	

The weight of a bushel of barley, oats, etc., varies slightly in different States, but the weights here given are those generally adopted in business transactions.

WEIGHTS.

1 bbl. of flour = 196 lb.
1 bbl. of pork or beef = 200 lb.
1 cask of lime = 240 lb.
1 cental of grain = 100 lb.
1 quintal of fish = 100 lb.

BOOKS.

A book of sheets folded in:
2 leaves is a folio.
4 leaves is a quarto.
8 leaves is an octavo.
12 leaves is a duodecimo.
16 leaves is a 16mo.

Denominate Fractions.

332. Examples. 1. Express $\frac{2}{3}$ rd. in yards, feet, and inches.

SOLUTION. $\frac{2}{3}$ rd. = $\frac{2}{3}$ of $5\frac{1}{2}$ yd. = $3\frac{2}{3}$ yd.
$\frac{2}{3}$ yd. = $\frac{2}{3}$ of 3 ft. = 2 ft.

Hence, $\frac{2}{3}$ rd. = 3 yd. 2 ft.

2. Find the value of $\frac{3}{8}$ of £3 2s. 4d.

	£	s.	d.
8	3	2	4
		7	$9\frac{1}{2}$
			3
	1	3	$4\frac{1}{2}$

SOLUTION. We divide by 8 to get $\frac{1}{8}$ of £3 2s. 4d. and multiply the quotient by 3 to get $\frac{3}{8}$ of £3 2s. 4d. Hence, $\frac{3}{8}$ of £3 2s. 4d. is £1 3s. $4\frac{1}{2}$d.

3. Find the value of 0.3975 of a mile.

0.3975	
320	
79500	**Solution.** 0.3975 mi. = 0.3975 of 320 rd.
11925	= 127.2 rd.
127.2	0.2 rd. = 0.2 of $16\frac{1}{2}$ ft. = 3.3 ft.
$16\frac{1}{2}$	0.3 ft. = 0.3 of 12 in. = 3.6 in.
3.3	Hence, 0.3975 mi. = 127 rd. 3 ft. 3.6 in.
12	
3.6	

Exercise 82.

Find the value of:

1. $\frac{4}{5}$ of a mile.
2. $\frac{3}{16}$ of an acre.
3. $\frac{5}{8}$ of a hundred weight.
4. $\frac{2}{3}$ of a pound sterling.
5. $\frac{9}{11}$ of a mile.
6. $\frac{7}{11}$ of an acre.
7. $\frac{4}{9}$ of a degree.
8. $\frac{1}{3}$ of a year.
9. 0.15625 of a bushel.
10. 0.625 of a gallon.
11. 0.875 of a leap year.
12. 0.325 of a pound troy.
13. $6\frac{2}{5}$ of 3 A. $101\frac{1}{3}$ sq. rd.
14. $1\frac{3}{7}$ of 7 hr. 21 min. 27 sec.
15. 10.0175 of 1 dy. 13 hr.
16. $17\frac{7}{12}$ of 10 yd. 2 ft. $3\frac{1}{5}$ in.
17. 0.01284 of 14 mi.
18. 0.42776 of 12 t. 10 cwt.

Find the value of:

19. $\frac{2}{5}$ of 1 lb. + $3\frac{2}{3}$ oz. + $5\frac{2}{3}$ dwt.
20. 0.35 of 4 lb. 5 oz. 6 dwt. 16 gr.
21. 3.726 mi. — 33.57 rd.
22. $\frac{3}{73}$ of a year + $\frac{9}{56}$ of a week + $\frac{7}{12}$ of an hour.
23. 5.268 of 2 dy. + 2.829 of 16 hr. + 0.9528 of 25 min.
24. $\frac{3}{16}$ of a mile + $\frac{2}{3}$ of 40 rd. + $\frac{3}{8}$ of a yard.
25. $\frac{3}{4}$ of 2 cwt. 84 lb. + $\frac{3}{7}$ of 5 cwt. 98 lb. + $\frac{2}{5}$ of $7\frac{1}{2}$ lb.
26. $\frac{3}{8}$ of 21 ft. 7 in. + 0.855 of 16 ft. 2 in. + 0.365 of 1 ft.
27. 0.9 of 4 A. 17 sq. rd. — $\frac{1}{12}$ of 3 A. 15 sq. rd.
28. 0.652 of 2 cu. yd. 7 cu. ft. — 0.888 of 1 cu. yd. 2 cu. ft.
29. 0.456 of 12 bu. 3 pk. — 0.654 of 5 bu. 2 pk.

333. Examples. **1.** Express 10 hr. 33 min. 36 sec. as the fraction of a day.

SOLUTION. 36 sec. $= \frac{36}{60}$ min. $= \frac{3}{5}$ min.

$$33\tfrac{3}{5}\text{ min.} = \frac{33\frac{3}{5}}{60}\text{ hr.} = \tfrac{168}{300}\text{ hr.} = \tfrac{14}{25}\text{ hr.}$$

$$10\tfrac{14}{25}\text{ hr.} = \frac{10\frac{14}{25}}{24}\text{ dy.} = \tfrac{264}{600}\text{ dy.} = \tfrac{11}{25}\text{ dy.}$$

Hence, 10 hr. 33 min. 36 sec. $= \frac{11}{25}$ dy.

2. Express 127 rd. 0 ft. 7.92 in. as the decimal of a mile.

$$\begin{array}{r|l}
12 & 7.92 \text{ in.} \\
16\frac{1}{2} & 0.66 \text{ ft.} \\
320 & 127.04 \text{ rd.} \\
\hline
 & 0.397 \text{ mi.}
\end{array}$$

SOLUTION. $7.92 \div 12 = 0.66.$

$0.66 \div 16\frac{1}{2} = 0.04.$

$127.04 \div 320 = 0.397.$

Hence, 127 rd. 0 ft. 7.92 in. = 0.397 mi.

3. Express 1 yd. 2 ft. 4 in. as the common fraction, and as the decimal, of 5 yd. 1 ft. 4 in.

SOLUTION. 1 yd. 2 ft. 4 in. = 64 in.; 5 yd. 1 ft. 4 in. = 196 in.

$$\frac{64\text{ in.}}{196\text{ in.}} = \frac{64}{196} = \frac{16}{49} = 0.32653+.$$

Hence, 1 yd. 2 ft. 4 in. is $\frac{16}{49}$, or 0.32653+ of 5 yd. 1 ft. 4 in.

EXERCISE 83.

Express:

1. A pound avoirdupois as the fraction of a pound troy.
2. An ounce avoirdupois as the fraction of an ounce troy.
3. 363 sq. yd. as the fraction of an acre.
4. $\frac{3}{5}$ of £2 1s. 3d. + $\frac{5}{11}$ of £1 4s. 9d. as the fraction of £2 14s.
5. 2 mi. 138 rd. 1 yd. as the fraction of 3 mi. 265 rd. 3 yd. 1 ft. 6 in.
6. $\frac{2}{7}$ of 560 lb. as the fraction of 5 long tons.
7. $\frac{2}{3}$ of 200 rd. as the fraction of 4 mi.
8. $\frac{10}{27}$ of 2 dy. 2 hr. 24 min. as the fraction of 2 wk. 1 dy.

9. $\frac{4}{5}$ of the difference between 3 yd. 2 ft. 11 in. and 10 yd. 7 in. as the fraction of 8 yd.

10. $\frac{10}{21}$ of the difference between $\frac{5}{8}$ of 7 hr. and $\frac{7}{25}$ of 15 min. as the fraction of 12 hr. 18 min.

11. $\frac{2}{5}$ pt. as the fraction of a gallon.

12. 16s. $3\frac{3}{4}$d. as the decimal of a pound.

13. 233 rd. 9 ft. 10.8 in. as the decimal of a mile.

14. 71 sq. rd. 54 sq. ft. 64.8 sq. in. as the decimal of an acre.

15. 15 hr. 14 min. 6 sec. as the decimal of 2 days.

16. 38 sq. rd. 21 sq. yd. 5 sq. ft. 108 sq. in. as the decimal of an acre.

17. 3 mi. 242 rd. 2 yd. 2 ft. 3 in. as the decimal of 7 mi. 160 rd.

18. 5 hr. 13 min. 30 sec. as the decimal of a week.

19. 27° 14′ 45″ as the decimal of 90°.

20. 54 dy. 2 hr. 40 min. as the decimal of $365\frac{1}{4}$ days.

21. 2 lb. avoirdupois as the decimal of 10 lb. troy.

22. 44,920.9025 hr. as the decimal of a year.

23. 14.52 sq. yd. as the decimal of a square chain.

24. 8 cwt. 77 lb. 9.6 oz. as the decimal of a ton.

25. What part of 4 lb. 1 oz. 8 dwt. 15 gr. is 1 lb. 1 oz. 9 dwt. 15 gr.?

26. What part of 2 mi. is $\frac{2}{3}$ of 6 rd. 3 yd. 2 in.?

27. What part of a bushel is 1 pk. 2 qt. 1 pt.?

28. What part of 20 acres is 19 A. 3.5 sq. ch.?

29. What part of 5 tons is 3 t. 240 lb.?

30. What part of an acre is 38 sq. rd. 194 sq. ft. 108 sq. in.?

31. Express 2 lb. 9 oz. 21 dwt. as the decimal of 4 lb. 7 oz. 19 dwt.

32. Express 17 wk. 6 dy. 22 hr. 39 min. as the decimal of 35 wk. 3 dy. 15 hr. 25 min.

33. What part of 61 ft. 3 in. is 8 ft. 7 in.?

Longitude and Time.

334. Meridians are imaginary lines drawn straight around the earth through both poles.

335. Longitude is reckoned in degrees, minutes, and seconds east or west from a *standard meridian*, as the meridian of Greenwich, near London. Since longitude is reckoned east and west from a given meridian the longitude of a place is never greater than 180°, half the distance round the earth.

336. When two places are both east or both west of the standard meridian, the difference of their longitudes is found by subtracting the one from the other.

When one place is east and the other west of the standard meridian, the difference of their longitudes is found by adding the two longitudes.

If the sum of the two longitudes is greater than 180°, this sum must be subtracted from 360° to obtain the correct difference of longitude.

Thus, if one longitude is 130° west and another 120° east, the difference of their longitudes is $360° - (130° + 120°)$, or 110°.

337. As the earth turns upon its axis once in twenty-four hours, a point on the earth's surface will describe a circumference (360°) in twenty-four hours. Therefore, longitude may be reckoned in *time* as well as in degrees.

In one hour a point on the earth's surface describes $\frac{1}{24}$ of $360° = 15°$; in one minute, $\frac{1}{60}$ of $15° = 15'$; and in one second, $\frac{1}{60}$ of $15' = 15''$.

Again, since it requires one hour (60 min.) for a point to pass over 15°, to pass over 1° it requires $\frac{1}{15}$ of 60 min. = 4 min.; and to pass over 1′ it requires $\frac{1}{60}$ of 4 min. = 4 sec.

338. Examples. 1. Express 20° 36′ 15″ of longitude in time.

15)20° 36′ 15″	
1 hr. 22 min. 25 sec.	

SOLUTION. Since 15° longitude give 1 hr. in time, 15′ longitude 1 min., and 15″ longitude 1 sec., divide 20° 36′ 15″ by 15, as in compound division, and the quotient will be the time required, 1 hr. 22 min. 25 sec.

2. Express 1 hr. 4 min. 4 sec. in degrees.

1 hr. 4 min. 4 sec.	
15	
16° 1′ 0″	

SOLUTION. Since 1 hr. of time equals 15° of longitude, 1 min. of time 15′, and 1 sec. of time 15″, multiply 1 hr. 4 min. 4 sec. by 15, as in compound multiplication, and the product will be the longitude required. Hence,

339. If longitude is expressed in **degree-measures, divide** by 15; the quotient gives the longitude in **time-measures.**

If longitude is expressed in **time-measures, multiply** by 15; the product gives the longitude in **degree-measures.**

EXERCISE 84.

Find the difference in longitude between two cities, if the difference in time is :

1. 1 hr. 15 min.
2. 2 hr. 11 min.
3. 5 hr. 10 min. 10 sec.
4. 3 hr. 25 min. 35 sec.
5. 6 hr. 12 min. 30 sec.
6. 4 hr. 8 min. 12 sec.
7. 18 hr. 10 min.
8. 15 hr. 15 min. 15 sec.

Find the difference in time between two cities, if the difference in longitude is :

9. 9° 20′.
10. 70° 30′.
11. 56° 36′ 12″.
12. 108° 32′ 36″.
13. 120° 14′ 30″.
14. 100° 45′ 54″.
15. 2° 2′ 2″.
16. 75° 10′.

17. Find the difference in time between New York, longitude 74° 0′ 3″ West, and San Francisco, longitude 122° 26′ 15″ West.

18. The difference in time between Berlin and New York is 5 hr. 49 min. 35 sec. What is the difference in longitude?

340. Since the sun *appears* to move from east to west, sunrise will occur earlier at all points east and later at all points west of a given place. Hence, clock-time will be later in all places east and earlier in all places west of a given meridian.

Therefore, if the time of a place is given,

To find the time of a place **east, add** to the given time the difference of time between the two places.

To find the time of a place **west, subtract** from the given time the difference of time between the two places.

341. To Find the Difference in Clock-time when the Difference in Longitude is Known.

When it is noon at Boston (long. 71° 3′ 30″ West), what is the time at Paris (long. 2° 20′ 22″ East)?

71°	3′	30″ W.	
2°	20′	22″ E.	
73°	23′	52″. . .	difference in longitude.

15)73° 23′ 52″
4 hr. 53 min. $35\frac{7}{15}$ sec. difference in time.

SOLUTION. Since Boston is west and Paris is east of the meridian of Greenwich, the difference between their longitudes is found by taking the sum of their longitudes.

The difference between their longitudes, 73° 23′ 52″, is equivalent to 4 hr. 53 min. $35\frac{7}{15}$ sec., and as Paris is *east* of Boston, the time at Paris is found by *adding* the 4 hr. 53 min. $35\frac{7}{15}$ sec. to the time at Boston, making 53 min. $35\frac{7}{15}$ sec. past 4 P.M.

EXERCISE 85.

The longitude of some public building in:

(1) Berlin is 13° 23′ 43″ E.
(2) Rome, 12° 27′ 14″ E.
(3) Constantinople, 28° 59′ E.
(4) Pekin, 116° 23′ 45″ E.
(5) San Francisco, 122° 26′ 15″ W.
(6) St. Louis, 90° 15′ 15″ W.
(7) Jerusalem, 35° 32′ E.
(8) Bombay, 72° 54′ E.
(9) Calcutta, 88° 19′ 2″ E.
(10) Chicago, 87° 35′ W.
(11) New York, 74° 0′ 3″ W.
(12) Montreal, 73° 25′ W.

What is the clock-time at each of the above cities:

1. When it is noon at Greenwich?
2. When it is half-past four P.M. at Chicago?
3. When it is eight o'clock A.M. at Constantinople?

When it is noon at Greenwich the time at:

(1) Boston, Mass., is 7 hr. 15 min. 46 sec. A.M.
(2) Columbia, S. C., 6 hr. 35 min. 32 sec. A.M.
(3) Salt Lake, 4 hr. 30 min. A.M.
(4) Albany, N. Y., 7 hr. 5 min. 1 sec. A.M.
(5) Harrisburg, Penn., 6 hr. 52 min. 40 sec. A.M.
(6) New Orleans, La., 6 hr. A.M.
(7) Columbus, O., 6 hr. 27 min. 48 sec. A.M.
(8) Washington, D. C., 6 hr. 51 min. 44 sec. A.M.
(9) Springfield, Ill., 6 hr. 1 min. 48 sec. A.M.

4. What is the longitude of each of the above cities?

NOTE. **Standard time** is the clock-time of some selected meridian. **Eastern standard time** is the clock-time of the meridian 75° west of Greenwich, and is five hours slower than Greenwich time. **Central standard time** is the clock-time of 90° west of Greenwich, and is just one hour slower than Eastern standard time. **Mountain standard time** is the clock-time of the meridian of 105°, and is one hour slower than that of 90°. **Western standard time** is the clock-time of the meridian of 120°, and is one hour slower than that of 105°. The railroads and many cities and towns of the United States have adopted standard time.

Places not more than $7\frac{1}{2}$° east or west of the meridians of 75°, 90°, 105°, 120° are reckoned to have the same time respectively as places on these meridians.

EXERCISE 86. — REVIEW.

1. Reduce 7 gal. 3 qt. 1 pt. to gallons and the decimal of a gallon.

2. Reduce £4.375 to pounds, shillings, and pence.

3. Reduce 7.6875 gal. to gallons, quarts, and pints.

4. If $4.85 is equal to a pound, reduce to pounds, shillings, and pence $5.875; $7.38; $17.85; $21.75.

5. How many square yards in 3.7156 A.?

6. If 2 qt. of linseed oil are mixed with ½ pt. spirits of turpentine, what fraction of the mixture is turpentine? How much turpentine in one pint of the mixture?

7. Reduce 5.1732 mi. to yards, feet, and inches.

8. If a man walks 88 mi. in 26 hr., how many feet does he walk in a second?

9. Of a mixture of sand and lime 0.27 of the weight is lime. How many ounces of lime in a pound of the mixture? How many troy grains of lime in an avoirdupois pound of the mixture?

10. A gill of water is put into a quart measure, and the measure then filled with milk. What part of the mixture is water?

11. Reduce 555 ft. to the decimal of a mile.

12. Reduce 1 mi. 13 rd. 2 yd. 2 ft. 6 in. to inches.

13. How many cubic inches in 2½ cu. ft.?

14. How many pounds avoirdupois does a cubic yard of water weigh if a cubic foot weighs 1000 oz.?

15. Express the weight of a cubic yard of water as the decimal of a ton.

16. What is the weight of 7 bu. 3½ pk. of potatoes?

17. A farmer sowed 5 bu. 1 pk. 1 qt. of seed, and harvested from it 103 bu. 3 pk. 5 qt. How much did he raise from a bushel of seed?

18. How many bushels in 5 tons of oats?

19. How many bottles, each holding 1 pt. 3 gi., can be filled from a barrel of cider?

20. If a steamer makes 13 mi. 6 rd. an hour, how far will she go between 6 A.M. and 6 P.M.? How many hours will she require to make 113 mi.?

21. If a train runs at the average rate of 111 rd. a minute, how many hours will it require to run from Boston to Buffalo, 498 mi.?

22. What is the cost of 12 A. 146 sq. rd. of land at $16.25 an acre?

23. What is the cost of 8 t. 3 cwt. 27 lb. of coal at $5.75 a ton?

24. What is the cost of 7 t. 1560 lb. of hay at $15.50 a ton?

25. What is the cost of a car load of wheat weighing 20,000 lb. at $1.05 a bushel?

26. Reduce 5 rd. 4 yd. $2\frac{1}{2}$ ft. to the decimal of a mile.

27. Reduce 9 sq. ch. 11.25 sq. rd. to the decimal of an acre.

28. Reduce 0.09375 bu. to quarts.

29. Reduce 7560 chains to miles.

30. How many gross are 2000 pens?

31. Find the cost of 27.248 A. at $93.75 an acre.

32. Which is the greater, 2.8 of 3 ft. 11 in. or 3.11 of 2 ft. 8 in., and by how much?

33. Reduce 171 lb. 6 oz. troy to the decimal of a ton avoirdupois.

34. Express 14.52 sq. yd. as the decimal of a square chain.

35. If a sovereign is equal to 25.22 francs, or to $4.85, what decimal of a dollar is a franc?

36. If 0.327 of some work is done in 3 hr. 38 min., how long will the whole work require?

37. A can run a mile in 7.68 min.; B can run at the rate of 7.68 mi. an hour. Which is the faster runner?

38. How many miles an hour does a person walk who takes 2 steps a second and 1900 steps in a mile?

39. If an ounce troy of gold is worth $20, what is the value of a pound avoirdupois?

40. Two stars cross the meridian at 6 hr. 4 min. 42.3 sec. and 7 hr. 2 min. 57.21 sec., respectively. What is the interval between the observations?

41. How long will it take to fill $\frac{13}{25}$ of a cistern, when the whole requires 6 hr. 10 min.?

42. The circumference of a circle is 6 yd. 1 ft. 5.1 in. What is the length of an arc of 55°?

43. Multiply 2 t. 16 cwt. $63\frac{2}{5}$ lb. by $1\frac{4}{9}$.

44. Into how many shares has £120 been divided when each share is £3 8s. $6\frac{6}{7}$ d.?

45. If $\frac{14}{15}$ of one line is equal to $\frac{8}{9}$ of another line, which is the greater? What fraction of the greater is the less?

46. Multiply 5 mi. 206 rd. 2 ft. 2 in. by 786.

47. The returns of a gold mine are 241 t. of ore yielding 2 oz. 1 dwt. 15 gr. of fine gold a ton, and 193 t. yielding 1 oz. 12 dwt. 9 gr. a ton. Find the value of the whole yield, at $19.45 an ounce.

48. Divide 93 long tons 56 lb. by 23 lb. 5 oz.

49. Telegraph poles on railroads are generally erected at intervals of 88 yd. Show that if a passenger counts the number of poles which the train passes in three minutes, that number will express the number of miles an hour the train is going.

50. If Greenwich time is 5 hr. 8 min. 16 sec. later than Washington time, and Chicago is 87° 35′ W., what is the difference between Washington time and Chicago time?

51. What fraction of 21 cu. yd. 11 cu. ft. 1215 cu. in. is 3 cu. yd. 1 cu. ft. 1161 cu. in.?

52. How many minutes in the first three months of 1895? How many in the first three months of 1896?

53. A knot is $\frac{1}{60}$ of a degree, and a mile is 0.01477 of a degree. Find in miles the value of a knot to five decimals.

54. The captain of a steamer, sailing from Liverpool, found on taking an observation that the sun crossed his meridian at 42 min. 5 sec. past one o'clock P.M. by Greenwich time. Find his longitude.

55. If a walk 6 ft. wide is made round a park 600 ft. square within the enclosure, how many square yards will the walk contain?

56. How many pickets 3 in. wide, placed 3 in. apart, will be required to fence a rectangular lot 231 ft. long and 99 ft. wide? What will they cost at $3.25 per hundred?

57. The length of a year is 365.242218 mean solar days. Express the length of a year in days, hours, minutes, and seconds.

58. The Flying Dutchman Express runs from London to Exeter, a distance of $193\frac{1}{8}$ mi., in $4\frac{1}{4}$ hr., making one stop of 10 min., two of 5 min. each, and one of 3 min. What is its average speed per hour when in motion?

59. The Scotch Express runs from London to Edinburgh, a distance of $393\frac{3}{8}$ mi., in 9 hr., making one stop of 30 min., three of 5 min. each, and one of 3 min. What is its average speed per hour when in motion?

60. The Empire State Express runs from New York to Buffalo, a distance of 439 mi., in 8 hr. 15 min., making two stops of 3 min. each and two stops of 2 min. each. What is its average speed per hour when in motion?

61. How many dollars worth 4s. 2d. each will pay a bill of £11 17s. 6d.?

62. The lunar month is 29.53059 days. Express the length of a lunar month in days, hours, minutes, and seconds.

CHAPTER IX.

PROBLEMS.

342. Arithmetical Analysis. If the value of any number of units is given, we may by division find the value of *one unit* of the same kind, and by multiplication the value of *any number of units* of the same kind. The solution which combines these two processes is called *analysis*.

Example. If 18 yards of cloth cost $45, what will be the cost of 27 yards of cloth?

SOLUTION. If 18 yd. of cloth cost $45, 1 yd. will cost $\frac{1}{18}$ of $45, or $2½. If 1 yd. of cloth costs $2½, 27 yd. will cost 27 × $2½, or $67½.

EXERCISE 87.

1. If 15 yards of silk cost $18.75, what will be the cost of 20½ yards?

2. If 3¾ pounds of tea cost $3.80, how many pounds can be bought for $21.89?

3. If $\frac{3}{14}$ of a ton of coal costs $1.12, what is the price of 5¼ cwt.?

4. If $\frac{2}{11}$ of a piece of work is done in 25 days, what fraction of the work will be done in 11⅞ days?

5. A bankrupt's debts are $2520, and the value of his property is $1890. How much can he pay on a dollar?

6. If a bankrupt's debts are $4264, and he pays 62½ cents on a dollar, what are his assets?

7. If an ounce of gold is worth \$20.67, what is the value of 0.04 of a pound?

8. A man spent $\frac{3}{8}$ of his money for dry goods, $\frac{7}{9}$ of the remainder for groceries, and had \$15 left. How much had he at first?

SOLUTION. After spending $\frac{3}{8}$ of his money he had $\frac{5}{8}$ left. After spending $\frac{7}{9}$ of $\frac{5}{8}$ of his money he had left $\frac{2}{9}$ of $\frac{5}{8} = \frac{5}{36}$. Then, $\$15 = \frac{5}{36}$ of the money he had at first.

9. Sampson & Reed sold $\frac{5}{8}$ of a lot of wheat to one man, $\frac{3}{4}$ of the remainder to another, and had 93 bushels left. How much had they at first?

10. In a certain school $\frac{9}{16}$ of the scholars are girls; $\frac{5}{7}$ of the boys are over 16 years old, and 6 boys are under 16. How many girls and how many scholars are there in the school?

11. In a certain school $\frac{13}{24}$ of the scholars are boys; $\frac{9}{22}$ of the girls are under 16, and 13 girls are over 16. How many boys and how many girls are there in the school?

12. If from a certain number $\frac{3}{4}$ of it is subtracted, then $\frac{1}{5}$ of the remainder, then $\frac{1}{7}$ of that remainder, 6 still remains. What is the number?

13. A ship's cargo sold for \$45,000 belongs to three partners. A owns $\frac{7}{9}$ of $\frac{3}{5}$ of it, B's share is equal to $3\frac{3}{14}$ of $\frac{2}{9}$ of A's share, and C owns the remainder. What does each receive from the sale?

14. A man bequeathed $\frac{5}{12}$ of his property to A, $\frac{1}{4}$ of it to B, $\frac{1}{6}$ to C, $\frac{1}{8}$ to D, and the remainder, \$550, to E. What was the value of his whole property?

15. A farmer raised 321 bu. 3 pk. of corn from 9 acres of land. At the same rate, what would be the yield from 25 acres?

16. If 7 horses eat 21 bushels of oats in 16 days, how many days will 99 bu. 3 pk. last them?

17. If 12 horses can plow 96 acres in 6 days, how many horses will plow 64 acres in 8 days?

SOLUTION. In 6 days 96 acres can be plowed by 12 horses.
In 1 day 96 acres can be plowed by 6×12 horses.

In 1 day 1 acre can be plowed by $\frac{6 \times 12}{96}$ horses.

In 8 days 1 acre can be plowed by $\frac{6 \times 12}{8 \times 96}$ horses.

In 8 days 64 acres can be plowed by $\frac{64 \times 6 \times 12}{8 \times 96}$ horses.

18. If 40 acres of grass is mowed by 8 men in 7 days, how many acres will be mowed by 24 men in 28 days?

SOLUTION. 24 men will mow *three times* as much as 8 men in the same time; the same number of men will mow *four times* as much in 28 days as in 7 days. Hence, 24 men in 28 days will mow 3×4 or 12 times as much as 8 men in 7 days.

19. How many bushels of wheat will serve 72 people 8 days when 4 bushels serve 6 people 24 days?

20. If 2 horses eat 8 bushels of oats in 16 days, how many horses will eat 3000 bushels in 24 days?

21. If a man travels 150 miles in 5 days of 12 hours, in how many days of 10 hours will he travel 500 miles?

22. If 939 soldiers consume 351 bu. of wheat in 21 days, how many soldiers will consume 1404 bu. in 7 days?

23. If 5 men, working 16 hours a day, can reap a field of $12\frac{1}{2}$ acres in $3\frac{1}{2}$ days, in how many days can 7 men, working 12 hours a day, reap a field of 15 acres?

24. If 7 men in 8 days of 11 hours mow 22 acres, in how many days of 10 hours will 12 men mow 360 acres?

25. If 44 cannon, firing 30 rounds an hour for 3 hours a day, use 300 barrels of powder in 5 days, how many days will 400 barrels last 66 cannon, firing 40 rounds an hour for 5 hours a day?

Areas.

343. If the length and breadth of a rectangle are expressed in the same linear unit, the product of these two numbers will express its area in square units of the same name (§ 145).

Also, the number of square units in a rectangle divided by the number of linear units in one dimension gives the number of linear units in the other dimension (§ 145).

Exercise 88.

1. Find the area of a floor 16 ft. 3 in. long and 12 ft. 6 in. wide.

Solution. 16 ft. 3 in. = $16\frac{1}{4}$ ft.; 12 ft. 6 in. = $12\frac{1}{2}$ ft.
$16\frac{1}{4} \times 12\frac{1}{2} = 203\frac{1}{8}$, the area of the floor in square feet.

2. A rectangle contains 672 sq. ft. 108 sq. in., and is 19 ft. 6 in. wide. Find its length.

Solution. 672 sq. ft. 108 sq. in. = $672\frac{3}{4}$ sq. ft.; 19 ft. 6 in. = $19\frac{1}{2}$ ft.
$672\frac{3}{4} \div 19\frac{1}{2} = 34\frac{1}{2}$, the length of the rectangle in feet.

3. What length of board 15 in. wide will contain 11 sq. ft. 36 sq. in.?

4. What length of road 44 ft. wide will contain an acre?

5. Find the area of a rectangular field 13.12 chains long, 10.35 chains broad.

6. A path 216 ft. long measures 72 sq. yd. Find its breadth.

7. A rectangular field of 21.66 A. is 250.8 yd. broad. Find its length.

8. What is the area of a table, if its length and breadth are 4 ft. $3\frac{3}{7}$ in. and 2 ft. $9\frac{3}{8}$ in., respectively?

9. From each corner of a square, each side of which is 2 ft. 5 in. long, a square measuring 5 in. on a side is cut out. Find the area of the remainder of the figure.

10. The length and breadth of a map are $4\frac{1}{2}$ ft. and $3\frac{1}{8}$ ft., respectively. If the map represents 77,760 sq. mi. of country, how many square miles are there to a square inch?

11. In rolling a grass plot that is 24 yd. long and contains 400 sq. yd., how many times must a roller 3 ft. 4 in. wide be drawn over it lengthwise that the whole plot may be rolled?

12. How many sods, each 2 ft. $3\frac{1}{2}$ in. long and $8\frac{1}{4}$ in. broad, will be required to turf an acre of ground?

13. Find the area of a picture frame $2\frac{1}{4}$ in. broad, if the outside measurement is 4 ft. $6\frac{1}{2}$ in. in length and 2 ft. 8 in. in width.

14. Find the expense of glazing four windows, each containing 12 panes, if the panes are each 1 ft. long and 10 in. wide, and the price of the glass is 38 cents per square foot.

15. A field 76 yd. long and 56 yd. broad, enclosed by a wall, has a border 4 ft. wide within the wall, and within this a path 5 ft. wide. If the remainder of the field is grass, find the area of the border, of the path, and of the grass.

16. A square plot of land 127 yd. long has a path 1 yd. wide running round the inside of it. Find the cost of graveling this path at 15 cents per square yard.

17. A street $\frac{3}{4}$ of a mile long has on each side a sidewalk $7\frac{1}{2}$ ft. wide. What will it cost to pave the sidewalks with stones, each measuring 2 ft. 9 in. by 1 ft. 8 in., if the stones cost, including the laying, 75 cents each?

18. How many planks 11 ft. by 9 in. are needed to cover a platform 27 ft. 6 in. long and 8 yd. wide? What will be the cost at 20 cents a square foot?

19. How many tiles 9 in. long and 4 in. wide will be required to pave a walk 8 ft. wide that surrounds a rectangular court 60 ft. long and 36 ft. wide?

344. If the diameter of a circle is multiplied by 3.1416, the product is the length of the circumference (§ 138).

If the circumference of a circle is multiplied by 0.31831, the product is the length of the diameter (§ 139).

345. If the square of the radius of a circle is multiplied by 3.1416, or if the square of the diameter is multiplied by 0.7854 ($\frac{1}{4}$ of 3.1416), the product is the area of the circle (§ 146).

20. How many times will a wheel $2\frac{1}{8}$ ft. in diameter turn in going a distance of 110 yards?

21. What distance will a wheel $\frac{5}{11}$ yd. in diameter pass over in making $4\frac{1}{2}$ revolutions?

22. Find the diameter of a wheel that makes 9 revolutions in going $7\frac{1}{3}$ yards?

23. If the circumference of a wheel is $\frac{22}{7}$ of 1 yd. $1\frac{1}{8}$ ft., how many times will the wheel turn in going $3\frac{3}{7}$ miles?

24. If the wheel of a locomotive is $3\frac{1}{7}$ times 5.52 ft. in circumference, how many times does it turn in a minute when the locomotive is running at the rate of 13.34 mi. an hour?

25. Find the area of a circle that has a radius of 3 feet.

26. What is the area of a circular field that has a radius of 400 yards?

27. The radius of the rotunda of the Pantheon at Rome is 71 ft. 6 in. Find the area of the floor.

28. The diameter of a cylindrical cistern is 13 ft. What is the area of the bottom?

29. The two dials of the clock of St. Paul's, London, are each $18\frac{1}{7}$ ft. in diameter. What is the area of each in square feet?

30. At 20 cents a square yard, what will it cost to gravel a walk 6 ft. wide running round a circular fish pond 70 yd. in diameter?

346. If the square of the diameter of a sphere is multiplied by 3.1416, the product is the area of the surface of the sphere (§ 149).

31. How many square inches on the surface of a ball 3 inches in diameter?

32. How many square inches of surface on a spherical blackboard 12 inches in diameter?

33. What is the interior surface of a hemispherical vase whose interior diameter is 20 inches?

34. Find the external and the internal surface of a spherical shell whose external and internal diameters are 8 in. and 5 in., respectively.

35. How many square feet of tin are required to make 16 hemispherical bowls, each 2 ft. 4 in. in diameter?

347. The line joining the centres of the bases of a cylinder is called the *axis* of the cylinder (§ 163).

348. Right Cylinder. A cylinder whose axis is perpendicular to the bases is called a *right cylinder.*

349. To Find the Area of the Lateral Surface of a Right Cylinder,

Multiply the height of the cylinder by the perimeter of its base.

36. Find the lateral surface of a right cylinder if its height is 10 in. and the radius of its base is 7 in.

37. Find the lateral surface of a right cylinder if its height is 12 ft. and the diameter of its base is 9 ft. 4 in.

38. At 32 cents a square foot, what is the cost of cementing a cylindrical cistern 20 ft. deep and 18 ft. in diameter?

39. The diameters of two right cylinders of the same height are as 6 to 1. Compare the lateral surfaces.

Carpeting Rooms.

350. Carpeting is of various widths, and is sold by the yard. Oilcloth and linoleum are sold by the square yard.

To find the number of yards of carpeting required for a room, we decide whether the strips shall run lengthwise or across the room, and then find the number of strips required (§ 150). The number of yards in a strip, including the waste in matching the pattern, multiplied by the number of strips will give the number of yards required.

Exercise 89.

1. How many yards of carpeting 27 in. wide will be required for a floor 26 ft. long, $15\frac{3}{4}$ ft. wide, if the strips run lengthwise? How many if the strips run across the room? How much will be turned under in each case?

2. How many yards of carpeting $\frac{7}{8}$ yd. wide will be required for a room $8\frac{1}{2}$ yd. by 17 ft., if the strips run lengthwise, and if there is a waste of $\frac{1}{18}$ yd. a strip?

3. How many square yards of oilcloth will be required for a hall floor $5\frac{1}{4}$ yd. long and 10 ft. wide?

4. At \$0.92 a yard, what is the cost of a carpet 27 in. wide for a room $28\frac{1}{2}$ ft. by $18\frac{3}{4}$ ft., if the strips run lengthwise?

5. Find the cost of carpet 30 in. wide, at \$1.25 per yard, for a room 18 ft. by 14 ft., if the strips run lengthwise.

6. Find the cost of carpeting 27 in. wide, at $\$1.12\frac{1}{2}$ per yard, for a room 29 ft. 9 in. by 23 ft. 6 in., if the strips run across the room.

7. Find the cost of carpeting 27 in. wide, at \$2.75 per yard, for a room 34 ft. 8 in. by 13 ft. 3 in., if the strips run lengthwise, and if there is a waste of $\frac{1}{4}$ yd. a strip.

8. Which way must the strips of carpet 27 in. wide run to carpet most economically a room $20\frac{1}{2}$ ft. by $19\frac{1}{2}$ ft.?

Papering Rooms.

351. Wall paper is 18 in. wide, and is sold in single rolls 8 yd. long, or in double rolls 16 yd. long.

In estimating the number of rolls of paper required for a room of ordinary height, we find the number of feet in the perimeter of the room, leaving out the widths of the doors and windows, and allow a double roll or two single rolls for every seven feet.

NOTE. A room is considered of ordinary height if the distance from the base board to the border is not more than 8 ft.

9. How many double rolls of paper will be required for a room of ordinary height, 15 ft. long and 12 ft. wide, if the room has one door and three windows, each $3\frac{1}{2}$ ft. wide?

SOLUTION. Perimeter of room $= 2 \times (15 + 12)$ ft. $= 54$ ft.
Width of door and windows $= 4 \times 3\frac{1}{2}$ ft. $= \underline{14}$ ft.
Perimeter less door and windows $= 40$ ft.

$40 \div 7 = 5\frac{5}{7}$. Hence, 6 double rolls will be required.

10. At $2.25 a double roll, put on, what is the cost of papering a room of ordinary height, 16 ft. by 14 ft., if the room has two doors each 4 ft. wide, and four windows each 3 ft. 6 in. wide?

11. At 75 cents a single roll, put on, what is the cost of papering a room of ordinary height, 20 ft. 6 in. long and 17 ft. 4 in. wide, if the room has two doors each 3 ft. 6 in. wide, and five windows each 3 ft. 3 in. wide?

12. What is the cost of the border for the room of Ex. 11 at $0.45 a running yard?

NOTE. Border is sold by the yard, and no allowance is made for doors or windows.

13. At $1.75 a double roll, put on, what is the cost of papering a room of ordinary height, 18 ft. 6 in. by 14 ft. 4 in., if the room has three doors 4 ft. wide, and three windows 3 ft. 9 in. wide?

Plastering, Painting, and Paving.

352. The unit of plastering, painting, and paving is the *square yard.*

The rule for estimating these kinds of work is:

Measure the total area; deduct from this total area half the area of the doors, windows, and other openings, and express the result to the nearest square yard.

14. Find at 20 cents a square yard the cost of plastering the walls and ceiling of a room 18 ft. by 16 ft. by 10 ft., if the room has two doors 7 ft. 6 in. by 4 ft., three windows 6 ft. 6 in. by 4 ft., and a base board of 10 in.

15. Find at 25 cents a square yard the cost of plastering the walls and ceiling of a room 16 ft. by 15 ft. by 10 ft., if the room has two doors 7 ft. by 3 ft. 9 in., three windows 5 ft. 6 in. by 3 ft. 6 in., and a base board of 10 in.

16. Find at 20 cents a square yard the cost of plastering the walls and ceiling of a room 15 ft. by 14 ft. by 9 ft. 6 in., if the room has two doors 7 ft. 4 in. by 4 ft., two windows 5 ft. 6 in. by 3 ft. 6 in., and a base board of 9 in.

17. Find at 15 cents a square yard the cost of painting the outside of the walls of a cottage-roofed house 36 ft. by 32 ft. by 13 ft., if the house has three doors 7 ft. 6 in. by 4 ft., and eleven windows 6 ft. by 4 ft.

18. Find at 20 cents a square yard the cost of painting the walls of a room 16 ft. by 15 ft. by 10 ft., if the room has two doors 7 ft. 6 in. by 4 ft., four windows 6 ft. by 3 ft. 9 in., and a base board of 9 in.

19. How many bricks 8 in. long and 4 in. wide will be needed to pave a rectangular court 60 ft. by 30 ft.?

20. How many bricks 8 in. long and $2\frac{1}{2}$ in. thick, laid on edge, will be needed to pave the court of Ex. 19?

Clapboards and Shingles.

353. Clapboards. Clapboards are usually cut 4 ft. long and 6 in. wide, and laid $3\frac{1}{2}$ in. to the weather. Therefore, each clapboard covers $1\frac{1}{6}$ sq. ft. of surface.

NOTE. In estimating the number of clapboards required, we deduct the area of all openings.

354. Shingles. Shingles are 16 in. long, and are estimated to average 4 in. wide, so that a shingle laid $4\frac{1}{2}$ in. to the weather will cover 18 sq. in., and 8 shingles will cover 1 sq. ft. At this rate, 800 shingles would cover a *square*, or 100 sq. ft. It is found, in practice, that 1000 shingles, laid $4\frac{1}{2}$ in. to the weather, will cover about 120 sq. ft.

Shingles are put up in bunches of 250, and therefore it takes four bunches to make a thousand.

21. How many clapboards will be required for the front of a house 40 ft. long and 20 ft. high, allowing 120 sq. ft. for doors and windows?

22. How many clapboards will be required for a house 44 ft. long, 35 ft. wide, and 22 ft. high to the eaves, if the gables extend 14 ft. above the end walls, the two gables to be reckoned as one full wall, and 500 sq. ft. to be allowed for doors and windows?

23. Allowing 1000 shingles for 120 sq. ft., how many thousand will be required for the pitched roof of a house 60 ft. long, if the width of each side of the roof is $24\frac{1}{2}$ ft.?

24. Allowing 1000 shingles for 110 sq. ft., how many thousand will be required for the pitched roof of a barn 40 ft. long, if the width of each side of the roof is 24 ft.?

25. Allowing 1000 shingles for 120 sq. ft., how many thousand will be required for the pitched roof of a house 28 ft. long, if the width of each side of the roof is 18 ft.?

Board Measure.

355. Boards *one inch or less* in thickness are sold by the square foot.

Boards *more than one inch* in thickness, and all squared lumber, are sold by the number of square feet of boards *one inch* in thickness to which they are equivalent.

Thus, a board 16 ft. long, 1 ft. wide, and 1 in. thick contains 16 ft. *board measure.* If only $\frac{7}{8}$, $\frac{3}{4}$, or $\frac{1}{2}$ of an inch thick, it still contains 16 ft.; but if $1\frac{1}{4}$ in. thick, it contains $1\frac{1}{4} \times 16$, or 20 ft. board measure.

356. To Find the Number of Feet Board Measure in Boards an Inch or More in Thickness, and Squared Lumber,

Express the length and width in feet, and the thickness in inches; take the product of these three numbers for the number of feet board measure.

NOTE 1. In practice, the width of a board, unless sawed to order, is reckoned only to the next smaller half-inch. Thus, a width of $11\frac{3}{8}$ in. is reckoned 11 in.; of $13\frac{5}{8}$ or $13\frac{3}{4}$ in. is reckoned $13\frac{1}{2}$ in.

NOTE 2. If a board tapers regularly, its average width is found by taking one half the sum of its end widths.

How many feet board measure in:

26. A board 18 ft. long, 9 in. wide, $\frac{7}{8}$ in. thick?

27. A board 16 ft. long, 11 in. wide, 1 in. thick?

28. Twenty boards averaging 14 ft. long, 10 in. wide, $1\frac{1}{2}$ in. thick?

29. Three joists 13 ft. long, 8 in. wide, 3 in. thick?

30. A stick of timber 8 in. by 9 in., and 27 ft. long?

31. Two beams, each 6 in. by 9 in., and 23 ft. long?

32. Three joists, each 3 in. by 4 in., and 11 ft. long?

33. Five joists, each 6 in. by 4 in., and 14 ft. long?

34. A stick of timber 10 in. square, and 36 ft. long?

35. Ten planks, each 13 ft. long, 15 in. wide, 2 in. thick?

Find the cost of:

36. Nine joists, each 15 ft. long, $3\frac{1}{2}$ in. by 5 in., at \$12 per M.

NOTE. The abbreviation *per M* means *by the thousand.*

37. Thirty planks, each 12 ft. long, 11 in. wide, 3 in. thick, at \$15 per M.

38. Four sticks of timber, each 8 in. by 9 in. and 23 ft. long, at \$18 per M.

39. A board 24 ft. long, 23 in. wide at one end and 17 in. at the other, and $1\frac{1}{2}$ in. thick, at \$30 per M.

40. A stick of timber 29 ft. long, 10 in. by 12 in., at \$13.50 per M.

41. The flooring for two floors, each 23 ft. by 17 ft., each floor double, and of boards $\frac{7}{8}$ in. thick; the under floors at \$18, and the upper at \$24, per M.

42. The flooring timbers for a room 23 ft. by 17 ft., at \$18 per M, if they are 2 in. by 10 in., 17 ft. long, and are placed on edge, one close to each wall and the others with spaces $\frac{39}{40}$ ft. wide between them.

357. Round Logs. Round logs are sold by the number of feet board measure that can be cut from them. If a log is not more than 16 ft. long, we measure the length of the log and the diameter of the *smaller end,* and find the number of feet board measure as follows:

Subtract twice the diameter expressed in inches from the square of the diameter, and take $\frac{21}{40}$ of the remainder for the number of feet board measure in a log 10 ft. long.

43. Find the number of feet board measure in a log 12 ft. long, and 20 in. in diameter at the smaller end.

SOLUTION. $20^2 - 2 \times 20 = 400 - 40 = 360.$

$\frac{21}{40}$ of $360 = 189.$

As the log is 12 ft. long, we must take $\frac{12}{10}$ of 189 ft. to obtain the number of feet in the whole log; that is, 226.8 ft.

By this rule find the number of feet board measure in:

44. A log 14 ft. long, smallest diameter 17 in.

45. A log 11 ft. long, smallest diameter 13 in.

46. A log 16 ft. long, smallest diameter 20 in.

47. A log 12 ft. long, smallest diameter 15 in.

Find the value at $9 per M of:

48. A log 15 ft. long, smallest diameter 11 in.

49. A log 16 ft. long, smallest diameter 13 in.

50. A log 13 ft. long, smallest diameter 16 in.

51. A log 14 ft. long, smallest diameter 12 in.

358. Large, heavy timber of hard wood is generally sold by the ton, signifying 50 cu. ft., or 600 ft. board measure.

Volumes.

359. If the length, breadth, and height of a rectangular solid are expressed in the same linear unit, the product of these three numbers will express its volume in cubic units of the same name (§ 161).

Also, the number of cubic units in a rectangular solid divided by the product of the numbers of linear units in any two dimensions gives the number of linear units in the third dimension (§ 161).

Exercise 90.

1. Find the volume of a rectangular solid 7 ft. long, 2 ft. 6 in. wide, and 11 in. thick.

2. How many cubic feet of air in a hall 54 ft. long, 33 ft. wide, and 21 ft. 4 in. high?

3. Find the volume of a cube whose edge is $2\frac{1}{2}$ yd.

4. A cellar was dug 21 ft. long, 17 ft. 3 in. wide, and 9 ft. deep. How many cubic yards of earth were taken out?

5. Find the volume of a brick 8 in. long, $3\frac{1}{4}$ in. wide, and $2\frac{1}{4}$ in. thick.

6. How many cubic feet of water will a rectangular cistern hold whose length, breadth, and height are 5 ft. 4 in., 3 ft. 6 in., and 2 ft. 10 in., respectively?

7. Find the volume in cubic inches of a bar of iron 21 ft. long, 3 in. wide, and 2 in. thick.

8. What is the value at $190 a cubic inch of a bar of gold 8 in. long and $\frac{3}{4}$ of an inch square?

9. A rectangular reservoir 15 yd. long, 12 yd. wide, holds 330 cu. yd. of water. What is its depth?

10. What length must be cut off a beam 9 in. by 15 in. that the part cut off may contain $2\frac{1}{2}$ cu. ft.?

11. How high is a room, if it is 31 ft. 3 in. long, 24 ft. broad, and contains 10,000 cu. ft. of air?

12. A piece of wood 5 ft. long, 1 ft. broad, and 9 in. thick is cut up into matches $2\frac{1}{2}$ in. long and 0.1 of an inch square. How many matches will there be, if no allowance is made for waste in cutting?

13. How long a wall 6 ft. high, $12\frac{3}{4}$ in. thick, can be built with the bricks forming a rectangular pile 17 ft. 6 in. long, 5 ft. wide, and 4 ft. 3 in. high?

14. Find the surface of a cube whose edge is 3 ft. $5\frac{3}{4}$ in.

15. Find the surface of a rectangular block of stone 4 ft. long, $2\frac{1}{2}$ ft. broad, and $1\frac{1}{2}$ ft. thick.

16. A lake whose area is 45 A. is covered with ice 3 in. thick. Find the weight of the ice in tons, if a cubic foot of ice weighs 920 oz.

17. How many bricks will be required to build a wall 75 ft. long, 6 ft. high, and 16 in. thick, if each brick is 8 in. long, 4 in. wide, and $2\frac{1}{4}$ in. thick?

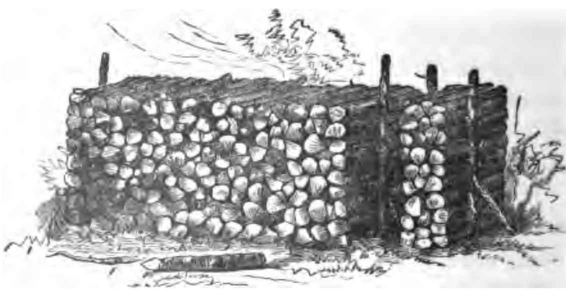
18. The ceiling of a room 27 ft. long, 24 ft. broad, and 10 ft. high is to be raised so as to increase the space by 84 cu. yd. What will then be the height of the room?

19. Find the cost of making a road 110 yd. long and 18 ft. wide, if the soil is first removed to the depth of 1 ft. at a cost of 25 cents a cubic yard, rubble then laid 8 in. deep at 25 cents a cubic yard, and gravel placed on top 9 in. thick at $62\frac{1}{2}$ cents a cubic yard.

20. If a rectangular block of wood 5 ft. 4.8 in. long, 1 ft. 9 in. wide and thick, weighs 7.56 cwt., find in pounds its weight per cubic foot.

360. A **cord** of wood or stone is a pile 8 ft. long, 4 ft. wide, and 4 ft. high, making 128 cu. ft.

A **cord foot** is a pile 1 ft. long, 4 ft. wide, and 4 ft. high, and is therefore *one eighth* of a cord, or 16 cu. ft. **Hence,**



Cord. Cord Foot.

361. To Find the Number of Cords in a Pile of Wood,

Divide the product of the length, breadth, and height, expressed in feet, by $8 \times 4 \times 4$.

21. How many cords of wood in a pile 40 ft. long, 4 ft. wide, and 5 ft. 4 in. high?

22. A pile of wood containing $67\frac{1}{2}$ cords is 270 ft. long and 4 ft. wide. How high is it?

23. What will be the cost of a pile of wood 25 ft. long, 4 ft. wide, and 4 ft. 8 in. high, at $3.75 a cord?

24. What must be the length of a load of wood $3\frac{1}{2}$ ft. high and 5 ft. wide to contain a cord?

25. How high must manure be piled in a cart 6 ft. by 4 ft., that the load may contain half a cord?

26. How many cords of wood in a pile 32 ft. long, 8 ft. wide, and 6 ft. high?

27. How many cords of wood in a pile 40 ft. long, 4 ft. wide, and 8 ft. high?

28. Find the cost of the wood at $3.75 a cord that can be piled in a shed 18 ft. long, 16 ft. wide, and 7 ft. high.

362. By § 162, to find the volume of a sphere, we multiply the cube of the diameter by 0.5236 ($\frac{1}{6}$ of 3.1416).

29. Find the number of cubic inches in a sphere 11 in. in diameter.

30. How many cubic inches of water can be poured into a hollow sphere whose inner diameter is $16\frac{1}{2}$ in.?

31. What is the volume of the ball on top of St. Paul's in London, which is 6 ft. in diameter?

32. If 30 cu. in. of powder weigh 1 lb., how many ounces of powder will just fill a shell, inner diameter 3 in.?

363. To find the volume of a cylinder, we multiply the number of square units in its base by the number of linear units of the same name in its height (§ 164).

33. Find the volume of a cylinder whose height is 5 ft. and the radius of whose base is 1 ft. 2 in.

34. Find the volume of a cylinder whose height is 4 ft. 6 in. and the diameter of whose base is 8 ft. 2 in.

35. How many cubic yards of earth must be excavated to make a well 3 ft. in diameter and 20 ft. deep?

36. How many cubic yards in a tunnel 800 ft. long, if a cross section is a semicircle with a radius of 10 ft.?

Capacity of Bins and Cisterns.

37. Find the number of cubic feet in a bushel.

SOLUTION. Since a bushel contains 2150.42 cu. in. (§ 294), and a cubic foot contains 1728 cu. in., therefore, a bushel contains $\frac{2150.42}{1728}$ cu. ft., or 1.24445 cu. ft.

If we add $\frac{1}{2}$ of 0.01 of 1.24445 to 1.24445, we have 1.25+. Hence,

364. To Find the Approximate Number of Bushels a Bin will Hold,

Take $\frac{4}{5}$ of the number of cubic feet in the bin, and add to the product $\frac{1}{2}$ of 0.01 of the product.

365. To Find the Number of Cubic Feet Required for a Given Number of Bushels,

Take $\frac{5}{4}$ of the number of bushels, and subtract from the product $\frac{1}{2}$ of 0.01 of the product.

38. Find the number of bushels a bin will hold that is 6 ft. long, 5 ft. wide, and 4 ft. deep.

SOLUTION.

$$\begin{aligned} \tfrac{4}{5} \text{ of } 6 \times 5 \times 4 &= 96. \\ \tfrac{1}{2} \text{ of } 0.01 \text{ of } 96 &= \underline{\ 0.48} \\ &\ 96.48 \end{aligned}$$

Therefore, the bin will hold 96.48 bu.

39. Find the number of cubic feet required for 1000 bu.

SOLUTION.

$$\begin{aligned} \tfrac{5}{4} \text{ of } 1000 &= 1250. \\ \tfrac{1}{2} \text{ of } 0.01 \text{ of } 1250 &= \underline{\quad 6.25} \\ &\ 1243.75 \end{aligned}$$

Hence, 1243.75 cu. ft. are required for 1000 bu.

40. Find the number of bushels a bin will hold that is 8 ft. long, 4 ft. wide, 3 ft. deep.

41. Find the number of bushels a bin will hold that is 9 ft. long, 6 ft. 6 in. wide, 3 ft. 4 in. deep.

42. Find the depth of a bin that will hold 360 bu., if its length is 12 ft. and its width 6 ft.

43. Find the length of a bin that is 6 ft. wide and 5 ft. deep, if it will hold 400 bu.

44. Find the number of bushels that will fill a bin 8.5 ft. long, 4.5 ft. wide, 3.5 ft. deep.

45. A bin 20 ft. long, 12 ft. wide, and 6 ft. deep is full of wheat. What is its value at $0.75 a bushel?

46. If a ton of coal occupies 40 cu. ft., how many tons of coal will fill a bin 21 ft. long, 10 ft. wide, 5 ft. deep?

47. If a ton of Lehigh coal occupies 35 cu. ft., how many tons of Lehigh coal will fill a bin 8 ft. long, 5 ft. 9 in. wide, 3 ft. 6 in. deep?

48. How many bushels will a bin hold that is 22 ft. long, 12 ft. 6 in. wide, 9 ft. 9 in. deep?

49. Find the number of gallons in a cubic foot.

SOLUTION. Since a cubic foot contains 1728 cu. in., and a gallon contains 231 cu. in., therefore, a cubic foot contains $\frac{1728}{231}$ gal., or 7.48052 gal.

If we add $\frac{1}{4}$ of 0.01 of 7.48052 to 7.48052, we have 7.5 nearly. Hence,

366. To Find the Approximate Number of Gallons a Cistern will Hold,

Multiply the number of cubic feet by $7\frac{1}{2}$, *and from the product subtract* $\frac{1}{4}$ *of 0.01 of the product.*

367. To Find the Exact Number of Gallons a Cistern will Hold,

Divide the number of cubic inches in the contents of the cistern by 231.

50. Find the exact number of gallons a cistern will hold that is 5 ft. square, and 6 ft. deep.

51. Find the exact number of gallons a cistern will hold that is 13 ft. long, 6 ft. wide, 7 ft. 4 in. deep.

52. Find the exact number of gallons a tank will hold that is 4 ft. long, 2 ft. 8 in. wide, 1 ft. 8 in. deep.

53. Find the capacity in cubic feet of a cistern that will hold 200 bbl. of water.

54. Find the approximate number of gallons a cylindrical cistern will hold that is 6 ft. in diameter and 7 ft. deep.

55. Find the approximate number of gallons a cylindrical vessel will hold that is 12 in. in diameter and 10 in. deep.

56. How many quarts will a cylindrical vessel hold $5\frac{1}{8}$ in. in diameter and 6 in. deep?

57. How many quarts will a hollow sphere hold whose interior diameter is 12 in.?

58. What part of a bushel will a hemispherical bowl hold that is 13 in. in diameter?

59. If a cubical box 2 ft. on an edge contains a solid sphere 2 ft. in diameter, how many gallons of water can be poured into the box?

60. If 64 qt. of water are poured into a vessel that will hold 2 bu. of wheat, what part of the vessel will be filled?

Specific Gravity.

368. The specific gravity of a substance is the number found by dividing the weight of the substance by the weight of an equal bulk of water (§ 165).

NOTE. If a substance is in water, the water buoys it up by just the weight of the water displaced by it.

EXERCISE 91.

1. Find the number of cubic inches in 1 oz. (av.) of water.
2. Find the weight in ounces (av.) of 1 cu. in. of water.
3. Find the weight in ounces (av.) of 1 pt. of water.
4. Find the number of pints in 1 lb. of water.
5. Find the weight in grains of 1 cu. in. of water.
6. A bar of iron 5 in. long and 2 in. square weighs 5 lb. What is the specific gravity of the iron?

7. If a bar of iron 18 in. long, $2\frac{1}{8}$ in. wide, $1\frac{3}{4}$ in. thick, weighs 18 lb. 9 oz., what is the specific gravity of the iron?

8. If the specific gravity of iron is 7.48, find the number of cubic inches of iron to the pound.

9. If the specific gravity of gold is 19.36, find the number of cubic inches in 2 lb. $6\frac{1}{2}$ oz. of gold.

10. How many pounds does a boy lift in raising a cubic foot of stone under water, if its specific gravity is $2\frac{1}{2}$?

11. A square-built scow 12 ft. long, $6\frac{1}{2}$ ft. wide, sinks 5 in. into the water. What does it weigh, and how many pounds will be required to sink it 7 in. deeper?

12. A square-built scow 11 ft. long, $5\frac{1}{4}$ ft. wide, weighs 320 lb., and is loaded with 750 lb. of stone. How deep does it sink in the water?

13. How many tons of ice, specific gravity 0.93, can be packed in a building 50 ft. long, 40 ft. wide, 20 ft. high?

14. If the specific gravity of an iceberg is 0.9, how many cubic yards does an iceberg contain that is 40 rd. long, 6 yd. wide, and rises 160 ft. out of the sea?

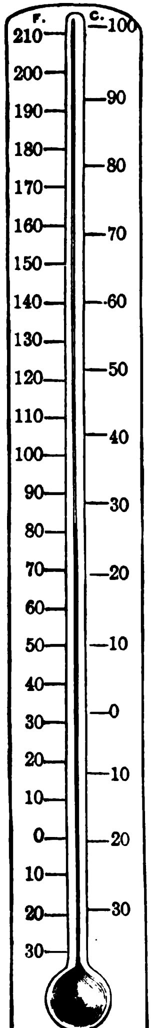
15. If a cubic foot of brick wall weighs 90 lb. and contains 22 bricks, with the mortar, what is the weight and the specific gravity of a brick and its share of mortar?

16. What is the weight of a brick wall 40 ft. long, 20 ft. high, and 1 ft. thick, if the specific gravity of a brick with its mortar is 1.46? How many thousand bricks will be required for the wall, allowing 22 for a cubic foot?

17. If the specific gravity of iron is 7.48, what is the weight of a cylindrical iron shell 1 in. thick and 2 ft. long, whose inner radius is 7 in.?

18. If a piece of marble weighs 37.78 oz. in air, and 23.89 oz. in water, what is its volume and its specific gravity?

19. If a mass of lead weighs $1986\frac{1}{4}$ lb. in air, and $1811\frac{1}{4}$ lb. in water, what is its volume and its specific gravity?



Thermometer.

Temperature.

369. **A thermometer** is an instrument for registering temperature.

There are three scales for registering temperature by means of the thermometer.

Fahrenheit's has the freezing point of water marked 32°, and boiling point *212°*.

The **Centigrade** has the freezing point 0°, and the boiling point 100°.

Réaumur's has the freezing point 0°, and the boiling point 80°.

Temperature below 0° is indicated by prefixing the minus sign.

Thus, — 20° means 20° *below* zero.

370. Examples. **1.** Express 80° C. in Réaumur's scale.

100° C. = 80° R.

Therefore, 80° C. = $\frac{80}{100}$ of 80° R.
= 64° R.

2. Express 50° F. in Centigrade scale.

The number of degrees F. between the freezing and boiling points is 180.

Therefore, 1° F. = $\frac{100}{180}$° C. = $\frac{5}{9}$° C.

But 50° F. is 18° F. above freezing point,

and $\frac{5}{9}$ of 18° = 10°.

That is, 50° F. = 10° C.

3. Express 60° C. in Fahrenheit's scale.

100° C. = 180° F.

Therefore, 60° C. = $\frac{60}{100}$ of 180° F.
= 108° F.

This is the height above the freezing point, and is marked 32° + 108° = 140°.

Exercise 92.

Express:

1. 59° F. in Centigrade scale; in Réaumur's scale.
2. 77° F. in Centigrade scale; in Réaumur's scale.
3. 950° F. in Centigrade scale; in Réaumur's scale.
4. −40° F. in Centigrade scale; in Réaumur's scale.
5. −4° F. in Centigrade scale; in Réaumur's scale.
6. 10° C. in Fahrenheit's scale; in Réaumur's scale.
7. 22° C. in Fahrenheit's scale; in Réaumur's scale.
8. −30° C. in Fahrenheit's scale; in Réaumur's scale.
9. $-11\frac{3}{7}$° C. in Fahrenheit's scale; in Réaumur's scale.

Time and Work Problems.

Exercise 93.

1. If one man can do a piece of work in 9 days and another man can do the same work in 8 days, in how many days can the men working together do the work?

Solution. If a man can do a piece of work in 9 days, in 1 day he can do $\frac{1}{9}$ of the work; and if another man can do the same work in 8 days, in 1 day he can do $\frac{1}{8}$ of it.

Both men together in 1 day can do $\frac{1}{9}+\frac{1}{8}$, or $\frac{17}{72}$ of the work.

Therefore, if the whole work is considered as divided into 72 equal parts, they together can do 17 of these parts in 1 day, and the number of days required to do the whole work will be $\frac{72}{17}$, or $4\frac{4}{17}$.

2. A cistern can be filled by a water-pipe in 30 min. and emptied by a waste-pipe in 20 min. If the cistern is full and both pipes are opened, in how many minutes will the cistern be emptied?

Solution. In 1 min. the waste-pipe empties $\frac{1}{20}$ of the cistern.

In 1 min. the water-pipe fills $\frac{1}{30}$ of the cistern.

When both are open $\frac{1}{20}-\frac{1}{30}$, or $\frac{1}{60}$ of the cistern, is emptied in 1 min.

Therefore, the cistern will be emptied in 60 min.

3. If A can mow a certain meadow in 4 days, and B in 3 days, how long will it take both together?

4. If A can lay a certain wall in $4\frac{1}{2}$ days, and B in $5\frac{1}{2}$ days, how long will it take both together?

5. If one pipe will fill a cistern in $4\frac{1}{2}$ hr., and another pipe in $3\frac{1}{2}$ hr., how long will it take both together to fill the cistern?

6. If A can go from Boston to Albany in $9\frac{1}{4}$ hr., and B from Albany to Boston in $11\frac{1}{8}$ hr., and they start at the same time, in how many hours will they meet?

7. If it takes A working alone 4 days, B 3 days, and C $4\frac{1}{2}$ days to do a piece of work, how long will it take to do the work if all three work together?

8. A can mow $\frac{5}{8}$ of a field in 3 days; and B $\frac{2}{3}$ of it in 4 days. How long will it take both together to mow the field?

9. One pipe can fill a cistern half full in $\frac{3}{4}$ of an hour, and another can fill it three quarters full in $\frac{1}{2}$ an hour. How long will it take both pipes together to fill the cistern?

10. A pipe can fill a cistern one third full in $\frac{1}{4}$ of an hour; a waste-pipe can empty one fourth of the cistern in 20 minutes. If both pipes are opened, in what time will the cistern be filled?

11. A cistern that will hold 100 gallons can be filled by a pipe in 25 minutes and emptied by a waste-pipe in 45 minutes. If the cistern is empty and both pipes are opened, how long will it take to fill the cistern, and how much water will be wasted?

12. If water runs into a cistern by one pipe at the rate of 2 gal. in 3 min., by another at the rate of 5 gal. in 4 min., and runs out by a third at the rate of 4 gal. in 5 min., how long will it take to gain 71 gal. in the cistern?

13. A can do a piece of work in 6 days, and B can do it in 7 days. If they work together 2 days, and A then leaves, how long will it take B to finish the work?

14. A cistern that will hold 200 gal. has two pipes; one will supply 0.15 gal. a second, the other $1\frac{3}{5}$ qt. a second. If the first is turned on for 10 minutes and afterwards both run together, in what time will the cistern be filled?

15. A and B together can do a piece of work in 15 days. After working together 6 days, A leaves and B finishes the work in 30 days more. In how many days can each alone do the work?

16. A and B together can do a piece of work in 12 days. After working together 9 days, however, they call in C to help them, and the three finish the work in 2 days. In how many days can C alone do the work?

17. A and B together can do a piece of work in $2\frac{1}{2}$ days; A and C in $3\frac{1}{3}$ days; B and C in $3\frac{3}{4}$ days. How long will it take the three working together to do the work, and how long will it take each alone?

NOTE. By working *two days each* they can do $\frac{1}{2\frac{1}{2}} + \frac{1}{3\frac{1}{3}} + \frac{1}{3\frac{3}{4}}$ of the work, that is, $\frac{2}{5} + \frac{3}{10} + \frac{4}{15}$ or $\frac{29}{30}$ of the work. Hence, by working one day each, they can do $\frac{1}{2}$ of $\frac{29}{30}$, or $\frac{29}{60}$ of the work.

In one day A can do $\frac{29}{60} - \frac{4}{15}$ of the work.

18. A and B together can do a piece of work in 48 days; A and C together in 30 days; B and C together in $26\frac{2}{3}$ days. How long will it take each alone to do the work?

19. A cistern has three pipes. The first and second will fill it in 1 hr. 10 min.; the first and third in 1 hr. 24 min.; the second and third in 2 hr. 20 min. How long will it take each alone to fill the cistern?

20. A, B, and C together can do a piece of work in 10 days; A and B together in 12 days; B and C together in 20 days. How long will it take each alone to do the work?

Rate and Time Problems.

Exercise 94.

1. A train travels 24 miles in 0.8 of an hour. Find its rate per hour.

SOLUTION. If the question had read, a train travels 70 mi. in 2 hr., its rate per hour would be found by dividing the whole distance, 70 mi., by 2. The application of the same method to this question gives 24 mi. ÷ 0.8, or 30 mi., for the rate per hour.

2. A train runs from New York to Philadelphia, 90 miles, in 1 hr. 33 min. What is its rate per hour?

SOLUTION. 1 hr. 33 min. = $1\frac{11}{20}$ hr. Therefore, the rate per hour is 90 mi. ÷ $1\frac{11}{20}$, or $58\frac{2}{31}$ mi. Hence,

371. To Find the Rate when the Distance and the Time are Known,

Divide the distance by the number of units of time.

3. A train runs from New York to Philadelphia, 90 miles, in 2 hr. 5 min. What is its rate per hour?

4. Winlock, in 1869, found that electricity went through 7200 miles of wire in $\frac{3}{8}$ of a second. What was its rate per second?

5. If the time required for a signal to pass through the cable from Brest to Duxbury, 3799 miles, is 0.816 of a second, what is the rate per second?

6. If the report of a gun $1\frac{1}{4}$ miles distant is heard in $5\frac{5}{8}$ seconds after the flash is seen, what is the velocity of sound in feet per second?

7. If a man walks $3\frac{1}{5}$ miles in 46 minutes, what is his rate per hour?

8. If a horse goes 48 miles in 10 hr. 40 min., what is his average rate per hour?

9. If a stone on a glacier is carried $95\frac{1}{2}$ feet in 188 days, what is its rate in inches per day?

10. If a horse went $5\frac{1}{2}$ miles in 33 minutes, how long did it take him to go a mile?

SOLUTION. 33 min. $\div 5\frac{1}{2} = 6$ min. Hence,

372. To Find the Rate for a Unit of Distance when the Distance and the Time are Known,

Divide the time by the number of units of distance.

11. If a horse can trot $\frac{5}{8}$ of a mile in $2\frac{1}{3}$ minutes, in what time can he trot a mile?

12. If a train runs 18 miles in 39 minutes, how long does it take to run one mile?

13. If sound travels 1125 feet a second, how long will it take to travel one mile?

14. If a train requires 3 hours to run $104\frac{1}{4}$ miles, find its average time for running a mile.

15. If a man cuts $7\frac{1}{2}$ A. of grass in $3\frac{1}{2}$ days, what part of a day will it take him to cut an acre? If 10 hr. makes a day, what part of an acre will he cut in an hour?

16. If a mower cuts $3\frac{1}{2}$ square rods in $\frac{1}{8}$ of an hour, how many acres will he cut in a day of 10 hours?

17. If a fountain yields $117\frac{1}{2}$ gallons of water in $\frac{3}{4}$ of an hour, at what rate per hour is the water flowing?

18. If a merchant's profits are $3147 in $7\frac{1}{2}$ months, what will be his profits at the same rate for a year?

19. If a wheel turns $17° 30'$ in 35 minutes, in how many hours does it make a complete revolution?

20. If a man's expenditures are $4358 in $13\frac{1}{3}$ months, what is his yearly rate of expenditure?

21. If a cistern loses by leakage 7 gal. 1 pt. in 49 hr. 40 min., what is its hourly rate of loss?

22. If a man travels $3\frac{3}{5}$ miles in $7\frac{1}{2}$ minutes, how many miles will he travel in 50 minutes? How long will it take him to travel 50 miles?

Clock Problems.

Exercise 95.

1. At what time between 5 and 6 o'clock do the hour and minute hands of a clock coincide?

Solution. Since in one hour the hour hand moves through 5 minute-spaces, and the minute hand through 60 minute-spaces, the minute hand moves 12 times as fast as the hour hand, and in moving through 12 minute-spaces gains 11 minute-spaces.

When the hour hand is at V, the minute hand, being at XII, is 25 minute-spaces behind. Since to gain 11 minute-spaces the minute hand must move through 12 minute-spaces, to gain 1 minute-space the minute hand must pass through $\frac{12}{11}$ of 1 minute-space, and to gain 25 minute-spaces, it must pass through $25 \times \frac{12}{11}$, or $27\frac{3}{11}$ minute-spaces.

Hence, the hands coincide when the minute hand has moved through $27\frac{3}{11}$ minute-spaces; that is, at $27\frac{3}{11}$ min. after 5 o'clock.

2. At what time between 10 and 11 o'clock do the hour and minute hands of a watch coincide?

3. At what time between 1 and 2 o'clock do the hour and minute hands of a clock coincide?

4. At what time between 8 and 9 o'clock are the hands of a clock exactly opposite each other?

5. At what time between 11 and 12 o'clock are the hands of a clock exactly opposite each other?

6. At what time between 4 and 5 o'clock are the hands of a clock exactly opposite each other?

7. At what time between 2 and 3 o'clock do the hands of a clock make right angles with each other?

8. At what times between 6 and 7 o'clock do the hands of a watch make right angles with each other?

9. At what time between 7 and 8 o'clock do the hands of a watch make an angle of 120° with each other?

10. At what time between 12 and 1 o'clock do the hands of a watch make an angle of 60° with each other?

Bills.

373. Bills. A bill is a written statement of goods sold, or services rendered, giving the price of each item and the total cost, as well as the date of each transaction, and the names of the parties concerned.

The party who owes is called the *Debtor*, and the party to whom a debt is owed is called the *Creditor*.

(Specimen of a Bill.)

Boston, Mass., March 9, 1896.

Mr. George Brown,

BOUGHT OF JAMES BATES.

1896					
Jan.	*15*	*10 lb. Coffee*	*@ 35¢*	*$3*	*50*
	22	*11 lb. Lard*	*@ 9¢*	*0*	*99*
Feb.	*5*	*25 lb. Sugar*	*@ 5¢*	*1*	*25*
	12	*2 lb. Tea*	*@ 65¢*	*1*	*30*
				$7	*04*

(Specimen of a Receipted Bill.)

Boston, Mass., March 17, 1896.

Mr. John Jones,

TO JAMES BROWN, DR.

1896							
Jan.	*22*	*To 40 t. Coal*	*@ $4.75*	*$190*	*00*		
	29	*To 20 cd. Wood*	*@ 3.25*	*65*	*00*		
						$255	*00*
		Cr.					
Jan.	*29*	*By 40 bbl. Apples*	*@ $3.50*	*140*	*00*		
Feb.	*10*	*By 50 bu. Potatoes*	*@ 0.80*	*40*	*00*	*180*	*00*
		Balance due				*$75*	*00*

1896, March 26. *Received payment,*

James Brown.

Exercise 96.

Make out receipted bills for the following **accounts, supplying dates**:

1. James Hardy bought of C. H. Mills 275 bbl. flour, at \$6.75; 324 bbl. flour, at \$6.25; 300 bu. potatoes, at 48 cents; 1578 lb. butter, at 32 cents; 2000 bbl. apples, at \$1.25; a car-load (20,000 lb.) of oats, at 42 cents a bushel; a car-load (28,575 lb.) of corn, at 55 cents a bushel.

2. James Harlow bought of John Pike 12 bales, 480 lb. each, Texas cotton, at $9\frac{1}{4}$ cents; 7 bales, 502 lb. each, upland, at $10\frac{1}{4}$ cents; 3 bales, 492 lb. each, low middling, at $9\frac{3}{4}$ cents; 18 bales, 490 lb. each, good ordinary, at 9 cents.

3. Richard Rowe bought of John Doe 125 lb. sugar, at 5 cents; 1 bag coffee, 115 lb., at 32 cents a pound; 25 gal. molasses, at 38 cents; 8 lb. Japan tea, at 92 cents; 28 lb. crackers, at 8 cents; 2 bbl. flour, at \$7.50.

4. William Litchfield bought of John Garvin 8 bags cracked corn, at 75 cents; 4 bags oats, at 80 cents; 16 lb. sweet potatoes, at $3\frac{1}{4}$ cents; 2 bu. potatoes, at \$1.10; 100 lb. wire nails, at $2\frac{1}{4}$ cents; 5 lb. coffee, at 35 cents.

5. Amos Tuck sold to Aaron Young 11 lb. ham at 15 cents, 22 lb. beefsteak at 24 cents, 18 lb. mutton at 13 cents, 14 lb. veal at 11 cents; and took in exchange 5 dozen eggs at 18 cents, 15 lb. butter at 26 cents, 9 bu. potatoes at 40 cents, and 2 bbl. apples at \$1.35.

6. W. G. Fernald sold to John Waldron 35 lb. sugar at 5 cents, 18 lb. coffee at 35 cents, 20 lb. rice at 8 cents, 4 tons hay at \$15.75, 3 cords pine wood at \$2.75, 4 cords hard wood at \$3.50, 8 tons furnace coal at \$6.75, 5 tons stove coal at \$7.25, 8 rolls wall paper at 35 cents; and took in exchange 25 bbl. apples at \$1.15, 32 bu. pears at 60 cents, and 42 bu. blueberries at 8 cents a quart.

7. C. A. Colton bought of Green, Fisk & Co. 4 doz. No. 7 teakettles, at 85 cents each; 2 safety ash barrels, at \$2.50; 3 doz. common scrapers, at 50 cents a dozen; 8 eagle shovels, at 10 cents; $\frac{1}{2}$ doz. 8 by 12 black registers, at \$1.50 each; $\frac{1}{2}$ doz. spice boxes, at 55 cents each; $\frac{1}{2}$ doz. 14-qt. dish pans, at \$6.00 a dozen; 2 doz. common stove lifters, at 50 cents a dozen; $\frac{1}{2}$ doz. 12 by 14 drip pans, at \$4.00 a dozen; $\frac{1}{2}$ gross retinned teaspoons, at 25 cents a dozen; 1 doz. ash sifters at \$1.00 each.

8. R. M. Hanson bought of W. F. Fox & Co. 2 bbl. flour, at \$5.75; $\frac{1}{2}$ bbl. fine sugar, 153 lb., at \$4.81 a cwt.; 25 lb. coffee, at 33 cents; 3 lb. Oolong tea, at 50 cents; 15 pint bottles olives, at 25 cents; 2 boxes graham wafers, at 40 cents; $\frac{1}{2}$ doz. cans tomatoes, at \$1.20 a dozen; $\frac{1}{2}$ doz. cans J. H. F. peaches, at \$3.50 a dozen; 4 Ferris hams, 48 lb., at $12\frac{1}{2}$ cents a pound; 6 strips Ferris bacon, 19 lb. 9 oz., at 13 cents a pound; 3 lb. rice, at 9 cents; 3 lb. tapioca, at 5 cents; 40 lb. rye meal, at $2\frac{1}{2}$ cents; 5 lb. boneless codfish, at 14 cents; $\frac{1}{2}$ doz. cans plums, at \$2.90 a dozen.

9. G. B. Cook bought of Gray, Higginson & Co. 1 No. 8–20 Glenwood B range, at \$35.00; 1 No. 12 Rockford heater, at \$20.00; 4 lb. Eng. stovepipe, at 15 cents; 3 lb. Rus. stovepipe, at 25 cents; 8 lb. sheet zinc, at 8 cents; 1 stove board, at \$2.00; 1 set kitchen knives and forks, at \$1.50; 2 washtubs, at 85 cents; 1 washboard, at 25 cents; 1 set Mrs. Potts' nickel sad-irons, at 75 cents; 2 milk cans, at 35 cents; 1 hand lamp complete, at 30 cents; 1 stand lamp, at \$3.50; 1 granite iron washbowl, at 50 cents; 1 tea canister and 1 coffee canister, at 20 cents each; 1 carving knife and fork, at \$2.00; 1 corn popper, at 25 cents; 1 rolling-pin, at 20 cents; 2 8-qt. porcelain kettles, at 70 cents; 1 granite iron coffeepot, at 75 cents.

CHAPTER X.

METRIC AND COMMON SYSTEMS.

374. **Table of Equivalents.**

LENGTH.

Meter = { 39.37043 in., or 1.09362 yd.	Inch = 2.53998^{cm}.
	Yard = 0.91439^{m}.
Kilometer = 0.62138 mi.	Mile = 1.60933^{km}.

SURFACE.

Sq. meter = { 1550.031 sq. in., or 1.19601 sq. yd.	Sq. inch = 6.45148^{qcm}.
	Sq. yard = 0.83611^{qm}.
Hektar = 2.47110 A.	Acre = 0.40468^{ha}.

VOLUME.

Cu. centimeter = 0.06103 cu. in.	Cu. inch = 16.38662^{ccm}.
Cu. meter = 1.30799 cu. yd.	Cu. yard = 0.76453^{cbm}.
Ster = 0.27590 cd.	Cord = 3.62446^{st}.

CAPACITY.

Liter = { 1.05671 liquid qt., or	Liquid quart = 0.94633^{l}.
0.90810 dry qt.	Dry quart = 1.10119^{l}.

WEIGHT.

Milligram = 0.015432 gr.	Grain = 0.06480^{g}.
Gram = 15.43235 gr.	Ounce av. = 28.34954^{g}.
Kilogram = 2.20462 lb. av.	Ounce troy = 31.10350^{g}.
Metric ton = 2204.62 lb. av.	Pound av. = 0.45359^{kg}.

375. **Table of Approximate Equivalents.**

Meter	$= 1.1$ yd.	Yard	$= 0.9^{m}$.
Kilometer	$= \frac{5}{8}$ mi.	Mile	$= 1.6^{km}$.
Sq. meter	$= 1\frac{1}{5}$ sq. yd.	Sq. yard	$= \frac{5}{6}^{qm}$.
Hektar	$= 2\frac{1}{2}$ A.	Acre	$= \frac{2}{5}^{ha}$.
Cu. centimeter	$= \frac{1}{16}$ cu. in.	Cu. inch	$= 16^{ccm}$.
Cu. meter	$= 1.3$ cu. yd.	Cu. yard	$= \frac{10}{13}^{cbm}$.
Ster	$= \frac{3}{11}$ cd.	Cord	$= 3\frac{2}{3}$ sters.
Liter	$= \begin{cases} 1\frac{1}{16} \text{ liq. qt., or} \\ \frac{9}{10} \text{ dry qt.} \end{cases}$	Liquid quart	$= \frac{15}{16}$ liter.
		Dry quart	$= 1\frac{1}{9}$ liters.
Hektoliter	$= 2\frac{5}{6}$ bu.	Bushel	$= \frac{6}{17}^{hl}$.
Gram	$= 15\frac{1}{2}$ gr.	Pound av.	$= \frac{5}{11}^{kg}$.
Kilogram	$= 2\frac{1}{5}$ lb. av.	Pound troy	$= \frac{5}{13}^{kg}$.

NOTE. These tables are given for reference, and are not to be committed to memory. It may be well, however, to remember that a meter is 39.37 in.; a liter is 1.0567 liquid qt., or 0.908 dry qt.; and that a gram is 15.432 gr.; as by these equivalents all measures expressed in one system may be converted into the corresponding measures of the other system.

EXERCISE 97.

In the following problems take the equivalents from the Table of Equivalents, using three places of decimals, or four when the first decimal figure is zero, and add one to the last decimal figure when the next is 5 or more.

1. Reduce 25.55^{kg} to pounds avoirdupois.

SOLUTION. Since $1^{kg} = 2.205$ lb., $25.55^{kg} = 25.55 \times 2.205$ lb., or 56.33775 lb., that is, 56 lb. 5.4 oz.

2. Reduce 5 sq. yd. 6 sq. ft. 108 sq. in. to square meters.

SOLUTION. 5 sq. yd. 6 sq. ft. 108 sq. in. $= 5.75$ sq. yd. Since 1 sq. yd. $= 0.836^{qm}$, 5.75 sq. yd. $= 5.75 \times 0.836^{qm}$; that, is 4.807^{qm}.

3. Reduce 24 gal. to liters.
4. Reduce 10 lb. troy to kilograms.
5. Reduce 50.5 cu. yd. to cubic meters.

6. Reduce $69\frac{17}{100}$ mi. to kilometers.
7. Reduce 12 A. 12 sq. rd. to hektars.
8. Reduce 10 cd. to sters.
9. Reduce 4 cwt. 24 lb. to kilograms.
10. Reduce 25 bu. 2 pk. to hektoliters.
11. Express 15^{km} in the common system.
12. Express 3^{ha} in the common system.
13. Express 12.125^{cbm} in the common system.
14. Express 101.25^{l} in the common system.
15. Reduce 20.25^{hl} to liquid quarts; to dry quarts.
16. Express 5^{kg} in troy weight.
17. Express 24^{st} in the common system.
18. Express 62.5^{qm} in the common system.
19. Express 1001^{kg} in avoirdupois weight.
20. Express 42 A. 100 sq. rd. in the metric system.
21. Find in acres, etc., the area of a rectangular field if it is 100^{m} long and 75^{m} broad.
22. Find the number of cubic meters in a rectangular box 2 yd. long, 3 ft. wide, and $2\frac{1}{2}$ ft. deep.
23. Find the number of cubic yards in a rectangular box 2^{m} long, 75^{cm} wide, and 50^{cm} deep.
24. If a man walks 75^{m} a minute, what is his rate in miles per hour?
25. If a cubic centimeter of cast iron weighs 7.113^{g}, how many pounds does a cubic foot weigh?
26. How many steps 2 ft. 6 in. long will a man take in walking a kilometer?
27. Find the value of a carboy (17 qt.) of sulphuric acid, specific gravity 1.841, at $4\frac{3}{4}$ cents a kilogram.
28. Find the value of a carboy ($17\frac{1}{2}^{l}$) of nitric acid, specific gravity 1.451, at 15 cents a pound.
29. If the specific gravity of sea water is 1.026, and that of olive oil is 0.915, what is the weight of a hektoliter of each in pounds and in kilograms?

30. Find the weight in pounds and in kilograms of $31\frac{1}{4}$ gal. of the best alcohol, specific gravity 0.792.

31. Find the weight in pounds and in kilograms of the air, specific gravity 0.00129206, in a room 7^{m} long, 5^{m} wide, and 3.5^{m} high.

32. Find the weight in pounds and in kilograms of the air, specific gravity 0.00129206, in a room 23 ft. long, 16 ft. wide, and 10 ft. high.

33. What is the lifting force in kilograms and in pounds of a balloon that weighs 2^{kg}, and contains $10{,}000^{l}$ of hydrogen gas, specific gravity 0.00008929?

NOTE. The lifting force is the weight of the air displaced by the balloon diminished by the weight of the hydrogen and the balloon.

34. What is the value at \$4.50 a cord of a pile of wood 1.2^{m} wide, 7^{m} long, and 2^{m} high?

35. How many miles will a train run in 1 hr. 28 min. 21 sec., at the rate of 50^{km} an hour?

36. Find the time it takes a train to run 31 mi. 180 yd. at the rate of 1 min. 25 sec. per kilometer.

37. What is the weight of 12 cu. yd. 16 cu. ft. 720 cu. in. of earth, if a cubic meter weighs 1 t. 17 cwt.?

38. Find the weight in grams of a liter of mercury, if a cubic inch weighs 0.4925 of a pound avoirdupois.

39. How many yards of cloth, at \$3.12½ a meter, should be given in exchange for 15^{m} at \$2.75 a yard?

40. If a wine merchant buys 3^{hl} of wine for 1600 francs, what does a gallon cost him in United States money, if 25 francs are equivalent to \$4.825?

41. A mill wheel is turned by a stream of water running at the rate of a yard a second in a channel 5 ft. wide and 9 in. deep. Find the weight in metric tons and in tons avoirdupois of the water supplied in 12 hr., if a cubic foot of water weighs 1000 oz.

Exercise 98.

In the following problems take the equivalents from the Table of Approximate Equivalents, and use $\frac{22}{7}$ for 3.1416.

1. When water is heated from the freezing point to the boiling point it expands $\frac{1}{24}$ in volume. Find in kilograms the weight of a cubic foot of water at the freezing point and at the boiling point.

2. A circular plate of lead 8 in. in diameter and 2 in. thick is changed without loss into spherical shot each 1.25^{mm} in radius. How many shot does it make?

3. If $\frac{3}{4}$ of a yard of velvet costs $3, how many francs will $\frac{5}{7}$ of a meter cost?

4. Water expands $\frac{1}{10}$ in freezing, and a floating body displaces an amount of water equal in weight to the body. What is the volume in cubic meters and the weight in metric tons of an iceberg floating in the ocean, if the specific gravity of sea water is 1.026, and the part of the iceberg above the water is a rectangular solid 200 ft. long, 60 ft. wide, and 12 ft. high?

5. How many hektoliters of wheat will a rectangular bin hold 14 ft. long, 10 ft. wide, and 6 ft. high?

6. How many hektoliters of water will a cylindrical stand-pipe hold 70 ft. high and 35 ft. in diameter?

7. How many bushels of wheat will a rectangular bin hold 4^m long, 3^m wide, and 2.5^m high?

8. How many gallons of water in a well 1.2^m in diameter, if the depth of the water is 2^m?

9. If 1 lb. troy of silver is worth $6.20, what is the value of a lump of silver weighing 2.64^{kg}?

10. A pound of brass contains 3.3 cu. in., and a pound of antimony contains 6.27 cu. in. Find the weight in kilograms of a mass of $313\frac{1}{2}$ cu. in. that contains equal volumes of the two metals.

11. If 2 cu. in. of mercury weighs 1 lb., and 100 cu. in. of air weighs 31 gr., how many kilometers high must a column of air be to weigh as much as a column of mercury 29.388 in. high, standing on a base of the same area?

12. If a sprinter can run 0.00645 of a mile in 1.08 sec., how many meters can he run in a second? How many seconds will it take him to run 100^{m}?

13. Two trains going in opposite directions pass each other in $3\frac{1}{2}$ sec. If their lengths are 260 ft. and 200 ft., respectively, and the first train is going at the rate of 80^{km} an hour, what is the rate of the second train?

14. If a cubic inch of water converted into steam will produce mechanical force sufficient to raise a weight of 2200 lb. one foot high, how many meters high would the conversion into steam of a cubic centimeter of water raise a weight of one kilogram?

15. If a man takes 100 steps of 0.7^{m} each in a minute, how long will it take him to walk a distance of 28^{km}?

16. A lot of land containing 63^{a} 21^{ca}, worth $0.35 a square yard, is exchanged for a second lot containing 1^{ha} 5^{a}. What is the cost per ar of the second lot?

17. Light travels in 8 min. 13 sec. from the sun to the earth, $153{,}624{,}000^{km}$. What is the velocity of light in miles per second?

18. How many square feet of surface has a rectangular table that is 1.1^{m} long and 0.85^{m} wide?

19. How many square meters of surface has a circular table that is $3\frac{1}{2}$ ft. in diameter?

20. If sound travels 340^{m} a second, how many feet distant is a cannon from a man who hears the report 13 sec. after he sees the flash?

21. How many square meters of zinc will be required to line a rectangular cistern, open at the top, 12 ft. long, 10 ft. wide, and 8 ft. deep?

22. A rectangular tank is 3^m long, $2\frac{1}{2}^m$ wide, and $1\frac{1}{2}^m$ high, external measurement. If its sides are 0.1^m thick, how many gallons of water will the tank hold?

23. If a cube of pine wood 11.2^{cm} on an edge weighs 2 lb., what is the specific gravity of the pine?

24. Find in kilograms the weight of water a cubical cistern will hold, 6 ft. on an edge.

25. Rain has fallen to the depth of half an inch. How many cubic meters of water has fallen on an acre of land?

26. How many centimeters will the water sink in a cylindrical cistern 7 ft. in diameter, if 310 gallons of water is pumped out?

27. How many square yards of tin are required to cover the roof of a hemispherical dome 12^m in diameter?

28. If a cubic inch of iron weighs $4\frac{1}{2}$ oz., what is the weight in kilograms of an iron ball 10^{cm} in diameter?

29. If a cubic inch of lead weighs 7 oz., what is the weight in kilograms of a lead pipe 3^m long, 6^{cm} in external diameter, if the pipe is 1^{cm} thick?

30. Find the cost at \$7.25 per meter of building a wall around a rectangular garden 90 ft. long and 55 ft. wide.

31. The minute hand of a clock is 0.5^m long. How many feet does its point move in an hour?

32. A spherical shot 3 in. in diameter is melted and then cast into a cylinder 9^{cm} in diameter. What is the height in centimeters of this cylinder?

33. What is the cost at \$18 per 1000 ft. board measure of 4 beams, each 4.5^m long, 7.5^{cm} wide, and 5^{cm} thick?

34. The radius of a cylindrical roller is 0.4^m and its length is 2.15^m. Find its volume in cubic feet.

35. A cylindrical cistern, the circumference of whose base is 2.2^m, and whose depth is 2.1^m, is four fifths filled with water. Find in gallons the volume of the water, and in pounds the weight of the water.

CHAPTER XI.

RATIO AND PROPORTION.

376. Ratio. The *relative magnitude* of two numbers is called their *ratio*, when expressed by the fraction that has the first number for its numerator and the second number for its denominator.

Thus, the ratio of 2 to 3 is expressed by the fraction $\frac{2}{3}$.

377. Antecedent and Consequent. The terms of this fraction are called the *terms* of the ratio. The first term of a ratio is called the *antecedent;* the second term, the *consequent.*

Thus, in the ratio of 2 to 3, commonly written $2:3$, the first term 2 is the antecedent, and the second term 3 is the consequent.

378. *If both terms of a ratio are multiplied or both divided by the same number, the value of the ratio is not changed.*

Thus, if the ratio $2\frac{1}{2}:3\frac{1}{3}$ is multiplied by 6, the resulting ratio is $15:20$, and the ratio $2\frac{1}{2}:3\frac{1}{3}$ is equal to $15:20$; for $\frac{2\frac{1}{2}}{3\frac{1}{3}}=\frac{15}{20}$. Since $\frac{15}{20}$ reduced to its lowest terms $=\frac{3}{4}$, the simplest expression for the ratio of $2\frac{1}{2}:3\frac{1}{3}$ is $3:4$.

379. If the numerator and denominator of a fraction are interchanged, the fraction is said to be *inverted;* likewise, if the antecedent and consequent of a ratio are interchanged, the resulting ratio is the *inverse* of the given ratio.

Thus, if the fraction $\frac{3}{4}$ is inverted the resulting fraction is $\frac{4}{3}$; and the inverse of the ratio $3:4$ is $4:3$.

380. If two *quantities* are expressed in the *same unit*, their ratio is the same as the ratio of the two *numbers* by which they are expressed.

Thus, the quantity \$5 is the same fraction of \$11 as 5 is of 11; and, therefore, the ratio \$5 : \$11 equals the ratio 5 : 11.

381. Since ratio is simply *relative magnitude*, two quantities *different in kind* cannot form the terms of a ratio; and two quantities the same in kind must be expressed in a *common unit* before they can form the terms of a ratio.

Thus, no ratio exists between 5 *tons* and 30 *days*; and the ratio of 5 tons to 3000 pounds can be expressed only when *both* quantities are written as tons or as pounds.

382. Since ratios are mere *numbers*, they may be compared.

383. Example. Which is the greater ratio, 5 : 7 or 12 : 18?

SOLUTION. $5:7=\frac{5}{7}$, and $12:18=\frac{12}{18}=\frac{2}{3}$.
Now, $\frac{5}{7}=\frac{15}{21}$, and $\frac{2}{3}=\frac{14}{21}$.
As $\frac{15}{21}$ is greater than $\frac{14}{21}$,
the ratio 5 : 7 is greater than the ratio 12 : 18.

EXERCISE 99.

Which is the greater ratio:

1. 5 : 8 or 6 : 9?
2. 7 : 10 or 9 : 12?
3. 8 : 9 or 10 : 12?
4. 6 : 12 or 8 : 14?
5. 10 cwt. : 15 cwt. or \$7 : \$9?
6. 5 dy. : 7 dy. or 8 ft. : 11 ft.?
7. 9 yd. : 6 yd. or 5 : 3?
8. $\frac{2}{3}$ lb. : $\frac{1}{2}$ lb. or $\frac{5}{8}$ yd. : $\frac{3}{8}$ yd.?
9. Find the ratio of 3 dry quarts to 2 pecks.
10. Find the ratio of 2500 lb. to 1 ton.
11. Find the ratio of a rectangular field 16 rd. long, 14 rd. wide to a rectangular field 14 rd. long, 12 rd. wide.
12. Find the ratio of a circle 1 in. in diameter to a circle 1 in. in radius.

384. When two ratios are equal the four terms are said to be *in proportion*, and are called *proportionals*.

Thus, 5, 3, 15, 9 are proportionals; for $\frac{5}{3} = \frac{15}{9}$.

385. Proportion. An expression of equality between two ratios is called a *proportion*.

A proportion is written by putting the sign of equality or a double colon between the ratios.

Thus, $5 : 3 = 15 : 9$, or $5 : 3 :: 15 : 9$, means, and is read, the ratio of 5 to 3 is equal to the ratio of 15 to 9.

386. Means and Extremes. The first and last terms of a proportion are called the *extremes*, and the two middle terms are called the *means*.

387. Test of a Proportion. *When four numbers are in proportion, the product of the extremes is equal to the product of the means.*

This is seen to be true by expressing the ratios in the form of fractions, and multiplying both by the product of the denominators.

Thus, the proportion $5 : 3 = 15 : 9$ may be written $\frac{5}{3} = \frac{15}{9}$; and if both are multiplied by 3×9, the result is $5 \times 9 = 3 \times 15$.

388. Either extreme, therefore, is equal to the product of the means divided by the other extreme; and either mean is equal to the product of the extremes divided by the other mean. Hence, if three terms of a proportion are given, the fourth may be found.

389. Examples. 1. Find the missing term of the proportion $18 : 32 = 45 : ?$.

SOLUTION. $$\frac{32 \times 45}{18} = 80.$$

2. Find the missing term of $20 : 24 = ? : 30$.

SOLUTION. $$\frac{20 \times 30}{24} = 25.$$

Exercise 100.

Find the missing term of:

1. 24 : 18 :: 16 : ?.	6. 18 : ? :: 32 : 45.
2. 35 : ? :: 15 : 21.	7. ? : 12 :: 5 : 18.
3. 45 : 40 :: ? : 32.	8. 8 : 17 :: ? : 119.
4. 30 : 27 :: 40 : ?.	9. 9 : 16 :: 12 : ?.
5. ? : 36 :: 4 : 3.	10. 17 : 3 :: ? : 12.

390. Rule of Three. When three terms of a proportion are given, the method of finding the fourth term is called the *Rule of Three*.

It is usual to arrange the quantities (that is, to ***state*** the question), so that the quantity required for the answer may be the *fourth* term. Hence, the quantity which ***corresponds*** to that of the required answer is the *third* term.

391. Examples. 1. If 5 tons of hay cost $87.50, what will 21 tons cost?

Solution. Since *cost* is required, $87.50 is the third term.

Since 21 tons will cost *more* than 5 tons, 21 tons is the second term and 5 tons the first term.

That is, 5 t. : 21 t. :: $87.50 : ?.

A difficulty presents itself here, inasmuch as no meaning can be given to the product of the means ($87.50 multiplied by 21 t.). Since, however, the ratio of 5 t. : 21 t. = the ratio of 5 : 21, the ratio 5 : 21 may be substituted for the ratio 5 t. : 21 t.

Then, 5 : 21 :: $87.50 : ?.

Therefore, the fourth term required is $\frac{21 \times \$87.50}{5}$, or $367.50.

2. When a post 11.5 ft. high casts a shadow on level ground 17.4 ft. long, a neighboring steeple casts a shadow 63.7 yd. long. How high is the steeple?

Solution. Since *height* is required, the height 11.5 ft. is the third term.

Since the *shadow* of the steeple is the longer, the *height* of the steeple must be the greater; therefore, the second term must be the greater of the two remaining quantities expressed in the *same unit.* 63.7 yd. = 191.1 ft. Therefore,

	Shadow.	Shadow.	Height.	Height.
	17.4 ft.	: 191.1 ft.	:: 11.5 ft.	: What?
or,	17.4	: 191.1	:: 11.5 ft.	: What?

Hence, the height of the steeple is $\frac{191.1 \times 11.5 \text{ ft.}}{17.4}$, or 126.3 ft.

392. To Solve Problems by the Rule of Three,

Make that quantity which is of the same kind as the required answer the third term.

Make the numbers by which the two remaining quantities are represented when expressed in the same unit the first and second terms.

If, from the nature of the question, the answer will be greater than the third term, make the greater of these two numbers the second term; if the answer will be smaller than the third term, make the smaller of these numbers the second term, and the larger the first term.

Divide the product of the second and third terms by the first term, and the quotient will be the answer required.

Exercise 101.

1. If 24 men can do a piece of work in 14 days, how long will it take 21 men to do it?

2. A well is dug in 13 days of 9 hours each. How many days of 10 hours each would it have taken?

3. A man who steps 2 ft. 5 in. takes 2480 steps in walking a certain distance. How many steps of 2 ft. 7 in. will be required for the same distance?

4. If $\frac{4}{13}$ of a ton of hay costs $6, what will $7\frac{5}{8}$ cwt. cost, at the same rate?

5. If 42 yd. of carpet 2 ft. 3 in. wide are required for a room, how many yards of carpet 2 ft. 4 in. wide will be required?

6. A court was paved with 950 stones, each containing $1\frac{5}{8}$ sq. ft., and is repaved with 836 stones of a uniform size. Find the surface of each.

7. If a train, at the rate of $\frac{5}{13}$ of a mile per minute, requires $3\frac{1}{4}$ hours to make a certain distance, how long will it require at the rate of $\frac{7}{15}$ of a mile a minute?

8. When a post 4 ft. 8 in. high casts a shadow 7 ft. 3 in. long, how long a shadow will a post 11 ft. high cast?

9. When a post 5 ft. 7 in. high casts a shadow 8 ft. 5 in. long, how high is a steeple that casts a shadow of 202 ft.?

10. If 4 men can mow a certain field in 10 hours, how many men will it take to mow it in 5 hours?

11. If a tap discharging 4 gal. a minute empties a cistern in 3 hours, how long will it take a tap discharging 7 gal. a minute to empty it?

12. If a pipe discharging 3 gal. 1 pt. a minute fills a tub in 4 min. 20 sec., how long will it take a pipe discharging 83 qt. a minute to fill it?

13. If both pipes of Ex. 12 discharge at the same time into the tub, how long will it take to fill it?

14. How long will it take to fill a cistern of 165 gal. by a pipe that fills one of 120 gal. in 7 min. 16 sec.?

15. If a ship sails 1800 mi. in a fortnight, how long will it take to make a voyage of 5000 mi.?

16. The wheels of a carriage are 6 ft. 9 in. and 9 ft. 6 in., respectively, in circumference. How many times will the larger turn while the smaller turns 3762 times?

17. If $\frac{3}{25}$ of a ship is worth $2167, what is $\frac{7}{17}$ of it worth?

18. What is the weight of 18 cu. ft. 432 cu. in. of stone, if 10 cu. ft. 864 cu. in. of the stone weighs 14 cwt. 7 lb.?

19. If 280 lb. of flour makes 360 lb. of bread, how many four-pound loaves can be made from 1 cwt. of flour?

20. If a column of mercury 27.93 in. high weighs 0.76 of a pound, what is the weight of a column of mercury of the same diameter 29.4 in. high?

21. How many francs will pay a bill of £100, when £42 10s. 8d. is equivalent to 1090.98 francs?

22. What is the weight of a cube of stone 2 ft. 2 in. on an edge, if a cube 1 ft. 4 in. on an edge weighs 537.6 lb.?

23. If a square field 50 yd. $10\frac{2}{7}$ in. on a side is worth $\$2710\frac{10}{17}$, what is a square field 62 yd. 1 ft. on a side worth?

24. A gains 4 yd. on B in running 30 yd. How many yards will he gain while B is running $97\frac{1}{2}$ yd.?

25. If 10 cu. in. of gold weighs as much as 193 cu. in. of water, how many cubic inches are there in a nugget of gold that weighs as much as a cubic foot of water?

26. If a garrison of 1500 men has provisions for 13 months, how long will the provisions last if the garrison is reënforced by 700 men?

27. If a tree 38 ft. high is represented by a drawing $1\frac{1}{2}$ in. high, what height on the same scale will represent a house 45 ft. high?

28. If a country 630 mi. long is represented on a raised map by a length of $5\frac{1}{4}$ ft., by what height ought a mountain of 15,750 ft. be represented on the map?

29. A train travels $\frac{1}{4}$ of a mile in 18 sec. How many miles an hour does it travel?

30. If $4\frac{1}{2}$ t. of coal fill a bin 9 ft. long, 5 ft. broad, 5 ft. high, how many cubic feet are required for the coal of a steamer that carries coal for 3 wk. at 20 t. a day?

31. If 2 lb. of rosin are melted with 5 oz. of mutton tallow to make a grafting wax, how many ounces of tallow will 20 oz. of the wax contain?

Compound Proportion.

393. Compound Ratio. A ratio is said to be *compounded* of two or more given ratios when it is expressed by a fraction that is the product of the fractions representing the given ratios.

Thus, the ratios 2 : 3 and 7 : 11 are represented by the fractions $\frac{2}{3}$ and $\frac{7}{11}$; and the ratio 14 : 33, which is represented by $\frac{14}{33}$ (the product of $\frac{2}{3}$ and $\frac{7}{11}$), is said to be compounded of the ratios 2 : 3 and 7 : 11.

394. Compound Proportion. A proportion which has one of its ratios a compound ratio is called a *compound proportion.*

In stating problems in compound proportion the quantity that corresponds to the answer required is made the third term. Each *pair* of the remaining quantities is then considered *separately* with reference to the answer required.

395. Example. If 4 men mow 15 A. in 5 dy. of 14 hr., in how many days of 13 hr. can 7 men mow $19\frac{1}{2}$ A.?

As the answer is to be in days, we make 5 dy. the third term.

I. It will require *less days* for 7 men to mow 15 A. than for 4 men. Therefore, we make 7 the first term, and 4 the second.

II. It will require *more days* for the same number of men to mow $19\frac{1}{2}$ A. than to mow 15 A.

Therefore, we make 15 the first term, and $19\frac{1}{2}$ the second.

III. It will require *more days* of 13 hr. than of 14 hr. for the same number of men to mow the same number of acres.

Therefore, we make 13 the first term, and 14 the second.

Hence, the statement is

$$\begin{array}{l} 7 : 4 \\ 15 : 19.5 :: 5 \text{ dy.} : ? \\ 13 : 14. \end{array}$$

Therefore, the fourth term, or the time required, is

$$\frac{4 \times 19.5 \times 14 \times 5 \text{ dy.}}{7 \times 15 \times 13} = 4 \text{ dy.}$$

Exercise 102.

1. In how many days of 8 hr. will 60 men do the same work that 24 men can do in 15 dy. of 10 hr.?

2. What is the expense of covering a room with drugget 4 ft. wide, at $91\frac{2}{3}$ cents a yard, if carpet 2 ft. 3 in. wide for the room costs \$70.50, at \$$1.37\frac{1}{2}$ a yard?

3. If 4418 tons of iron ore produce \$36,190 worth of metal, when iron is at \$37.50 a ton, what will be the value of the iron at \$47 a ton from 2275 tons of ore?

4. If a bar of iron $3\frac{1}{3}$ ft. long, 3 in. wide, and $2\frac{3}{4}$ in. thick weighs 93 lb., what will be the weight of a bar $3\frac{2}{3}$ ft. long, 4 in. wide, and $2\frac{1}{2}$ in. thick?

5. If 40 bu. of wheat can be grown on the same area as 48 bu. of barley, and 28 A. produce 840 bu. of wheat, how much barley will 38 A. produce?

6. If 18 men can dig a trench 150 ft. long, 6 ft. broad, and 4 ft. 6 in. deep in 12 days, in how many days will 16 men dig a trench 210 ft. long, 5 ft. broad, and 4 ft. deep?

7. A book of 810 pages, 40 lines to a page, and 60 letters to a line, is reprinted in pages of 50 lines, 72 letters to a line. How many pages will the new edition contain?

8. If 3280 42-lb. shot cost \$3000, how many 32-lb. shot can be bought for \$4200?

9. What is the rate of wages, if 12 men earn in 10 dy. as much as 9 men earn in 14 dy., at \$1.50 a day?

10. A rectangular reservoir 15 yd. long and 4 ft. deep holds 32,500 gal. What quantity of water will it hold if its length is increased by 18 ft. and its depth by 1 ft.?

11. What must be the length of a bar of silver $\frac{3}{4}$ in. square to weigh the same as a bar of gold $\frac{1}{2}$ in. square and $6\frac{3}{4}$ in. long, if the weight of a cubic inch of silver to that of a cubic inch of gold is in the ratio 47 : 88?

12. How far can A, who takes 3.1 ft. each step, walk, while B, who takes 2.3 ft. each step, walks 220 yd., if A takes 7 steps while B takes 11?

13. If 6 hr. are needed to go a given distance at a given rate, how many hours are needed when the distance is diminished by one fourth and the rate increased by one half?

14. How many hours a day must 5 men work to mow a field in 8 dy. that 7 men can mow in 6 dy. of 10 hr.?

15. If a bar of iron 10 ft. $6\frac{1}{2}$ in. long, $3\frac{5}{8}$ in. broad, and $3\frac{1}{2}$ in. thick weighs 4 cwt. 20.21 lb., what is the length of a bar of iron that weighs a long ton, if its breadth and thickness are $4\frac{3}{8}$ in. and $4\frac{1}{8}$ in., respectively?

16. If 27 men in 28 dy. of 10 hr. dig a trench 126 yd. long, $2\frac{1}{2}$ yd. broad, $1\frac{1}{2}$ yd. deep, how long a trench $2\frac{3}{4}$ yd. broad and $1\frac{3}{4}$ yd. deep will 56 men dig in 25 dy. of $8\frac{1}{4}$ hr.?

17. If 34^{kg} of wool makes 25^{m} of cloth 0.6^{m} wide, how long a piece of cloth 0.8^{m} wide will 108.8^{kg} of wool make?

18. If an oak beam 5.40^{m} long, 0.63^{m} thick, and 0.57^{m} wide weighs 1469.25^{kg}, what is the weight of an oak beam 4.87^{m} long, 0.58^{m} thick, and 0.53^{m} wide?

19. A certain quantity of air has a volume of 195.5 cu. ft. at 27.8° C. What will be its volume at 100° C.?

NOTE. The *coefficient of expansion* of a body is the increase of a unit of volume of the body when the temperature is increased 1° C. The coefficient of expansion for air is 0.00367. That is, a cubic foot of air at 0° C. occupies at 1° C. 1.00367 cu. ft., if the pressure remains the same. The increase in volume is found approximately by multiplying the increase of 1° by the number of degrees.

20. A quantity of air at a temperature of 15.6° C. has a volume of 4 cu. ft. under a pressure of 12 lb. per square inch. What will be its volume at 48.7° C. under a pressure of 14 lb. per square inch?

NOTE. The volumes occupied by the same quantity of air, the temperature remaining unchanged, are in inverse ratio to the pressures.

Cause and Effect.

396. Problems in compound proportion are readily solved by the *cause and effect* method.

397. The cause and effect method depends upon the following principle:

Like causes produce like effects; and the ratio between any two causes equals the ratio between the effects produced.

NOTE. Examples of *causes* are men at work, time, and goods bought or sold; examples of *effects* are work done, wages, and cost or selling price of goods.

398. Example. If 4 men mow 15 A. in 5 dy. of 14 hr., in how many days of 13 hr. can 7 men mow $19\frac{1}{2}$ A.?

SOLUTION. 1st cause. 2d cause. 1st effect. 2d effect.

$$\left.\begin{array}{l}4\text{ men}\\5\text{ dy.}\\14\text{ hr.}\end{array}\right\} : \left.\begin{array}{l}7\text{ men}\\?\text{ dy.}\\13\text{ hr.}\end{array}\right\} :: \left\{15\text{ A.}\right. : \left\{19.5\text{ A.}\right.$$

Since the product of the means equals the product of the extremes, the number of days required equals the product of the extremes divided by the product of the remaining means.

Hence, the number of days required is $\frac{4 \times 5 \times 14 \times 19.5}{7 \times 13 \times 15} = 4.$

Compare this solution with the solution of the example in § 395.

EXERCISE 103.

Solve the following problems by the cause and effect method.

1. If a man can mow $\frac{3}{11}$ of a field in a day, how long will it take another man to mow $\frac{5}{6}$ of a field $5\frac{1}{4}$ times as large, if the second man works $1\frac{3}{4}$ times as fast as the first but only $\frac{7}{8}$ as many hours each day?

2. If 4 men or 7 boys can do a piece of work in 6 days, how long will it take 6 men and 9 boys to do the work?

3. If 50 men working 9 hr. a day require 6 dy. to dig a trench 100 yd. long, 2 yd. wide, and 3 yd. deep, how many men working 10 hr. a day for 9 dy. will be required to dig a trench 50 yd. long, 6 yd. wide, and 5 yd. deep, in ground twice as hard to dig?

4. If 12 men in 9 dy. can harvest 40 A. of wheat, how many acres can 16 men harvest in 3 dy.?

5. If 120 men can make an embankment $\frac{3}{4}$ of a mile long, 30 yd. wide, and 7 yd. high, in 42 dy., how many men will it take to make an embankment 1000 yd. long, 36 yd. wide, and 22 ft. high, in 30 dy.?

6. If 7 women in 8 dy. of 11 hr. each can make 22 dozen shirts, in how many days of 10 hr. each can 12 women make 360 dozen shirts?

7. Twenty-five lamps used 5 hr. an evening for 40 dy. required a quantity of oil that cost $4.25. How many lamps used 4 hr. an evening for 30 dy. can be furnished with oil at a cost of $7.65?

8. If 8 horses can be kept 12 dy. for a certain sum when hay is worth $15 a ton, how many days can 6 horses be kept for the same sum when hay is worth $12 a ton?

9. Twenty horses working 14 wk., 6 dy. a week and 8 hr. a day, transport the output of a mine to the nearest wharf. In how many weeks will 24 horses do the same work, if they work 5 dy. a week and 7 hr. a day?

10. If 6 men can reap a field of rye 200 yd. long and 150 yd. wide in 4 dy. of 12 hr. each, in how many days of 10 hr. each will 8 men reap a field 300 yd. long and 250 yd. wide?

11. If a boy can do only half as much work as a man, how many hours a day must 42 boys work to accomplish as much in 45 dy. as 27 men, working 10 hr. a day, would accomplish in 28 dy.?

Proportional Parts.

399. If it is required to divide a quantity into parts proportional to 3, 4, 5; the *numbers* 3, 4, 5 may be taken to represent the *parts*, and then the *whole* will be represented by $3 + 4 + 5$; that is, by 12.

400. Examples. 1. Divide $391 into parts proportional to the numbers 5, 7, and 11.

SOLUTION. The whole quantity will be represented by $5+7+11=23$. Therefore, the respective parts will be $\frac{5}{23}$, $\frac{7}{23}$, $\frac{11}{23}$ of $391; that is, $85, $119, and $187.

Or, the parts of the quantity may be found by the proportions:

$$23 : 5 :: \$391 : ?$$
$$23 : 7 :: \$391 : ?$$
$$23 : 11 :: \$391 : ?$$

The required terms will be $85, $119, and $187, respectively.

2. Divide $248 into parts proportional to $\frac{1}{10}$, $\frac{1}{15}$, $\frac{1}{25}$.

SOLUTION. Multiply the fractions by 150, the L. C. M. of their denominators. The results are 15, 10, 6. Hence, the parts will be *represented* by the numbers 15, 10, 6, and the whole by 31.

Therefore, the respective parts will be $\frac{15}{31}$, $\frac{10}{31}$, $\frac{6}{31}$ of $248; that is, $120, $80, $48.

EXERCISE 104.

1. Divide $12,000 proportionally to the numbers 3, 4, 5.
2. Divide 815 tons proportionally to $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$.
3. Divide 6853 lb. of wool proportionally to $1\frac{3}{4}$, $2\frac{4}{5}$, $5\frac{5}{6}$; also proportionally to the reciprocals of these numbers.
4. Two men purchase some property together, one paying $1250 and the other $1000. If the value of the property rises to $3600, what will be the share of each?
5. Gun metal is composed by weight of 3 parts of tin to 100 parts of copper. What weight of each of these metals is there in a cannon weighing 721 lb.?

6. Bell metal contains by weight 78 parts of copper and 22 parts of tin. What weight of each of these metals is there in a bell weighing 937 lb.?

7. It takes 75^{kg} of saltpetre, 12.5^{kg} of charcoal, and 12.5^{kg} of sulphur to make 100^{kg} of powder. How many kilograms of each will be required to make 10,000,000 cartridges, each containing 5^{g} of powder?

8. Yellow copper contains by weight 2 parts of red copper and 1 part of zinc. How many ounces of red copper in an article of yellow copper that weighs 1 lb.?

9. Type metal is an alloy containing by weight 39 parts of lead to 11 parts of antimony. How many pounds of each are required to make 957 lb. of type?

10. Plumbers' solder contains by weight 2 parts of lead and 1 part of tin. How many pounds of each are required to make 100 lb. of solder?

11. The air is composed of oxygen and nitrogen. In 100 volumes of air there are 21 volumes of oxygen and 79 of nitrogen. If the weight of a liter of oxygen is 1.4295^{g}, and that of a liter of nitrogen is 1.2577^{g}, how many grams of each gas does 100^{g} of air contain?

12. At $20.67 an ounce for pure gold, what is the value of the gold in a chain that weighs 3 oz. 4 dwt., if it is 18 carats fine (that is, 18 parts of pure gold out of 24)?

13. Two men agree to do a piece of work for $63. They finish the work in 18 days, but one of them was absent 5 days of this time. How should the pay be divided?

14. Five men working together do a piece of work in 20 days and receive as pay $253. One of the men was absent 5 days, and another 2 days of this time. How should the pay be divided?

15. Standard silver consists of 37 parts of pure silver to 3 parts of copper. What weight of pure silver in the crown piece that weighs $\frac{19}{11}$ oz. troy?

Partnership.

401. Partnership. An association of two or more persons for the purpose of conducting business is called a *partnership.*

A partnership association is called a *firm*, *company*, or *house*, and the persons associated in business are called *partners.*

402. Assets and Liabilities. The property of all kinds of a firm or company together with all amounts due it is called its *assets*; the debts of a firm or company are called its *liabilities.*

403. Partnership is separated into *simple* and *compound.*

In simple partnership the capital of each partner is invested for *the same time.*

In compound partnership *the time* for which the capital of each partner is invested is taken into account, as well as *the amount of the capital.*

404. *The division of profits and losses is made proportionally to the amount of the capital and the time it is invested.*

405. Examples. 1. A and B entered into partnership, A furnishing \$4000, and B \$5000. If they gained \$1800, what was each partner's share of the profits?

SOLUTION. \$4000 + \$5000 = \$9000, the entire capital.

A's share of the profits was $\frac{4000}{9000}$ of \$1800, or \$800; and B's share of the profits was $\frac{5000}{9000}$ of \$1800, or \$1000.

2. A and B entered into partnership, A furnishing \$2000 for 2 yr., and B \$3000 for 1 yr. If their profits are \$1400, what is the share of each?

SOLUTION. The use of \$2000 for 2 yr. is equal to the use of 2 × \$2000, or \$4000, for 1 yr. Hence, the entire capital for 1 yr. equals \$4000 + \$3000, or \$7000. A's share of the profits is $\frac{4000}{7000}$ of \$1400, or \$800; and B's share is $\frac{3000}{7000}$ of \$1400, or \$600.

Exercise 105.

1. A, B, and C entered into partnership, A furnishing $18,150; B, $19,360; and C, $10,890. If their profits were $12,100, what was each man's share of the profits?

2. Four men engaged in business together and made a profit of $1200. How much of it should each man receive, if the first put in $3000, the second $5000, the third $4200, and the fourth $2400?

3. A man dies owing three creditors $8050, $2970, and $7170, respectively. If his assets, after deducting expenses, are $13,646, how much will each creditor receive?

4. Three heirs receive from an estate $4700, $3200, and $12,500, respectively, on condition that they together pay a debt of $2000. What amount will each have?

5. Arnold and Baker enter into partnership. Arnold puts in $6000 for 8 mo., and Baker $4000 for 6 mo. Their profits are $2000. What is each man's share?

6. Dobson furnishes the firm of Dobson & Fogg with $5000 for 13 mo.; Fogg furnishes $7000 for 9 mo. Their profits are $1700. What is the share of each?

7. In a business partnership, A furnishes $800, and after 3 mo. $250 more; B furnishes $950, and at the end of 2 mo. withdraws $200; C furnishes $650, and at the end of 6 mo. $400 more. At the end of the year their profit is $2516. How shall it be divided among them?

8. Two partners, A and B, enter into partnership with capitals of $3500 and $8700, respectively, and A is to have 0.12 of the profits for managing the business. How shall a profit of $1906.25 be divided between them?

9. A puts $2100 into a business, and B $1750. At the end of a year each puts in $700 more, and C joins them with $2500. How shall a profit of $2166.50 be divided 18 months after C enters the firm?

10. Three graziers hire a pasture, for which they pay \$132.50. One puts in 10 oxen for 3 months, another 12 oxen for 4 months, and the third 14 oxen for 2 months. How much of the rent ought each to pay?

11. A begins business, with a capital of \$2400, on the 19th of March; and on the 17th of July admits B as a partner, with a capital of \$1800. December 31 the profits are \$943. What is the share of each?

12. A and B join capitals in the ratio 7 : 11. At the end of 7 months A withdraws $\frac{1}{2}$ of his, and B $\frac{1}{3}$ of his; and, after 11 months more, they divide a profit of \$5148.50. What is the share of each?

13. Divide £65 9s. among three men, so that the first may have as many half-crowns as the second has shillings; and the second as many guineas as the third has pounds.

14. A and B begin business each with a capital of \$2000. A adds \$500 at the end of 2 months, and \$500 more at the end of 7 months; B adds \$800 at the end of 3 months. If the profits are \$3605.25 at the end of a year, what is the share of each?

15. Three partners in a restaurant furnish respectively \$500 for 7 months, \$600 for 8 months, and \$900 for 9 months. If they lose \$410, what is each one's share of the loss?

16. Two capitalists contribute, one \$10,000, the other \$12,000, to an enterprise which continues in operation for 10 years. 10 months after starting, a third man becomes a partner and contributes \$15,000; and 2 years after this a fourth man contributes \$17,400. If the total profits are \$45,600, what amount does each partner receive?

17. A began business with a capital of \$2500. After three years he invested \$1250 more, and took as a partner B, who invested \$5000. At the end of four years more the profits amounted to \$9562.50. What was the share of each?

Averages, or Alligation.

406. The **average** of several numbers is the number that can be put in place of each of them without altering their sum.

Thus, the average weight of four turkeys, weighing respectively 10 lb., 11 lb., 12 lb., and 13 lb., is $11\frac{1}{2}$ lb.; for the four turkeys together weigh 10 lb. + 11 lb. + 12 lb. + 13 lb., or 46 lb., and the average weight of the four turkeys is $\frac{1}{4}$ of 46 lb., or $11\frac{1}{2}$ lb.

407. Alligation is the process of finding the average value of a compound or mixture composed of quantities of different values; or of finding the proportion of several quantities of different values that must be used to form a compound or mixture of a given average value.

NOTE. The first process is called *alligation medial;* the second process, *alligation alternate.*

408. Examples. 1. A grocer mixed 11 lb. of coffee, costing \$0.25 a pound, with 6 lb. of chicory, costing \$0.08 a pound. What was the cost of the mixture a pound?

$$\begin{array}{rcr} 11 \times \$0.25 & = & \$2.75 \\ 6 \times \ \ 0.08 & = & 0.48 \\ \hline 17 \times \ \ \$\ ? & = & \$3.23 \end{array}$$

SOLUTION. 11 lb. at \$0.25 cost \$2.75, and 6 lb. at \$0.08 cost \$0.48. Therefore, 17 lb. of the mixture cost \$3.23, or \$0.19 a pound.

2. In what proportion may a grocer mix syrups costing respectively 42, 56, 65, and 75 cents a gallon to make a mixture worth 60 cents a gallon?

SOLUTION. 1 gal. of the 42-cent syrup gains in value \$0.18, and 1 gal. of the 75-cent syrup loses in value \$0.15. Hence, to make a mixture of these two syrups worth \$0.60 a gallon, the grocer must mix them in the ratio 15 : 18; that is, 5 : 6.

1 gal. of the 56-cent syrup gains in value \$0.04, and 1 gal. of the 65-cent syrup loses in value \$0.05. Hence, to make a mixture of these two syrups worth \$0.60 a gallon, he must mix them in the ratio 5:4.

Therefore, the grocer may take 5 gal. at \$0.42, 5 gal. at \$0.56, 4 gal. at \$0.65, and 6 gal. at \$0.75.

EXERCISE 106.

1. There were 125 pupils at school on Monday, 130 on Tuesday, 128 on Wednesday, 132 on Thursday, and 125 on Friday. What was the average daily attendance?

2. A spring of water that yields 250 gal. an hour supplies a town containing 360 families. What is the average daily supply of water for each family?

3. A wine merchant put into an empty cask 15 qt. of brandy costing $1.10 a quart, 66 qt. costing $1.20 a quart, and 43 qt. costing $1.40 a quart. At what price per quart must he sell the brandy to gain one fifth of the cost?

4. A grocer mixed 120 lb. of tea costing 50 cents a pound with 180 lb. costing 40 cents a pound. At what price per pound must he sell the mixture to make a profit of $30 on the whole?

5. A grocer buys two kinds of tea at 40 cents a pound and 56 cents a pound, respectively, and mixes them in the ratio of 5 to 3. What is his profit, if he sells 56 lb. of the mixture at 84 cents a pound?

6. The average length of ten sticks is 2 ft. $10\frac{1}{2}$ in.; one stick is $27\frac{1}{2}$ in. long, another $37\frac{1}{2}$ in. long, and the remaining eight are of the same length. What is the length of one of the remaining eight?

7. The average age of the boys in the four classes of a school is 18.4 yr., 17.9 yr., 16.8 yr., and 15.7 yr. The classes contain 29, 33, 34, and 33 boys, respectively. What is the average age of the boys in the school?

8. Seven boys weigh respectively 119.7 lb., 105 lb., 178.3 lb., 165.3 lb., 142.8 lb., 109 lb., 154.2 lb. What is their average weight?

9. In what proportion should tea costing 60 cents a pound be mixed with tea costing 45 cents a pound that the cost of the mixture should be 54 cents a pound?

10. A merchant has teas that cost 80 cents, 60 cents, and 40 cents a pound, respectively. How many pounds of each kind shall he take to make a mixture of 1000 lb., so that in selling it at 70 cents a pound he may make a profit of 8 cents a pound?

11. A grocer mixed black tea that cost him 28 cents a pound with green tea that cost him 42 cents, and by selling the mixture at 35 cents a pound he gained $\frac{1}{6}$ of its cost. What was the actual cost of the mixture a pound? In what ratio were the teas mixed?

12. A dealer has an order for 1000 bu. of wheat at 70 cents a bushel. In what proportion shall he mix three kinds of wheat at 66, 69, and 72 cents a bushel to fill the order?

13. A wine merchant mixes wines that cost \$0.95, \$1.05, \$1.10, and \$1.20 a gallon to make a mixture costing \$1.00 per gallon. How many gallons of each kind of wine does he take?

14. A merchant wishes to fill a barrel that will hold 240 lb. of sugar with sugar costing $4\frac{1}{2}$, $4\frac{3}{4}$, and $5\frac{1}{8}$ cents a pound, respectively, so that the mixture may cost $4\frac{7}{8}$ cents a pound. How many pounds of each kind shall he take?

15. A grocer wishes to mix 12 lb. of coffee at 40 cents a pound and 20 lb. at 35 cents a pound with coffee at 28 cents a pound, so that the mixture may be worth 30 cents a pound. How many pounds at 28 cents must he use?

16. A grocer mixed 14 lb. of coffee costing 32 cents a pound, 18 lb. costing 35 cents a pound, 22 lb. costing 38 cents a pound, and 40 lb. costing 30 cents a pound. What is the cost of the mixture per pound, and at what price must he sell it to gain 0.25 of the cost?

17. In what proportion may oils costing \$1.20, \$0.80, and \$0.60 a gallon be mixed that the mixture may cost \$0.70 a gallon?

CHAPTER XII.

PERCENTAGE.

409. A **percentage** of a number is the result obtained by taking a stated number of *hundredths* of it.

One *hundredth* of a number is called one *per cent* of it; two *hundredths,* two *per cent;* and so on.

410. A **rate per cent** is a *fraction* whose denominator is 100, and whose numerator is the *given number of hundredths.* The methods of common fractions or of decimals are used in the solution of all examples in Percentage.

The *shortest* method is the *best* method.

411. The symbol % stands for the words *per cent.*

Thus, 13% is 0.13; $2\frac{1}{4}\%$ is $0.02\frac{1}{4}$; 867% is 8.67.

412. Example. Express $37\frac{1}{2}\%$ as a common fraction.

SOLUTION. $37\frac{1}{2}\% = \frac{37\frac{1}{2}}{100} = \frac{3}{8}$. Hence,

413. To Express a Rate Per Cent as a Common Fraction,

Write the rate for the numerator and 100 for the denominator, and reduce this fraction to its lowest terms.

EXERCISE 107.

Reduce to a common fraction:

1. 20%.	**6.** 5%.	**11.** $62\frac{1}{2}\%$.	**16.** $18\frac{3}{4}\%$.
2. 80%.	**7.** 10%.	**12.** $87\frac{1}{2}\%$.	**17.** 95%.
3. 25%.	**8.** $12\frac{1}{2}\%$.	**13.** $66\frac{2}{3}\%$.	**18.** 70%.
4. 50%.	**9.** $16\frac{2}{3}\%$.	**14.** $37\frac{1}{2}\%$.	**19.** $144\frac{4}{9}\%$.
5. 75%.	**10.** $11\frac{1}{9}\%$.	**15.** $83\frac{1}{3}\%$.	**20.** $262\frac{1}{2}\%$.

414. Example. Express $\frac{2}{5}$ as a rate per cent.

SOLUTION. $1 = \frac{100}{100} = 100\%$.
$\frac{2}{5} = \frac{2}{5}$ of $100\% = 40\%$. Hence,

415. To Express a Common Fraction as a Rate Per Cent,

Divide 100 by the denominator of the fraction and multiply the quotient by the numerator.

416. Examples. 1. Express 0.4 as a rate per cent.

SOLUTION. $0.4 = 0.40$, or 40%.

2. Express 0.4575 as a rate per cent.

SOLUTION. $0.4575 = 0.45\frac{75}{100} = 0.45\frac{3}{4}$, or $45\frac{3}{4}\%$.

3. Express 0.00375 as a rate per cent.

SOLUTION. $0.00375 = 0.00\frac{375}{1000} = 0.00\frac{3}{8} = \frac{3}{8}\%$. Hence,

417. To Express a Decimal as a Rate Per Cent,

Write the decimal as hundredths, and the number that expresses the hundredths is the rate per cent required.

NOTE. If the decimal has more than two places, the figures that follow the hundredths' place signify a fraction of 1%.

EXERCISE 108.

Express as a rate per cent:

1. $\frac{1}{2}$.	8. $\frac{4}{5}$.	15. 0.25.	22. 0.33333.
2. $\frac{1}{4}$.	9. $\frac{8}{25}$.	16. 0.6.	23. 0.16667.
3. $\frac{3}{8}$.	10. $\frac{7}{20}$.	17. 0.75.	24. 0.83333.
4. $\frac{1}{3}$.	11. $\frac{2}{9}$.	18. 0.9.	25. 0.875.
5. $\frac{1}{6}$.	12. $\frac{7}{16}$.	19. 0.65.	26. 1.375.
6. $\frac{5}{6}$.	13. $\frac{4}{11}$.	20. 0.45.	27. 2.66667.
7. $\frac{2}{3}$.	14. $\frac{9}{32}$.	21. 0.2.	28. 4.2525.

418. Problems in Percentage are conveniently divided into **three classes,** as follows:

Class I. To find a certain fraction of a number.

Class II. To find the fraction that one number is of another.

Class III. To find a number when a fraction of it is given.

The following examples illustrate the three classes:

Class I. What number is $\frac{2}{3}$ of 300?

Class II. What fraction of 300 is 200?

Class III. What is the number, if 200 is $\frac{2}{3}$ of it?

The fraction in each class is expressed in *hundredths* for the sake of a uniform standard; the phrase *per cent* is used for the word *hundredths,* and the symbol % is written for the phrase *per cent.*

Thus, in common fractions 8 is $\frac{1}{2}$ of 16; in decimals 8 is 0.5 of 16; in percentage 8 is 50% of 16.

In common fractions 5 is $\frac{1}{8}$ of 40; in decimals 5 is 0.125 of 40; in percentage 5 is $12\frac{1}{2}$% of 40.

419. One hundred per cent of a number is the number itself.

Thus, 100% of 40 is 40.

CLASS I.

420. Examples.

1. Find $16\frac{1}{3}$% of 288.

SOLUTION. $16\frac{1}{3}\% = 0.16\frac{1}{3}$.

$0.16\frac{1}{3}$ of $288 = 47.04$.

Therefore, $16\frac{1}{3}$% of 288 is 47.04.

2. Find $16\frac{2}{3}$% of 288.

SOLUTION. $16\frac{2}{3}\% = 0.16\frac{2}{3} = \frac{1}{6}$.

$\frac{1}{6}$ of $288 = 48$.

Therefore, $16\frac{2}{3}$% of 288 is 48.

421. To Find a Percentage of a Number,

Multiply the number by the given rate per cent, expressed as a common fraction or as a decimal.

Exercise 109.

Find by using decimals:

1. 23% of 1728.	6. 2% of 846.	11. 0.5% of 144.
2. 44% of 1861.	7. 9% of 24.87.	12. 8752% of 2645.
3. 87% of 14.22.	8. 122% of 12.5.	13. 0.02% of 52.36.
4. 63% of 2.832.	9. 287% of 48.2.	14. 2% of 3.
5. 72% of 841.	10. 1% of 7854.	15. 2.06% of 312.

Find by using common fractions:

16. $33\frac{1}{3}$% of 363.	21. $62\frac{1}{2}$% of 216.	26. $14\frac{2}{7}$% of 81.9.
17. 20% of 545.	22. $37\frac{1}{2}$% of 360.	27. $22\frac{2}{9}$% of 8.19.
18. 25% of 1728.	23. $83\frac{1}{3}$% of 486.	28. $168\frac{3}{4}$% of 256.
19. 50% of 8642.	24. $66\frac{2}{3}$% of 456.	29. $143\frac{3}{4}$% of 288.
20. 75% of 432.	25. $12\frac{1}{2}$% of 2.56.	30. 70% of 8432.

31. The population of a town in 1880 was 12,275, and it increased 8% in the next ten years. Find the population of the town in 1890.

32. How much metal will be obtained from 365 tons of ore, if the ore contains 7% of metal?

33. If gunpowder contains 75% of saltpetre, 10% of sulphur, 15% of charcoal, how many pounds of each are there in a ton of powder?

34. Air is composed by volume of 20.0265% of oxygen and 79.9735% of nitrogen. How many cubic feet of oxygen in 1750 cu. ft. of air?

35. If 2% of a regiment of 750 men are killed in an engagement, 6% are wounded, and 4% are missing, what is the number still available for service?

36. A man sold a bicycle that cost him \$60, and lost $16\frac{2}{3}$% of the cost. For what price did he sell it?

37. A merchant sold hats that cost him \$1.50 each, and gained $33\frac{1}{3}$%. For what price did he sell them?

38. In a school of 80 children, $17\frac{1}{2}\%$ are girls. Find the number of boys.

39. The lead ore from a certain mine yields 60% of metal, and of the metal $\frac{3}{4}$ of 1% is silver. How much silver and how much lead will be obtained from 1200 t. of ore?

40. If 13% of a population of 27,000,000 are foreign born, how many of the population are foreign born?

41. If iron expands $\frac{1}{8}$ of 1% when heated 185° F., what will be the expansion of iron when heated from $-20°$ F. to $+120°$ F.?

42. A tubular iron bridge 740 ft. long has one end fast to a pier. How much play must be allowed at the other end for the expansion of the iron, if the climate varies from $-30°$ F. in winter to $+130°$ F. in a July sun?

43. How much longer is 100 miles of iron rails at 118° F. than at 20° below zero?

CLASS II.

422. Examples. 1. What per cent of 4 is 3?

SOLUTION. 3 is $\frac{3}{4}$ of 4, and $\frac{3}{4} = \frac{75}{100}$, or 75%.
Therefore, 3 is 75% of 4.

2. What per cent of 447 is 169.86?

SOLUTION. $4.47 = 1\%$ of 447.
Therefore, $169.86 = \frac{169.86}{4.47}\%$ of $447 = 38\%$ of 447. Hence,

423. To Find the Rate Per Cent when a Number and a Percentage of the Number are Given,

Write the number as the denominator and the percentage as the numerator of a fraction, and reduce this fraction to hundredths. Or,

Divide the percentage of the number by one per cent of the number.

Exercise 110.

1. What per cent of 64 is 16 ?
2. What per cent of 16 is 64 ?
3. What per cent of 450 lb. is 50 lb.?
4. What per cent of 50 lb. is 450 lb.?
5. What per cent of $465 is $130.20 ?
6. What per cent of $832 is $807.04 ?
7. What per cent of $987 is $2289.84 ?
8. A brick kiln contained 29,800 bricks, but after burning only 29,734 were found in good condition. What per cent had been spoiled in burning?
9. If a house worth $4000 rents for $360 a year, what per cent of its value is the rent ?
10. If 75 bu. of corn are raised from 1 pk. of corn, what per cent is the increase?
11. Ten years ago the population of a city was 26,275; its present population is 31,530. What is the increase per cent ?
12. If $3\frac{3}{4}$ tons of sulphur are required to make $31\frac{1}{4}$ tons of gunpowder, what per cent of gunpowder is sulphur ?
13. If a long ton of ore in a gold mine yields 5 oz. (troy) of gold, what is the yield per cent?
14. If $12\frac{1}{2}$ tons of iron are obtained from 235 tons of ore, what per cent of the ore is iron ?

Find the gain per cent in population in each of the following cities from 1880 to 1890 :

	Cities.	1880.	1890.
15.	New York,	1,206,594,	1,513,501.
16.	Chicago,	503,304,	1,099,850.
17.	Philadelphia,	846,981,	1,046,964.
18.	Brooklyn,	566,689,	806,343.
19.	Boston,	362,535,	448,477.

20. If 2 gal. of water are added to 25 gal. of alcohol, what per cent of the mixture is water? What per cent is alcohol?

21. If 5% of the present population of a town has been the increase in the preceding ten years, what per cent of the population ten years ago has been added?

22. A man gained in weight in January 3%, and in February lost 3%. What per cent of his weight the first day of January is his weight the first day of March?

23. If 7 lb. of a certain article loses 3 oz. in weight by drying, what per cent of its original weight is water?

24. If 7 lb. of a dry article has lost 3 oz. by drying, what per cent of its original weight was water?

25. If a dry article exposed to damp air absorbed 3 oz. of water, and then weighed 7 lb., what per cent of its present weight is water?

26. If rosin is melted with 20% of its weight of tallow, what per cent of tallow does the mixture contain?

27. If 20% of a mixture of tallow and rosin is tallow, what per cent of the weight of the rosin is the weight of the tallow?

28. Nitrogen gas, under standard pressure and temperature, is $\frac{1}{8}$ of 1% of the weight of an equal volume of water. What is the specific gravity of nitrogen? How many gallons of nitrogen will it take to weigh as much as a pint of water?

29. Oxygen gas is $\frac{1}{7}$ of 1% of the weight of an equal volume of water. What is its specific gravity? How many gallons of oxygen will it take to weigh as much as a pint of water?

30. If common air consists of 4 volumes of oxygen to 13 of nitrogen, what is its specific gravity?

31. How many gallons of air will it take to weigh as much as a pint of water?

CLASS III.

424. Examples. 1. If 17% of a number is 799, what is the number?

SOLUTION. Since 17% of a number is 799, 1% of the number is $\frac{1}{17}$ of 799, or 47, and 100% of the number is 100 × 47, or 4700.

2. If $16\frac{2}{3}$% of a number is 432, what is the number?

SOLUTION. 432 is $16\frac{2}{3}$%, or $\frac{1}{6}$ of the required number. Therefore, the required number is 6 × 432, or 2592.

3. 1400 is $16\frac{2}{3}$% more than what number?

SOLUTION.
100 % of the number = the number.
$16\frac{2}{3}$% of the number = the increase.
$116\frac{2}{3}$%, or $\frac{7}{6}$ of the number = 1400.
$\frac{1}{6}$ of the number = $\frac{1}{7}$ of 1400, or 200.
Therefore, the number = 6 × 200, or 1200.

4. 1200 is 25% less than what number?

SOLUTION.
100% of the number = the number.
25% of the number = the decrease.
75%, or $\frac{3}{4}$ of the number = 1200.

If 1200 is $\frac{3}{4}$ of the number, $\frac{1}{4}$ of the number is $\frac{1}{3}$ of 1200, or 400. Therefore, the number is 4 × 400, or 1600. Hence,

425. To Find a Number when a Percentage of the Number and the Rate Per Cent are Given,

Express the rate per cent as a fraction, divide the percentage by the numerator of this fraction and multiply the quotient by the denominator.

EXERCISE 111.

1. 15 is $\frac{3}{4}$ of what number? 15 is 75% of what number?
2. $500 is 4% of what sum of money?
3. Find the number of which 324 is 27%.

4. 288 is 20% more than what number?
5. 145 is 25% more than what number?
6. 1240 is 55% less than what number?
7. 260 is $33\frac{1}{3}$% less than what number?
8. 91 is 40% more than what number?
9. 901 is $6\frac{1}{4}$% more than what number?
10. If $8\frac{3}{4}$% of a number is 4140.15, what is the number?
11. If 3% of a number is $2\frac{5}{8}$, what is the number?
12. If 140% of a number is 630, what is the number?
13. If $6\frac{1}{4}$% of a number is 33.25, what is the number?

14. A town, after decreasing 11%, has 4539 inhabitants. Find its number at first.

15. In a certain school there are 200 girls, and the number of girls is 40% of the whole number of pupils. How many pupils in the school?

16. A manufactory uses 24 tons of coal a day, 20% of which is lost in smoke. How much coal would be needed if this waste could be prevented?

17. A town, after decreasing 25%, has 4539 inhabitants. Find its number at first.

18. If the ore from a mine yields $\frac{3}{80}$ of 1% of pure gold, how many long tons of ore must be taken to obtain 7 lb. (troy) of gold?

19. Goods were sold, at a loss of 3%, for $2667.50. What was the cost?

20. A tradesman, in selling goods, deducts from the marked price 5% for cash. What was the marked price of goods for which he received $14.25?

21. If an ore loses $41\frac{1}{2}$% of its weight in roasting, and $43\frac{3}{4}$% of the remainder in smelting, how much ore will be required to yield 1000 tons of metal?

22. How many pounds of tallow must be mixed with $8\frac{1}{2}$ pounds of rosin that the mixture may contain 15% of tallow?

Commercial Discount.

426. Commercial Discount. A reduction from the list price of an article, from the amount of a bill of goods, or from the amount of a debt is called *commercial discount.*

427. Discounts are reckoned at some common fraction of, or at some rate per cent of, *the amount from which the discount is made.*

If two or more discounts are quoted, the *first* denotes a discount off the *list price;* the *second,* a discount off *the remainder after the first discount is made;* the *third,* off *the remainder after the second discount is made;* and so on.

Thus, discounts of 20 and $\frac{1}{3}$ mean that 20% is to be deducted from the amount, and then from the remainder $\frac{1}{3}$ of it is to be taken.

NOTE. By varying the rate of discount the manufacturer can raise or lower the price of his goods without issuing a new catalogue.

428. Examples. 1. Find the net amount of a bill of \$150 after a discount of $33\frac{1}{3}\%$ is made.

SOLUTION. Discount is $33\frac{1}{3}\%$, or $\frac{1}{3}$ of \$150 = \$30.
Net amount is \$150 − \$30 = \$120.

2. Find the net amount of a bill of \$527.10 with $\frac{1}{3}$ and 20% off.

3	\$527.10
	175.70
5	\$351.40
	70.28
	\$281.12

SOLUTION. \$527.10 less $\frac{1}{3}$ of \$527.10 is \$351.40. $20\% = \frac{1}{5}$, and \$351.40 less $\frac{1}{5}$ of \$351.40 is \$281.12, the net amount of the bill.

EXERCISE 112.

1. Find the net amount of a bill of \$1550, if a discount of 5% is made for cash.
2. Find the net amount of a bill of \$88, if the discounts are 20 and 10.

3. Find the net cash amount of a bill of $800, if the discounts are 75, 5, and $2\frac{1}{2}$.

4. Find the net cash amount of a bill of $272, if the discounts are $\frac{1}{2}$, 10, and 5.

5. Find the net cash amount of a bill of $1440, if the discounts are 55, 10, and 5.

6. Find the net cash amount of a bill of $1125, if the discounts are $\frac{1}{2}$, 10, 10, 10, and 5.

7. Find the net amount of a bill of $872.29, if the discounts are $\frac{1}{3}$, 20, and 25.

8. Find the difference between a single discount of 50% and two successive discounts of 25% and 25% off a bill of $1272.36.

9. An agent bought 25 sewing machines with 15, 10, and 5 off the list price of $40 each, and sold them at a discount of 10% off the list price. What was the net amount he received for the sewing machines and his profit?

10. An agent bought a bicycle with 25 and 5 off the list price of $100. If he received an additional discount of $2\frac{1}{2}$% for cash, and sold the bicycle at a discount of $12\frac{1}{2}$% off the list price, what was the selling price and his profit?

11. A collector collects 65% of a debt of $727, and charges 5% of the amount he collected. What was the net amount for the creditor?

Gain and Loss.

429. The **gain** or **loss** in business transactions is often computed as a per cent of the cost.

430. In commercial discount the per cent is always reckoned upon the *price asked.* In gain or loss the per cent is always reckoned upon the *price paid;* that is, upon the *cost.*

EXERCISE 113.

1. If goods are bought for \$415, and sold for \$500, what is the gain per cent?

2. If goods are bought for \$415, and sold for \$400, what is the loss per cent?

3. A farmer buys 24 head of cattle at \$80 a head. After losing 6 head, he sells the remainder at \$105 a head. What does he gain or lose per cent?

4. Teas at 68 cents, 86 cents, and 96 cents a pound are mixed in equal quantities, and sold at 90 cents a pound. Find the gain per cent.

5. By selling goods for \$1173.92 a merchant gains \$153.12. Find the gain per cent.

6. What was the cost, when $17\frac{1}{2}\%$ was gained by selling goods for \$253.80?

7. A wine merchant mixes 24 gal. of wine at \$7 a gallon, with 18 gal. at \$5 a gallon, and sells the whole at \$7 a gallon. What does he gain per cent?

8. By selling a horse for \$200, a dealer loses $12\frac{1}{2}\%$. What would he have gained or lost per cent if he had sold the horse for \$250?

9. A spirit merchant buys 75 gal. of spirits at \$3.25 a gallon, and, after drawing off 10 gal., sells the remainder so as to gain 5% on the cost of the whole. What is the selling price per gallon?

10. A man owns two city lots worth respectively \$9845 and \$12,155. If the first gains in value 32%, and the second loses 13%, what is the gain or loss per cent in the value of the two lots?

11. A tradesman marks a hat \$5, but takes off 5%. If his profit is 14%, what was the cost of the hat?

12. What would a dishonest dealer gain per cent by using a false weight of 15 oz. instead of a pound?

13. A dishonest dealer gains 12% by using false weights. What is the real weight of his pound?

14. What per cent above cost must a merchant mark his goods that he may take off 20% from the marked price, and still make 20% on the cost?

SOLUTION. Since the merchant is to make 20% on the cost of the goods, the selling price is 120% of the cost price.

Since the selling price is to be 20% below the marked price, the selling price is 80% of the marked price.

Therefore, the marked price will be $\frac{100}{80}$ of 120% of the cost price, or 150% of the cost price; that is, the goods must be marked 50% above cost.

15. What per cent above cost must a merchant mark his goods to take off 10%, and still gain 17%?

16. What per cent above cost must a merchant mark his goods to take off $12\frac{1}{2}$%, and still gain $12\frac{1}{2}$%?

17. What per cent above cost must a merchant mark his goods to take off 15%, and still gain 15%?

18. What per cent above cost must a merchant mark his goods to take off $33\frac{1}{3}$%, and still gain $33\frac{1}{3}$%?

19. A man bought a horse for $70, and sold him for $80. What per cent did he gain? What per cent of the selling price of the horse did he gain?

20. If a merchant clears $800 by selling goods for $12\frac{1}{2}$% profit, what was the cost of the goods, and for how much were they sold?

21. A man selling eggs at $0.40 a dozen gains $33\frac{1}{3}$%; what was the cost? Another, selling at the same price, gains $33\frac{1}{3}$% of his receipts; what did his eggs cost?

22. A man lost 10% by selling a carriage for $117. At what price should he have sold it to make 10%?

23. If a real estate dealer gained $600 by selling a farm for 20% profit, what was the cost of the farm, and for how much did he sell it?

Commission.

431. Commission is the payment made by one person, called the Principal, to another, called the **Agent** or **Factor**, for the transaction of business.

432. Commission is usually a percentage of the money involved in the transaction. If goods are bought, it is a percentage of the amount *paid;* if sold, a percentage of the amount *received;* if money is collected, a percentage of the amount *collected.*

Exercise 114.

1. Find the commission on \$2595, at $2\frac{1}{2}\%$.

2. An agent sells 200 bbl. of flour, at \$6.25, and 600 gal. of molasses, at 65 cents, and charges a commission of $1\frac{3}{4}\%$. What are the net proceeds?

Note. The sum left after the payment of the commission and of all other expenses is called the *net proceeds.*

3. A commission merchant received \$1640 to buy corn, and charged a commission of $2\frac{1}{2}\%$. What is his commission, and how many bushels of corn at $62\frac{1}{2}$ cents a bushel can he buy?

4. An agent sells a consignment of cotton for \$5216. He pays \$51 for storage, and charges a commission of $2\frac{1}{4}\%$. What are the net proceeds?

5. An agent sold butter for \$1570, and remitted \$1546.45. What was the rate per cent of commission?

6. What are the net proceeds from the sale of 2250 bbl. of flour at \$6.25 a barrel, if the charge for freight is 50 cents a barrel, the commission for selling 2%, and the commission for guaranteeing payment $1\frac{1}{2}\%$?

7. An agent sells 350 crates of peaches, at \$2.60. If the commission is $4\frac{1}{2}\%$, find the net proceeds.

8. An agent sells 420 acres of land, at \$40 an acre, and charges $1\frac{1}{4}\%$ commission. What is his commission?

9. An agent, charging $4\frac{1}{2}\%$ commission, receives for his services \$313. Find the amount of his sales.

10. A merchant buys 730 yd. of carpeting at \$1.25 a yard, and pays his agent $\frac{3}{4}$ of 1% commission. If the freight amounts to \$23.58, at what price per yard must he sell the carpeting to gain 20%?

11. An agent sells a consignment of goods for \$2100. He pays \$33.50 for freight, and remits \$2024.50. Find his rate of commission.

12. An agent sells 5000 lb. of cotton at 14 cents a pound, charging 2% commission. With the net proceeds he buys cotton cloth at 10 cents a yard, charging $1\frac{1}{2}\%$ commission. How many yards of cloth does he buy?

13. An agent sold 500 bbl. of flour at \$5.50 a barrel, and charged $2\frac{1}{2}\%$ commission; the expenses for freight, etc., were \$250. With the net proceeds he bought sugar at $4\frac{3}{8}$ cents a pound, charging $2\frac{1}{2}\%$ commission. How much sugar did he buy, and what was his total commission?

14. A collector's commission for collecting taxes, at $1\frac{1}{2}\%$, is \$206.55. What sum did he collect?

15. An agent received \$2961 to purchase goods, and charged 5% commission. What was his commission?

16. An agent buys 3100 bbl. of flour at \$4.50 a barrel, and charges $1\frac{1}{2}\%$ commission. What is his commission?

17. A broker receives \$6150 to invest in cotton, at $7\frac{5}{8}$ cents a pound. If his commission is $2\frac{1}{2}\%$, how many pounds of cotton can he buy?

18. An agent sells 1100 bbl. of flour at \$4.50 a barrel, and charges $2\frac{1}{2}\%$ commission. He invests the proceeds in steel at $1\frac{1}{2}$ cents a pound, charging $1\frac{1}{2}\%$ commission. What is his entire commission, and how many long tons of steel does he buy?

Insurance.

433. Insurance is the guarantee by an insurance company of the payment of a stated amount to the person insured, in the event of loss of property by fire, by storm at sea, or by other specified disaster; or in the event of the death of the person insured, or of an accident to him.

434. The **policy** is the written agreement.

The **face** of the policy is the sum named therein which is to be paid to the holder in case of loss.

435. The **premium** is the sum paid for the insurance. It is reckoned as a percentage on the face of the policy.

436. Insurance companies usually insure $\frac{2}{3}$ or $\frac{3}{4}$ of the value of the property; and, in case of partial loss, pay only the value of the property destroyed.

NOTE. Insurance agents are also called underwriters, and insuring is often called underwriting.

EXERCISE 115.

1. Find the premium of the fire insurance on a house for \$2650 at $\frac{1}{2}$ of 1%.

2. Find the premium for insuring a man's life for \$2500, at an age for which the rate is $2\frac{1}{4}$%.

3. At $6\frac{3}{4}$%, what premium will be paid on a vessel worth \$36,400, insured for $\frac{3}{4}$ its value?

4. A vessel worth \$16,000 is insured for $\frac{3}{4}$ its value at $7\frac{1}{4}$%. What is the premium?

5. The premium of insurance at $1\frac{1}{4}$% is \$150. What is the amount insured?

6. A vessel valued at \$128,000 is insured for $\frac{3}{4}$ its value at $3\frac{7}{8}$%. What is the net loss to the owners, if the vessel is destroyed during the third year after it is insured?

7. A building worth \$7500 is insured at $\frac{2}{3}$ its value, at $\frac{1}{8}$ of 1% per annum. What is the annual premium?

8. Four companies insure a store and contents for \$60,000. One company takes \$20,000, at $\frac{3}{8}$ of 1%; a second takes \$10,000, at $\frac{3}{4}$ of 1%; a third, \$15,000, at $\frac{5}{8}$ of 1%; a fourth, the remainder, at $\frac{1}{2}$ of 1%. What is the premium?

9. If the store of Ex. 8 is damaged to the extent of \$4500, what amount does each company pay?

10. A man insures his life for \$10,000, paying \$350 a year in advance, and dies the day before the fifth premium is due. The company pays his widow \$10,000. How much has the company lost by him, if the interest gained on the premiums paid amounts to \$175?

11. A merchant shipped a cargo to London, and took a policy of \$100,800 at $3\frac{1}{2}$%, to cover both the cargo and the premium. Find the value of the cargo.

HINT.
$$\begin{array}{r}
100\% \text{ of policy} = \text{policy (cargo and premium).} \\
\underline{3\tfrac{1}{2}\% \text{ of policy} = \text{premium.}} \\
96\tfrac{1}{2}\% \text{ of policy} = \text{cargo.}
\end{array}$$

12. Three companies insure, at $\frac{3}{4}$ its value, a building worth \$16,000. The first company takes $\frac{1}{3}$ the risk at $\frac{3}{4}$ of 1%; the second, $\frac{2}{5}$ at $\frac{7}{8}$ of 1%; and the third, the remainder at $\frac{3}{4}$ of 1%. Find the total premium.

13. S. Williams pays \$18.40 premium for insuring his house for $\frac{2}{3}$ its value at $1\frac{1}{2}$%. What is the value of his house?

14. Find the annual premium for an ordinary life policy of \$5000 issued to a man 30 years old, if the rate of insurance is 1.93%.

15. What is the annual premium for an ordinary life policy of \$12,000 issued to a man 40 years old, if the rate of insurance is 2.661%?

Direct Taxes.

437. **Taxes** are levied for the support of the government, for the support of schools, and for other purposes.

438. **Direct taxes** are laid on the person or on the value of the property he possesses.

439. A **poll tax** is a tax levied upon a person, and a **property tax** is a tax levied on property.

440. **Assessors** are officers whose duty it is to appraise the property belonging to each person to be taxed.

441. A **tax collector** is an officer who collects the taxes. He receives a salary, or a per cent of the sum collected.

442. The **treasurer** receives the money collected, and is usually paid a salary.

443. The tax to be raised on the assessed valuation of property is the total amount of tax voted, diminished by the poll taxes, and by such corporation taxes as are collected by the state and distributed to the several towns.

A tax of \$18,000 is levied upon a town which contains 800 polls, assessed at \$1.50 each, and which has taxable property assessed at \$1,200,000. The town receives from the state \$3600 as its share of corporation taxes. Find the rate of taxation, and the tax paid by Mr. Brown, if his property is assessed at \$5960, and he pays for one poll.

SOLUTION. The amount of poll taxes = 800 × \$1.50 = \$1200; and the amount from state and polls = \$3600 + \$1200 = \$4800.

The amount levied on property = \$18,000 − \$4800 = \$13,200.

The rate = \$13,200 ÷ 1,200,000 = \$0.011, or \$11 on \$1000.

To provide for contingencies, such as abatement of taxes, cost of collecting, etc., the assessors make the rate \$12 on \$1000.

Therefore, Mr. Brown's property tax is 0.012 of \$5960 = \$71.52, and his total tax = \$71.52 + \$1.50 = \$73.02.

444. To facilitate the making of taxes, the assessors usually prepare a table like the following, which is computed at 12 mills on a dollar:

TABLE.

PROP.	TAX.	PROP.	TAX.	PROP.	TAX.	PROP.	TAX.
$1	$0.012	$10	$0.12	$100	$1.20	$1000	$12.00
2	0.024	20	0.24	200	2.40	2000	24.00
3	0.036	30	0.36	300	3.60	3000	36.00
4	0.048	40	0.48	400	4.80	4000	48.00
5	0.060	50	0.60	500	6.00	5000	60.00
6	0.072	60	0.72	600	7.20	6000	72.00
7	0.084	70	0.84	700	8.40	7000	84.00
8	0.096	80	0.96	800	9.60	8000	96.00
9	0.108	90	1.08	900	10.80	9000	108.00

445. Example. Find by the above table the tax of Mr. Gay, who pays for one poll at $1.50, and has property assessed at $6545.

SOLUTION.

Tax on	$6000	=	$72.00
Tax on	500	=	6.00
Tax on	40	=	0.48
Tax on	5	=	0.06
Tax on poll		=	1.50
Total tax		=	$80.04

EXERCISE 116.

Make a table for a tax rate of 16 mills, and:

1. Find the tax on property assessed at $7500.
2. Find the tax on property assessed at $4825.
3. Find the tax on property assessed at $9685.
4. Find the tax on property assessed at $10,727.
5. Find the tax on property assessed at $12,863.
6. Find the tax on property assessed at $16,458.
7. Find the tax on property assessed at $38,249.

8. James Brown is assessed $2500 on his real estate and $5200 on his personal property, and pays for two polls at $1.50 each. If the rate is $12.18 on $1000, what is his total tax?

9. If the tax rate of a town is $12.25 on $1000, and the amount of the levy $11,788.50, what is the assessed valuation of the town?

10. If the assessed valuation of a town is $1,777,000, and the levy is $29,231.65, what is the rate on $1000?

11. What sum must be assessed that $15,000 may remain after paying 2% commission for collecting the taxes?

12. For building a schoolhouse a tax of $1857.60 was levied upon a school district, assessed valuation $1,935,000. What was the tax on property assessed at $6250?

13. In a certain town there are 1350 polls. The assessed valuation of the real estate is $713,250, and of the personal property is $738,954. The poll tax is $2 per poll, and the tax on property is $1\frac{1}{8}\%$. Only 96% of the whole tax can be collected, and the collector is paid $2\frac{1}{2}\%$ of the amount collected. How much does the town receive from the taxes? How much does the collector receive for his services?

Indirect Taxes.

446. **Indirect Taxes** are taxes levied upon articles of merchandise.

447. Indirect taxes are of two kinds: **Customs** or **Duties,** taxes levied on certain imported goods; **Excises** or **Internal Revenue,** taxes levied on certain domestic goods.

NOTE. These are called Indirect Taxes, because, while paid to the government by the person in whose possession the goods are first found, this tax forms a part of the price finally paid by the consumer, and is, therefore, a tax paid by him.

448. Duties are of two classes : *Specific* and *Ad Valorem.*

449. Specific Duties. A specific duty is a definite sum levied on each unit by which the article is measured or weighed, without regard to the value of the unit.

450. Ad Valorem Duties. An ad valorem duty is levied as a certain per cent of the cost of the article in the country where purchased. The cost of the article includes the cost of transportation, commissions, etc.

451. On merchandise imported at a specific duty the following allowances are made :

Tare is an allowance for the weight of the box, cask, bag or other wrapping.

Breakage is an allowance of a certain per cent on liquors in bottles; also on glassware and china.

Leakage is an allowance of a certain per cent on liquors in barrels or casks.

Duties are computed on the sum left *after all deductions are made.*

NOTE. The *long ton* only is used in computing duties.

452. Excises are taxes or licenses for the manufacture or sale of certain domestic articles, as tobacco, whiskey, beer, etc.

EXERCISE 117.

In the following examples the rates of duty are taken from the Dingley Tariff Law, in effect July 24, 1897.

1. What is the duty at $2\frac{1}{2}$ cents a pound on 320 boxes of raisins each containing 40 pounds ?

2. What is the duty at 6 cents a gallon on 420 hhd. of best molasses of 63 gal. each ?

3. What is the duty at $4 a dozen bottles on 50 cases of champagne, each containing 24 pint bottles, if breakage of 5% is allowed ?

4. Find the duty on 150 gross of spectacles, cost price $1.20 a dozen; specific duty 45 cents a dozen, breakage allowed $2\frac{1}{2}\%$; and 20% ad valorem.

5. Find the duty on 100 shotguns, cost price $8.50 each; specific duty of $4 each, and 15% ad valorem.

6. Find the duty at $1 per M on 12,500 ft. of white-wood boards, planed on one side, if an additional duty of 50 cents per M is collected for each side planed.

7. Find the duty on 500 boxes of cigars, gross weight 475 lb., tare 40%, costing $82\frac{1}{2}$ cents per box in Havana. Specific duty $4.50 per pound; and 25% ad valorem.

8. Find the duty on 400 pairs of woolen blankets, cost price $1.75 per pair; weighing $7\frac{1}{2}$ lb. per pair, tare 5%. Specific duty 33 cents per pound, ad valorem 40%.

9. Find the duty on 12 boxes of skein silk, each box weighing 40 lb.; cost price $2.125 per pound, tare 10%. Specific duty 50 cents per pound, ad valorem 15%.

10. Find the duty on 150 gross of clay tobacco pipes, cost price 55 cents a gross. Specific duty 15 cents a gross, and 25% ad valorem.

11. A New York merchant bought in London 400 gal. of cologne at $1.25 a gallon, and commission and other expenses amounted to $56.25. At what price per pint must he sell the cologne to gain 40% on the cost, if he paid a specific duty of 60 cents a gallon, and an ad valorem duty of 45%?

12. Find the duty on 750 lb. of glue, cost price 40 cents; specific duty of 15 cents a pound, tare 2%; and ad valorem duty of 25%.

13. A Boston merchant bought in Sheffield 50 gross of razors at a net price of $4.25 a dozen. At what price per dozen must he sell the razors to gain $33\frac{1}{3}\%$ on the net cost, if he paid a specific duty of $1.75 a dozen, and an ad valorem duty of 20%?

CHAPTER XIII.

INTEREST AND DISCOUNT.

Simple Interest.

453. Interest is money paid for the use of money.

454. Principal. The sum loaned is the *principal.*

455. Rate of Interest. The rate of interest is a specified per cent, called **rate per cent,** of the principal for **one year.**

456. Amount. The sum of the principal and interest is the *amount.*

NOTE. In the applications of percentage considered in Chapter XII no reference has been made to time. In all applications of percentage that relate to the *use of money* the elements of *time* and *rate* are considered.

457. Example. Find the interest on \$1024 at $5\frac{1}{2}\%$ for 2 yr. 8 mo.

SOLUTION. 2 yr. 8 mo. = $2\frac{2}{3}$ yr.
Interest on \$1024 for 1 yr. = $5\frac{1}{2}\%$ of \$1024 = \$56.32.
Interest on \$1024 for $2\frac{2}{3}$ yr. = $2\frac{2}{3} \times$ \$56.32 = \$150.19.

EXERCISE 118.

Find the interest on:

1. \$125.65 for 1 mo. at 6%.
2. \$1165 for 3 yr. at 5%.
3. \$1296.50 for 2 mo. at $5\frac{1}{2}\%$.
4. \$630.50 for 3 yr. at 4%.
5. \$231.50 for 3 yr. 8 mo. at $4\frac{1}{2}\%$.
6. \$580.40 for 2 yr. 4 mo. at 6%.
7. \$285.85 for 1 yr. 7 mo. at 4%.
8. \$1275.35 for 3 yr. 2 mo. at $3\frac{1}{2}\%$.

Six Per Cent Method.

458. The interest at six per cent on one dollar for one year is 6 cents; for one month is $\frac{1}{12}$ of 6 cents, or $\frac{1}{2}$ cent; for one day is $\frac{1}{30}$ of $\frac{1}{2}$ cent, or $\frac{1}{60}$ cent, that is, $\frac{1}{6}$ mill.

The interest at six per cent on any number of dollars is that number times the interest on one dollar. Hence,

459. To Find the Interest at Six Per Cent on any Principal for Time expressed in Years, Months, and Days,

Multiply six cents by the number of years, one half cent by the number of months, one sixth mill by the number of days. Take the sum of these products, and multiply this sum by the number of dollars in the principal.

460. Example. Find the interest on $320 for 3 yr. 9 mo. 20 dy., at 6%.

Int. on $1.	Int. on $320.	Condensed Work.
For 3 yr. = 3 × $0.06 = $0.18	0.228\frac{1}{3}$	3 yr. 9 mo. 20 dy.
For 9 mo. = 9 × 0.005 = 0.045	320	$0.18 0.045 0.003$\frac{1}{3}$
For 20 dy. = 20 × 0.000$\frac{1}{6}$ = 0.003$\frac{1}{3}$	106$\frac{2}{3}$	0.045
For 3 yr. 9 mo. 20 dy. = 0.228\frac{1}{3}$	456	0.003$\frac{1}{3}$
	684	0.228\frac{1}{3}$
	$73.07	320

EXERCISE 119.

Find the interest at 6% on:

1. $744.20 for 3 yr. 6 mo. 18 dy.
2. $625.44 for 6 yr. 7 mo. 12 dy.
3. $124.87 for 2 yr. 10 mo. 16 dy.
4. $847.64 from Jan. 12, 1896 to Aug. 7, 1899.
5. $84.84 from Mar. 22, 1895 to Jan. 1, 1898.
6. $1248.27 from Apr. 7, 1894 to May 17, 1897.

461. Short time notes generally run for 1, 2, 3, or 4 months; or for 30, 60, or 90 days.

462. To Find Interest at 6% for Time in Months,

Move the decimal point in the principal two places to the left, and multiply by half the number of months.

463. To Find Interest at 6% for Time in Days,

Move the decimal point in the principal three places to the left, and multiply by one sixth the number of days.

Thus, the interest at 6% on \$600 for 4 mo. is 2 × \$6.00, or \$12; and for 30 dy. is 5 × \$0.600, or \$3.

EXERCISE 120.

Find the interest at 6% on:

1. \$1278.75 for 1 mo.; 2 mo.; 3 mo.; 4 mo.
2. \$2265.50 for 1 mo.; 2 mo.; 3 mo.; 4 mo.
3. \$1840.25 for 30 dy.; 60 dy.; 90 dy.
4. \$1946.75 for 30 dy.; 60 dy.; 90 dy.

Six Per Cent Method for Other Rates.

464. Example. Find the interest on \$73.42 for 5 yr. 8 mo. 16 dy., at $7\frac{1}{2}\%$.

SOLUTION.	5 yr.	8 mo.	16 dy.		
	\$0.30	0.04	$0.002\frac{2}{3}$		\$$0.342\frac{2}{3}$
	0.04				73.42
	$0.002\frac{2}{3}$			6)	\$25.158
	\$$0.342\frac{2}{3}$				\$4.193
				Multiply by	$7\frac{1}{2}$
					\$31.45

465. To Find Interest at Rates other than 6%,

Divide the interest at six per cent by six and multiply the quotient by the given rate. Or,

Take such a part of the interest at six per cent as the given rate is of six per cent.

For 7%, we add $\frac{1}{6}$ of the interest at 6% to the interest at 6%; for $7\frac{1}{2}\%$, we add $\frac{1}{4}$; for 8%, we add $\frac{1}{3}$; for 9%, we add $\frac{1}{2}$; for 5%, we subtract $\frac{1}{6}$; for $4\frac{1}{2}\%$, we subtract $\frac{1}{4}$; for 4%, we subtract $\frac{1}{3}$.

Exercise 121.

Find the interest on:

1. \$680.40 for 2 yr. 4 mo. 6 dy., at 6%.
2. \$25.62 for 30 dy., at 6%.
3. \$85.85 for 1 yr. 7 mo. 21 dy., at 6%.
4. \$1100 for 3 yr. 4 mo., at 5%.
5. \$1275 for 3 yr. 2 mo. 15 dy., at 8%.
6. \$475.16 for 27 dy., at $4\frac{1}{2}$%.
7. \$1290.50 for 60 dy., at 6%.
8. \$125 for 1 yr. 2 mo. 2 dy., at 9%.
9. \$250.80 for 10 mo. 10 dy., at $3\frac{1}{2}$%.
10. \$258.85 from Mar. 6 to June 24, at 5%.
11. \$380 for 2 yr. 11 mo. 27 dy., at $4\frac{1}{2}$%.
12. \$475.05 for 1 yr. 9 mo. 14 dy., at $7\frac{3}{10}$%.
13. \$725.40 for 11 mo. 24 dy., at $5\frac{1}{4}$%.
14. \$680.50 for 2 yr. 6 dy., at 5%.
15. \$630.50 for 90 dy., at 6%.
16. \$547.60 from Feb. 20 to Dec. 5, at $6\frac{1}{2}$%.
17. \$875 from May 5, 1897 to June 21, 1898, at $5\frac{1}{2}$%.
18. \$758.50 from Jan. 5 to July 1, at $4\frac{1}{2}$%.
19. \$342.42 from Feb. 5, 1897 to Mar. 15, 1899, at 7%.
20. \$540 from Mar. 5 to Sept. 21, at $3\frac{1}{2}$%.

Find the amount of:

21. \$431.50 for 2 yr. 8 mo., at $4\frac{1}{2}$%.
22. \$476.50 from July 5, 1897 to Feb. 9, 1898, at 4%.
23. \$319.20 from Apr. 7 to Aug. 31, at $3\frac{1}{4}$%.
24. \$6460 from June 15, 1897 to May 7, 1899, at $4\frac{1}{4}$%.
25. \$150 from Aug. 5, 1897 to Mar. 17, 1899, at 7%.
26. \$527.20 from Jan. 1 to Nov. 20, at $4\frac{1}{2}$%.
27. \$1250 from Nov. 15, 1897 to Mar. 1, 1898, at 5%.
28. \$624.36 from Mar. 5 to Dec. 20, at $7\frac{3}{10}$%.
29. \$12,260 from May 6 to Oct. 24, at $3\frac{3}{4}$%.
30. \$11,216 from Oct. 20 to Dec. 31, at 1% a month.

Other Interest Problems.

466. In interest problems four elements are considered: *principal, rate per cent, time,* and *interest* or *amount.*

In the problems already considered the first three elements are given to find the fourth.

If p stands for the principal, r for the rate per cent, t for the time expressed in years, i for the interest, and a for the amount, we have

$$1.\ prt = i. \qquad 2.\ p + prt = a.$$

In either equation, if three of the elements are given, the other can be found.

467. Examples. 1. What principal will in 2 yr. 7 mo. 24 dy. produce \$1006.47 interest, at $4\frac{1}{2}\%$?

SOLUTION. Divide by rt both sides of the equation

$$prt = i;$$

then $$p = \frac{i}{rt}.$$

Here t, 2 yr. 7 mo. 24 dy., $= 2.65$ yr.; r, $4\frac{1}{2}\%$, $= 0.045$; $i = \$1006.47$.

Hence, $$p = \$\frac{1006.47}{0.045 \times 2.65} = \$8440.$$

2. What principal will in 3 yr. 6 mo. amount to \$748.41, at 4%?

SOLUTION. Divide by $1 + rt$ both sides of the equation

$$p + prt = a;$$

then $$p = \frac{a}{1 + rt}.$$

Here $a = \$748.41$; r, 4%, $= 0.04$; t, 3 yr. 6 mo., $= 3.5$ yr.; and $1 + rt = 1 + 0.04 \times 3.5 = 1 + 0.14 = 1.14$.

Hence, $$p = \$\frac{748.41}{1.14} = \$656.50.$$

3. At what rate per cent will \$8440 produce \$1006.47 interest in 2 yr. 7 mo. 24 dy.?

SOLUTION. Divide by pt both sides of the equation

$$prt = i;$$

then
$$r = \frac{i}{pt}.$$

Here $i = \$1006.47$; $p = \$8440$; t, 2 yr. 7 mo. 24 dy., $= 2.65$ yr.

Hence,
$$r = \frac{1006.47}{8440 \times 2.65} = 0.045.$$

Therefore, the rate required is $4\frac{1}{2}\%$.

4. In what time will the interest on \$8440 at $4\frac{1}{2}\%$ amount to \$1006.47?

SOLUTION. Divide by pr both sides of the equation

$$prt = i;$$

then
$$t = \frac{i}{pr}.$$

Here $i = \$1006.47$; $p = \$8440$; r, $4\frac{1}{2}\%$, $= 0.045$.

Hence,
$$t = \frac{1006.47}{8440 \times 0.045} = 2.65.$$

Therefore, the time required is 2.65 yr., or 2 yr. 7 mo. 24 dy.

468. From these examples we have the following rule for finding the principal, rate, or time:

Divide the given interest (or amount) by the interest (or amount) obtained when the required element is represented by a unit.

The unit of this rule is \$1 in finding the principal; 1% in finding the rate; 1 yr. in finding the time.

EXERCISE 122.

Find the rate per cent:

1. When the interest on \$326 for 15 yr. is \$220.05.
2. When the interest on \$745 for 18 yr. is \$603.45.
3. When \$980 amounts to \$1016.75 in 9 mo.

Find the rate per cent:

4. When the interest on \$470.50 is \$141.15 for 5 yr.

5. When \$3631.25 amounts to \$3715.98 for 7 mo.

6. When the interest on \$997.75 is \$199.55 for 5 yr. 4 mo.

7. When \$350 amounts \$406.70 for 3 yr. 7 mo. 6 dy.

8. When the interest on \$6875 is \$68.75 for 90 dy.

9. When the interest on \$642 is \$10.70 for 5 mo.

10. When the interest on \$8432 for 2 yr. 7 mo. 23 dy. is \$1339.28.

11. When a sum of money is doubled in 14 yr.

12. When an investment for 4 yr. 2 mo. produces a sum equal to $\frac{5}{24}$ of the capital.

13. When an investment for 3 yr. 1 mo. 15 dy. produces a sum equal to $\frac{1}{8}$ of the capital.

Find the time in which the:

14. Interest on \$450 will amount to \$72, at 4%.

15. Interest on \$487.50 will amount to \$39, at 4%.

16. Interest on \$238.75 will amount to \$64.46, at $4\frac{1}{2}$%.

17. Sum of \$1587.75 will amount to \$1611.68, at $5\frac{1}{2}$%.

18. Sum of \$1 will double itself at 4%.

19. Sum of \$10 will amount to \$17, at 6%.

20. Sum of \$502.67 will amount to \$578.07, at $4\frac{1}{2}$%.

21. Interest on \$537.50 will amount to \$80.62, at 4%.

22. Interest on \$6875 will amount to \$75.05, at $4\frac{1}{4}$%.

23. Interest on \$8520 will amount to \$1746.60, at 6%.

Find the principal that will:

24. Produce \$90 interest in 3 yr., at 4%.

25. Produce \$63 interest in 3 yr., at $6\frac{1}{4}$%.

26. Produce \$100 interest in 8 yr. 6 mo., at 5%.

27. Produce \$1746.60 interest in 3 yr. 5 mo., at 6%.

28. Produce \$12 interest in 7 mo., at 5%.

29. Produce \$50 interest in 228 dy., at $4\frac{1}{2}$%.

Find the principal that will:

30. Produce \$1339.28 interest in 2 yr. 7 mo. 24 dy., at 6%.

31. Produce \$1312.65 interest in 2 yr. 3 mo., at 6%.

32. Produce \$750 interest in 3 yr. 8 mo., at 5%.

33. Amount to \$840 in 3 yr., at 4%.

34. Amount to \$90,113.84 in 2 yr. 6 mo., at $4\frac{1}{8}$%.

35. Amount to \$6000 in 21 dy., at 5%.

36. Amount to \$297.60 in 8 mo., at 6%.

37. Amount to \$6378.75 in 1 yr. 1 mo., at 5%.

38. Amount to \$21,047.95 in 1 yr. 7 mo. 21 dy., at $4\frac{1}{2}$%.

39. Amount to \$185.09 in 2 yr. 3 mo. 18 dy., at 5%.

40. Amount to \$659.40 in 2 yr. 11 mo. 15 dy., at 6%.

41. Amount to \$94,375.16 in 2 yr. 7 mo. 24 dy., at $4\frac{1}{2}$%.

42. Amount to \$10,266.60 in 3 yr. 5 mo., at 6%.

43. Find the interest on \$195 for 2 yr. 2 dy., at $6\frac{1}{2}$%.

44. At what rate per cent will \$1025.20 produce \$25.72 interest in 4 mo. 9 dy.?

45. The principal is \$653; the interest, \$5.52; the rate, 8%. Find the time.

46. Find the amount of \$520 for 2 mo. 3 dy., at $4\frac{1}{2}$%.

47. What sum bearing interest at $4\frac{1}{2}$% will yield an annual income of \$1000?

48. In what time will \$4000 amount to \$4625, at $5\frac{1}{2}$%?

49. At what rate per cent will \$3000 produce \$250 interest in 1 yr. 10 mo. 7 dy.?

50. Find the interest on \$1721.84 from April 1 to Nov. 12, at $4\frac{1}{2}$%.

51. How long must \$3904.92 be on interest to amount to \$4568.76, at 5%?

52. Find the interest on \$137.60 from July 3 to Dec. 12, at $7\frac{3}{10}$%.

53. Find the interest on \$680.20, at $7\frac{1}{2}$%, for 73 dy., reckoning 365 dy. for a year.

Promissory Notes.

469. Promissory Notes. A written promise to pay a specified sum of money on demand or at a specified time is called a *promissory note*, or simply a *note*.

A note may be made payable *to bearer*, to a *person named in the note*, or to the person named or *his order*.

The **maker** or **drawer** is the person who signs the note.

The **payee** or **drawee** is the person to whom the note is payable.

The **holder** of a note is the person who has lawful possession of it.

The **face** of a note is the sum of money named in it.

A **negotiable note** is a note that can be sold and transferred to any one else by the holder. A note is not negotiable unless made payable to *bearer*, or to the *order of* the person named in the note.

470. **Forms of Notes.**

1. *$375.25.* *Boston, Mass., Nov. 1, 1897.*

Four months after date, I promise to pay Benjamin Parker Three Hundred Seventy-five and $\frac{25}{100}$ Dollars, with interest at 5%, for value received. *James Overton.*

2. *$375.25.* *Boston, Mass., Nov. 1, 1897.*

Four months after date, I promise to pay Benjamin Parker, or bearer, Three Hundred Seventy-five and $\frac{25}{100}$ Dollars, with interest at 5%, for value received. *James Overton.*

3. *$375.25.* *Boston, Mass., Nov. 1, 1897.*

Four months after date, I promise to pay Benjamin Parker, or order, Three Hundred Seventy-five and $\frac{25}{100}$ Dollars, with interest at 5%, for value received. *James Overton.*

4. *$375.25.* *Boston, Mass., Nov. 1, 1897.*

On demand, I promise to pay Benjamin Parker, or order, Three Hundred Seventy-five and $\frac{25}{100}$ Dollars, with interest at 5%, for value received. *James Overton.*

471. An **endorser** of a note is a person who writes his name on the back of a note.

An endorser of a note becomes responsible for its payment, unless he writes above his signature the words *without recourse.*

A note that contains the words *or bearer* can be collected by any one who has lawful possession of it.

A note that contains the words *or order* becomes payable to bearer when the payee merely writes his name across the back. This is called an *Endorsement in blank.*

In form 3, if Benjamin Parker sells the note to James Whitney he writes on the back *Pay to the order of James Whitney* and signs his name, or else he simply endorses in blank.

Notes that contain the words *on demand* are called *Demand Notes,* and are due whenever payment is demanded; see form 4. Other notes are called *Time Notes.* The day on which a time note is legally due is called the *day of its maturity.*

Notes containing the words *with interest* bear interest from date until paid. Notes not containing these words *begin to bear interest from the time they are due* at the legal rate, if not paid when due.

Legal rate of interest is the rate established by law in the state where the note is made.

Notes should contain the words *value received,* otherwise it may be necessary to prove the note was given for a valuable consideration. The *face* of a note, except the cents, should be expressed in words.

Exercise 123.

Find the day of maturity, and amount due, having given:

	Face of Note.	Date of Note.	Time.	Rate of Int.
1.	\$530.25,	Jan. 12, 1897,	60 dy.,	6%.
2.	\$687.45,	Mar. 22, 1897,	90 dy.,	5%.
3.	\$286.75,	Aug. 5, 1897,	4 mo.,	4%.
4.	\$944.40,	Oct. 20, 1897,	3 mo.,	4½%.
5.	\$1262.72,	Oct. 5, 1897,	30 dy.,	5½%.
6.	\$1875.44,	Dec. 16, 1897,	6 mo.,	4%.
7.	\$1521.87,	Apr. 30, 1897,	1 mo.,	6%.
8.	\$2849.65,	May 22, 1897,	2 yr.,	3½%.
9.	\$1968.10,	July 10, 1897,	2 mo.,	4½%.

Find the amount due Dec. 3, 1898, on the following demand notes:

10. $875.18. CONCORD, N. H., May 10, 1897.

On demand, I promise to pay George H. Chick, or order, Eight Hundred Seventy-five and $\frac{18}{100}$ Dollars, with interest at 5%. Value received. FREDERICK D. SIBLEY.

11. $642.75. LAKEWOOD, N. J., Oct. 25, 1897.

On demand, I promise to pay Harry Jones, or order, Six Hundred Forty-two and $\frac{75}{100}$ Dollars, with interest at $4\frac{1}{2}$%. Value received. GEORGE B. ATKINS.

12. $1286.50. ATLANTA, GA., Apr. 22, 1897.

On demand, I promise to pay Clarence E. Garland, or order, Twelve Hundred Eighty-six and $\frac{50}{100}$ Dollars, with interest at $5\frac{1}{2}$%. Value received. ROBERT PAGE.

13. $2548.25. ST. PAUL, MINN., June 17, 1897.

On demand, I promise to pay Fred Lacey, or order, Twenty-five Hundred Forty-eight and $\frac{25}{100}$ Dollars, with interest at 7%. Value received. WILLIAM P. WISSMAN.

14. $418.33. OAKLAND, CAL., Dec. 23, 1897.

On demand, I promise to pay Albert J. Farnham, or order, Four Hundred Eighteen and $\frac{33}{100}$ Dollars, with interest at $4\frac{1}{2}$%. Value received. AUSTIN C. WIGGIN.

15. $7486.45. WATERTOWN, IA., Apr. 16, 1898.

On demand, I promise to pay Harry D. Smith, or order, Seven Thousand, Four Hundred Eighty-six and $\frac{45}{100}$ Dollars, with interest at 5%. FRANK J. LEAVITT.

Bank Discount.

472. **Bank discount** is a deduction from the amount of a note or draft for cashing it before it is due.

473. The **term of discount** is the time from the day of discounting the note to and including the day of maturity of the note.

474. **Days of Grace.** *Three days* are allowed in addition to the time stated in the note before the note is legally due, unless the note contains the words *without grace*, or is made in states that have abolished days of grace by statute.

475. A **protest** is a notice in writing by a notary public to the endorsers that a note has not been paid. If a note is not protested at the end of the day of its maturity the endorsers are released from their obligation.

476. Bank discount is *the interest* for the term of discount at a specified rate on the *amount* of the note.

477. If a note draws interest, the amount of the note is the *face and interest.*

478. **Exchange.** A charge is sometimes made for collecting, if the place of payment of the note or draft is not the place of discount. This charge is called *exchange.*

The **rate of exchange** is a number of cents on $1000 for large amounts; and for small amounts from $\frac{1}{8}$ to $\frac{1}{2}$ of 1% of the *face value* of the note or draft, a fraction of $100 being reckoned as $100.

NOTE. The rate of exchange is never higher than the cost of sending the *money* by express.

479. A **bank draft** is a written order from a bank requesting another bank to pay a specified sum of money to the order of the person named in the draft. Thus,

No. 27861

First National Bank of Buffalo.

Buffalo, N.Y. Dec. 15, 1897.

Pay to the order of John R. Richards $172.15.

One hundred seventy-two and —— $\frac{15}{100}$ *Dollars.*

TO THE PARK NATIONAL BANK, NEW YORK CITY.

S. C. Whitcomb, *Cashier.*

480. A **check** is a draft upon a bank where the maker (or drawer) has money deposited. Thus,

No. 89

Boston, Mass., Dec. 15, 1897.

First National Bank of Boston.

Pay to the order of John Doe —— $86.45.

Eighty-six and —— $\frac{45}{100}$ *Dollars.*

A. S. Smythe.

481. Certificate of Deposit. A receipt for money deposited in a bank is called a *certificate of deposit.* Thus,

No. 28642 CERTIFICATE OF DEPOSIT.

Second National Bank of Boston.

Boston, Mass., Dec. 15, 1897.

James C. Leavitt *has deposited in this Bank*

Five hundred —— *Dollars*

payable to his *order in current funds on the return of this certificate properly endorsed.*

$500. Richard Roe, *Cashier.*

482. A **certified check** is a check upon the face of which the cashier of the bank has stamped the word *Certified*, with the date, the name of the bank, and has written his signature as cashier. The bank is then responsible for its payment.

483. A **commercial draft** is in effect a letter from one person to another requesting him to pay a stated sum of money to the bank named in the draft. The name of the person on whom the draft is drawn is written at the lower left-hand corner of the draft. These drafts are sent through banks instead of through the mail; and are payable at sight, or at a specified time after sight.

Commercial drafts are employed by creditors to demand payments and collect debts through banks.

No. 93 *Springfield, Mass.,* Dec. 15, 1897.

At sight pay to the order of

The Second National Bank of Springfield $100.^{00}$

One hundred ———————— *Dollars.*

To W. H. Foursome,
196 Green St., New York. } Geo. H. Hansen.

484. When the person on whom a time draft is drawn accepts the draft, he writes in red ink the word *Accepted* with the date across the face of the draft and signs his name. The draft is then called an *acceptance*, and the acceptor is responsible for its payment.

NOTE. An acceptance is like a promissory note, the acceptor and the maker of the draft taking the place respectively of the maker and the endorser of the note.

485. The **proceeds** or **avails** of a note or draft is the amount of the note or draft less the discount and exchange.

486. Examples. Find the day of maturity, the time to run, the discount, and the proceeds of the following notes:

1. \$680. MANCHESTER, N. H., Oct. 5, 1897.

Sixty days after date, without grace, I promise to pay F. R. Thompson, or order, Six Hundred Eighty Dollars, value received.

Payable at the Amoskeag National Bank.

Discounted at 6%, Oct. 14. B. F. SHAW.

SOLUTION. Counting 60 days from Oct. 5, we have 26 days in Oct., 30 days in Nov., and 4 days in Dec. Therefore, the note is due Dec. 4.

The time to run (from Oct. 14, the day of discount) is 17 days in Oct., 30 days in Nov., and 4 days in Dec., or 51 days.

The discount is the interest on \$680 for 51 days at 6%; or $8\frac{1}{2} \times$ \$0.680 = \$5.78 (§ 463).

The proceeds is \$680 − \$5.78 = \$674.22.

2. \$840. MANCHESTER, N. H., Oct. 5, 1897.

Three months from date, I promise to pay E. A. Jackson, or order, Eight Hundred Forty Dollars, value received, with interest at six per cent.

Payable at the Amoskeag National Bank.

Discounted at 6%, Nov. 2, 1897. G. A. BATCHELDER.

SOLUTION. The interest on the note for 3 mo. 3 dy. is $840 \times$ \$0.0155 = \$13.02; and the amount of the note is \$840 + \$13.02 = \$853.02.

The day of maturity is Jan. 8, 1898.

The time to run is 28 days in Nov., 31 days in Dec., and 8 days in Jan., or 67 days.

The discount on \$853.02 for 67 days at 6% is $11\frac{1}{6} \times$ \$0.85302, or \$9.53.

The proceeds is \$853.02 − \$9.53 = \$843.49.

NOTE. In Boston, and in many other places, when the time a note has to run is expressed in *months*, the term of discount is computed for this number of months, and not for the exact number of days contained in the months.

Exercise 124.

Find the day of maturity, the time to run, the discount, and the proceeds of the following notes, without grace:

1. \$750. New York, Jan. 1, 1897.

Four months from date, I promise to pay to the order of James Fay Seven Hundred Fifty Dollars, value received.

Payable at the National Bank of the Republic.

Discounted at 5%, Jan. 12. John Pray.

2. \$4325.50. Boston, Mar. 4, 1897.

Sixty days from date, I promise to pay to James Finn, *or* order, Four Thousand Three Hundred Twenty-five and $\frac{50}{100}$ Dollars, value received.

Payable at the Merchants National Bank.

Discounted at $5\frac{1}{2}$%, Mar. 8. George Bellows.

3. \$1300. Richmond, Va., July 14, 1897.

Ninety days from date, I promise to pay to the order of Peter Bright Thirteen Hundred Dollars, value received.

Payable at the First National Bank.

Discounted at 4%, Aug. 3. George Wright.

4. \$1456.30. Charleston, S. C., Aug. 27, 1897.

Three months after date, I promise to pay to the order of John George Fourteen Hundred Fifty-six and $\frac{30}{100}$ Dollars, value received.

Payable at the Second National Bank.

Discounted at 5%, Sept. 10. John Waldorf.

5. \$4550.36. Baltimore, Md., Nov. 10, 1897.

Four months after date, I promise to pay to the order of John Callender Four Thousand Five Hundred Fifty and $\frac{36}{100}$ Dollars, value received.

Payable at the National Mechanics Bank.

Discounted at $5\frac{1}{2}$%, Nov. 24. James Barton.

6. $5000. CHICAGO, ILL., Dec. 23, 1897.

Six months after date, we jointly and severally promise to pay to John Adams, or order, Five Thousand Dollars, value received, with interest at five per cent.

Payable at the Metropolitan National Bank.

Discounted at 4%, Jan. 21, 1898. WILLIAM DUNN,
F. R. CROCKETT.

Find the day of maturity, the time to run, the discount, and the proceeds of the following notes, with grace :

7. $4760. MILWAUKEE, WIS., Jan. 1, 1897.

Ninety days after date, I promise to pay to the order of James Pike Four Thousand Seven Hundred Sixty Dollars, value received.

Payable at the Wisconsin National Bank.

Discounted at $4\frac{1}{2}$%, Feb. 15. WILLIAM CLEMENT.

8. $2017.85. ST. PAUL, MINN., Jan. 14, 1897.

Three months after date, I promise to pay to the order of John Brown Two Thousand Seventeen and $\frac{85}{100}$ Dollars, value received.

Payable at the German-American National Bank.

Discounted at 7%, Mar. 1. TIMOTHY BRUCE.

9. $9040. GALVESTON, TEX., Jan. 19, 1897.

Sixty days from date, I promise to pay to the order of Charles Carroll Nine Thousand Forty Dollars, value received.

Payable at the First National Bank.

Discounted at $5\frac{1}{2}$%, Feb. 16. JAMES MONROE.

10. $215. AUGUSTA, ME., Jan. 28, 1897.

Thirty days after date, I promise to pay to the order of James Fogg Two Hundred Fifteen Dollars, value received.

Payable at the Maine National Bank.

Discounted at 6%, Feb. 3. JOHN MOSES.

11. \$2216.85. OMAHA, NEB., Dec. 15, 1897.

Ninety days after date, I promise to pay to the order of F. C. Green Two Thousand Two Hundred Sixteen and $\frac{85}{100}$ Dollars, value received.

Payable at the Omaha National Bank.

Discounted at 7%, Jan. 8, 1898.

W. C. COLBURN.

Find the proceeds of the following drafts, with grace :

12. Draft for \$620 at 60 days ; rate of discount 6% ; exchange $\frac{1}{8}$%.

SOLUTION. The discount for 63 days is $10\frac{1}{2} \times \$0.620$, or \$6.51 ; and the exchange is $\frac{1}{8}$% of \$700, or \$0.88 (every fraction of \$100 being reckoned as \$100). The total discount is, therefore, $\$6.51 + \$0.88 = \$7.39$, and the proceeds is $\$620 - \$7.39 = \$612.61$.

13. Draft for \$890 at 90 days ; rate of discount $4\frac{1}{2}$% ; exchange $\frac{1}{4}$%.

14. Draft for \$12,500 at 60 days ; rate of discount 5% ; exchange 15 cents on \$1000.

15. Draft for \$1260 at 30 days ; rate of discount $5\frac{1}{2}$% ; exchange $\frac{1}{8}$%.

16. Draft for \$1430 at 3 months ; rate of discount 6% ; exchange $\frac{1}{4}$%.

17. Draft for \$1875 at 4 months ; rate of discount 5% ; exchange $\frac{1}{8}$%.

18. Draft for \$22,843 at 60 days ; rate of discount $4\frac{1}{2}$% ; exchange 25 cents on \$1000.

19. Draft for \$18,000 at 2 months ; rate of discount 5% ; exchange $\frac{1}{8}$%.

20. Draft for \$3437.50 at 90 days ; rate of discount 5% ; exchange $\frac{1}{4}$%.

21. Draft for \$1287.50 at 60 days ; rate of discount $4\frac{1}{2}$% ; exchange $\frac{3}{8}$%.

22. Draft for \$866.65 at 3 months ; rate of discount 5% ; exchange $\frac{1}{8}$%.

Present Worth and True Discount.

487. The **present worth** of a debt is the sum which, if put at interest, will amount to the debt when due.

488. The **true discount** is the difference between a sum due at some future time and the present worth of that sum.

489. Example. Find the present worth and true discount of $515 due in 7 mo. 6 dy., if money is worth 5%.

SOLUTION. The amount of $1 at 5% for 7 mo. 6 dy. is $1.03. As $1.03 is the amount of $1, $515 is the amount of $\$\frac{515}{1.03}$, or $500. The true discount is $515 − $500 = $15. Hence,

490. To Find the Present Worth of a Given Sum of Money Due at a Stated Future Time,

Divide the given sum by the amount of $1 for the given rate and time.

EXERCISE 125.

1. Find the present worth of $500 due in 11 mo., if money is worth 5%.
2. Find the present worth and discount of $3334.62 due in 2 yr., if money is worth 4½%.
3. Find the present worth and discount of $4261.33 due in 1 yr. 6 mo., if money is worth 6%.
4. Find the present worth and discount of $2416.50 due in 7 mo., if money is worth 5%.
5. Find the present worth of $678.40 due in 16 mo., if money is worth 4½%.
6. Find the present worth and discount of $574.17 due in 2 yr. 3 mo., if money is worth 5⅛%.
7. Find the present worth and discount of $625.13 due in 8 mo., if money is worth 4%.
8. Find the present worth and discount of $715.20 due in 1 yr. 4 mo., if money is worth 3½%.

Exact Interest.

491. By the ordinary method interest is computed on a basis of 30 days for a month; that is, 360 days for a year.

A year has 365 days. The interest for a year counted by days would thus be $\frac{365}{360}$, or $\frac{73}{72}$, of the true interest.

The exact interest for any number of days is, therefore, found by diminishing interest found by the ordinary method by $\frac{1}{73}$ of itself.

492. To Find One Seventy-third of a Number,

Move the decimal point two places to the left, add a third three times, each time one place further to the right. Carry only to three decimal places.

493. Example. Find the exact interest on $12,762.44 from May 6 to Sept. 15, at 6%.

SOLUTION.	25	$12.76244	$280.77
	30	22	$2.8077
	31	2552488	0.9359
	31	2552488	0.0935
	15	$280.77368	0.0093
	6 | 132		$3.8464
	22		$276.92

494. Exact interest is used by trust companies, and in all calculations of national and state governments.

NOTE. This rule applies only to periods less than one year. For periods greater than one year, we find the interest for the years, then for the months and days by the above rule, and add the results.

EXERCISE 126.

Find the exact interest at 6%:

1. On $692.74 for 250 days.
2. On $1472.38 from Jan. 7, 1897 to Oct. 4, 1897.
3. On $1247.75 from Mar. 4, 1897 to Dec. 22, 1897.
4. On $1898.48 from Feb. 26, 1897 to Aug. 12, 1899.

Annual Interest.

495. Annual interest is simple interest on the principal and on each year's interest from the time each interest is due until settlement.

496. Example. Find the interest on $400 for 4 yr. 7 mo. 20 dy., at 5%, payable annually.

SOLUTION. Simple interest on $400 at 5% for 4 yr. 7 mo. 20 dy. is $92.78.

Interest due the 1st year, $20, draws interest	3 yr. 7 mo. 20 dy.
Interest due the 2d year, $20, draws interest	2 yr. 7 mo. 20 dy.
Interest due the 3d year, $20, draws interest	1 yr. 7 mo. 20 dy.
Interest due the 4th year, $20, draws interest	7 mo. 20 dy.
Interest upon the interest = interest on $20 for	8 yr. 6 mo. 20 dy.

Interest on $20 for 8 yr. 6 mo. 20 dy. at 5% = $8.56.
The annual interest = $92.78 + $8.56 = $101.34.

EXERCISE 127.

1. Find the amount at annual interest of $1247.75 for 3 yr. 5 mo. 10 dy. at 6%.

2. Find the interest due on $987.25 in 4 yr. 9 mo. 6 dy., interest at 4%, payable annually.

3. Find the interest due on $742.60 in 5 yr. 11 mo. 27 dy., interest at 4½%, payable annually.

4. Find the interest due May 19, 1898, on a note dated Dec. 26, 1894, for $1224.60, with interest payable annually, at 5%, if no interest has been paid.

5. Find the amount due May 27, 1898, on a note dated Jan. 4, 1896, for $215.50, with interest payable annually at 5½%, if no interest has been paid.

6. Find the amount due Jan. 16, 1897, on a note dated Jan. 8, 1895, for $3115.20, with interest payable annually at 5%, if no interest has been paid.

Compound Interest.

497. Interest is *compounded* when it is added to, and becomes a part of, the principal at specified intervals.

Interest is compounded annually, semi-annually, or quarterly, according to agreement. Interest is understood to be compounded annually unless otherwise stated.

498. Example. Find the amount of $500 at compound interest for 3 years at 4%. Find the compound interest.

Interest for 1st year is 4% of $500 = $20. Amount is $520.
Interest for 2d year is 4% of $520 = $20.80. Amount is $540.80.
Interest for 3d year is 4% of $540.80 = $21.63. Amount is $562.43.

The compound interest is the compound amount less the principal; that is, $562.43 − $500 = $62.43.

NOTE. If the time is not an integral number of years, the compound amount is found for the number of entire years, and the amount of this sum at simple interest for the portion of a year.

EXERCISE 128.

1. Find the amount of $356.25 for 4 yr., at 5% compound interest.
2. Find the amount of $637.50 for 2 yr. 6 mo., at 4% compound interest.
3. Find the compound interest on $800 for 3 yr. 9 mo., at 6%.
4. Find the compound interest on $39.35 for 4 yr. 9 mo., at 5%.
5. Find the compound interest on $300 for 2 yr., at 4%, interest being compounded semi-annually.

   HINT. The interest is 2% every six months.
6. Find the compound interest on $525 for 1 yr. 6 mo., at 5%, interest being compounded quarterly.
7. Find the compound interest on $10,000 for 6 mo., at 6%, interest being compounded monthly.

Partial Payments.

499. Partial payments, as the term implies, are payments of a part of a note.

There are several methods of computing interest upon such notes, all recognizing the general principle that interest should cease upon payments made.

500. Merchants' Rule. If a note that bears interest runs *one year or less,* and partial payments have been made, the interest is computed by the following rule:

1. *Find the amount of the note at date of settlement without regarding payments.*
2. *Find the amount of each payment with interest from date of payment to date of settlement.*
3. *Subtract the sum of the payment amounts from the total amount.*

501. Example. A man holds a note of $460, dated Jan. 20, 1897, on which the following payments are endorsed: $140, Mar. 26, 1897; $100, June 16, 1897; $160, Oct. 14, 1897.

Settlement is made Dec. 22, 1897. Find the balance due, reckoning interest at 6%.

SOLUTION.

Time from Jan. 20 to Dec. 22 is	11 mo. 2 dy.	Int. on $1 = $0.055⅓.
Time from Mar. 26 to Dec. 22 is	8 mo. 26 dy.	Int. on $1 = 0.044⅓.
Time from June 16 to Dec. 22 is	6 mo. 6 dy.	Int. on $1 = 0.031.
Time from Oct. 14 to Dec. 22 is	2 mo. 8 dy.	Int. on $1 = 0.011⅓.

Int. on $460 = 460 × $0.055⅓ = $25.45.	Amount is		$485.45
Int. on 140 = 140 × 0.044⅓ = 6.21.	Amount is	$146.21	
Int. on 100 = 100 × 0.031 = 3.10.	Amount is	103.10	
Int. on 160 = 160 × 0.011⅓ = 1.81.	Amount is	161.81	
	Total payment amounts		$411.12
	Balance due Dec. 22, 1897		$74.33

Exercise 129.

1. A note of $618.75, dated Apr. 17, 1897, payable on demand, bears the following endorsements: June 5, $126.50; Aug. 20, $137.25; Nov. 17, $210. What is due Jan. 1, 1898, reckoning interest at 6% ?

2. A note of $1000, dated Apr. 1, 1897, payable on demand, with interest at 5%, bears the following endorsements: May 6, $200; July 5, $225.37; Oct. 18, $322. What is due Jan. 1, 1898 ?

3. A note of $835.25, dated July 1, 1897, payable on demand, with interest at 4½%, bears the following endorsements: Aug. 20, $157.50; Sept. 21, $180.25; Oct. 5, $200; Dec. 1, $80. What is due Jan. 1, 1898 ?

4. A note of $1247.50, dated Mar. 10, 1897, payable on demand, with interest at 5%, has the following endorsements: $350.40, Apr. 14, 1897; $212.85, June 16, 1897; $316.45, Aug. 25, 1897. What is due Oct. 18, 1897 ?

5. A note of $1648.25, dated Jan. 22, 1897, payable on demand, with interest at 5%, has the following endorsements: $212.60, Mar. 1, 1897; $168.40, May 26, 1897; $244.40, Aug. 4, 1897; $744.80, Oct. 1, 1897. What is due Jan. 22, 1898 ?

502. United States Rule. If a note that bears interest runs *more than one year*, and partial payments have been made, the interest is computed by the following rule:

1. *Find the amount of the principal to the time when the payment, or sum of the payments, is equal to or greater than the interest.*

2. *From this amount deduct the payment, or sum of the payments.*

3. *Consider the remainder as a new principal, and proceed as before.*

503. Example. A note of $1520, dated May 20, 1896, bearing interest at 6%, had payments endorsed upon it as follows: Oct. 2, 1896, $300; Feb. 26, 1897, $25; Apr. 2, 1897, $570; Aug. 9, 1897, $600. Find the amount due Dec. 6, 1897.

yr.	mo.	dy.				
1896	10	2			$1520	1st principal.
1896	5	20			0.022	
	4	12	0.022		$33.44	1st interest.
					1520.00	
					$1553.44	
					300.00	1st payment.
1897	2	26			$1253.44	2d principal.
1896	10	2			0.024	
	4	24	0.024		$30.08	2d interest.

Since the interest is greater than the payment, we find the interest on the same principal from Feb. 26, 1897, to Apr. 2, 1897.

					$1253.44	2d principal.
					0.006	
					$7.52	3d interest.
1897	4	2			30.08	2d interest.
1897	2	26			1253.44	
	1	6	0.006		$1291.04	
				$25 + $570 =	595.00	2d & 3d payments.
					$696.04	3d principal.
					$0.021\frac{1}{6}$	
1897	8	9			$14.73	4th interest.
1897	4	2			696.04	
	4	7	$0.021\frac{1}{6}$		$710.77	
					600.00	
					$110.77	4th principal.
					$0.019\frac{1}{2}$	
1897	12	6			$2.16	5th interest.
1897	8	9			110.77	
	3	27	$0.019\frac{1}{2}$		$112.93	due Dec. 6, 1897.

Exercise 130.

1. A note of $2000, dated Jan. 22, 1896, and drawing interest at 6%, had the following endorsements: May 20, 1896, $100; July 20, 1896, $325; Nov. 2, 1896, $20; Dec. 23, 1896, $125. Find the balance due Mar. 1, 1897.

2. A note of $1662.50, dated Jan. 15, 1896, and drawing interest at 5½%, had the following endorsements: Apr. 30, 1896, $25; June 24, 1896, $25; Sept. 2, 1896, $625; Jan. 30, 1897, $700. Find the balance due May 12, 1897.

3. A note of $4560, dated Jan. 22, 1896, and drawing interest at 5%, had the following endorsements: Jan. 11, 1897, $2000; Aug. 31, 1897, $500; Jan. 15, 1898, $1200; Mar. 4, 1898, $860. Find the balance due June 15, 1898.

4. A note of $785.50, dated Jan. 30, 1896, and drawing interest at 5%, had the following endorsements: July 17, 1896, $100; Jan. 29, 1897, $100; Dec. 31, 1897, $20; Mar. 16, 1898, $300; June 18, 1898, $50. Find the balance due July 23, 1898.

5. A note of $300.25, dated Aug. 4, 1896, and drawing interest at 4½%, had the following endorsements: Oct. 14, 1896, $100; July 21, 1897, $100; Oct. 11, 1897, $50; Jan. 19, 1898, $50. Find the amount due July 22, 1898.

6. A note of $1475.40, dated Feb. 12, 1896, and bearing interest at 5%, had the following endorsements: July 22, 1896, $370; Dec. 26, 1896, $426.50; Aug. 24, 1897, $112.40; Oct. 6, 1897, $163.25; Apr. 14, 1898, $185.85. Find the balance due June 16, 1899.

7. A note of $5762.45 dated Jan. 2, 1896, and drawing interest at 5%, had the following endorsements: May 17, 1896, $500; Oct. 12, 1896, $750; Feb. 4, 1897, $1000; Aug. 25, 1897, $1250; Mar. 1, 1898, $1500; June 15, 1898, $1050. Find the balance due Oct. 2, 1898.

Special Rules for Partial Payments.

THE NEW HAMPSHIRE RULE.

504. When a note is written *with interest annually* or *with annual interest* the following is the **New Hampshire Rule:**

If in any year, reckoning from the time the annual interest began to accrue, payments have been made, compute interest upon them to the end of the year in which they are made.

The amount of the payments is to be then applied, first, to cancel interest upon annual interest; secondly, to cancel annual interest; and thirdly, to the extinguishment of the principal.

If, however, at the date of any payment there is no interest except the accruing annual interest, and the payment, or payments, do not exceed the annual interest at the end of the year, deduct the payment, or payments, without interest on the same.

THE VERMONT RULE.

505. If we omit the last paragraph of the New Hampshire Rule, we have the **Vermont Rule.**

THE CONNECTICUT RULE.

506. The **Connecticut Rule** for partial payments is:

Follow the United States Rule if a year's interest or more is due at the time of a payment, and in case of the last payment.

If less than a year's interest is due at the time of a payment except the last, find for a new principal the difference between the amount of the principal for an entire year and the amount of the payment for the remainder of that year.

If the interest due at the time of a payment exceeds the payment, find the interest on the principal only.

507. Examples. 1. Find, by the New Hampshire Rule, the amount due Sept. 22, 1896, on a note for $500, dated July 16, 1893, with interest annually at 6%, on which the following payments have been made: Oct. 16, 1894, $200; Dec. 16, 1895, $20.

Principal,		$500.00		
1st annual interest,			$30.00	
Int. on 1st annual int. for 1 yr.,				$1.80
2d annual interest,			30.00	
		$500.00	$60.00	$1.80
Payment Oct. 16, 1894,	$200			
Int. on payment July 16, 1895,	9			
Amt. of payment July 16, 1895,	$209 =	147.20	+ 60.00	+ 1.80
Principal July 16, 1895,		$352.80		
3d annual interest,			$21.17	
Payment Dec. 16, 1895,			20.00	
		$352.80	+ $1.17	
Principal July 16, 1896,		$353.97		
4th annual interest,		3.89		
Amt. due Sept. 22, 1896,		$357.86		

2. Find, by the Vermont Rule, the amount due Sept. 22, 1896, of the note in Example 1.

From Example 1, the principal July 16, 1895, is $352.80.

Principal July 16, 1895,		$352.80	
3d annual interest,			$21.17
Payment Dec. 16, 1895,	$20.00		
Int. on payment July 16, 1896,	0.70		
Amt. of payment July 16, 1896,	$20.70 =		20.70
		$352.80 +	$0.47
Principal July 16, 1896,		$353.27	
4th annual interest,		3.89	
Amt. due Sept. 22, 1896,		$357.16	

3. Find, by the Connecticut Rule, the amount due Sept. 22, 1896, on a note for $500 given Apr. 10, 1892, bearing interest at 6%, which has the following endorsements:

June 16, 1893, $100; Dec. 28, 1893, $100; May 10, 1895, $15; Mar. 4, 1896, $200.

Principal,		$500.00
Int. on principal to June 16, 1893,		35.50
Amt. of principal June 16, 1893,		$535.50
Payment June 16, 1893,		100.00
New principal June 16, 1893,		$435.50
Int. on principal to June 16, 1894,		26.13
Amt. of principal June 16, 1894,		$461.63
Payment Dec. 28, 1893,	$100.00	
Int. on payment to June 16, 1894,	2.80	
Amt. of payment June 16, 1894,		$102.80
New principal June 16, 1894,		$358.83
Int. on principal to June 16, 1895,		21.53
Amt. of principal June 16, 1895,		$380.36
Payment May 10, 1895 (less than interest),		15.00
New principal June 16, 1895,		$365.36
Int. on principal to Mar. 4, 1896,		15.71
Amt. of principal Mar. 4, 1896,		$381.07
Payment Mar. 4, 1896,		200.00
New principal Mar. 4, 1896,		$181.07
Int. on principal to Sept. 22, 1896,		5.98
Amt. due Sept. 22, 1896,		$187.05

Exercise 131.

1. Find, by the New Hampshire Rule and also by the Vermont Rule, the amount due Sept. 22, 1896, on a note for $1750, dated June 6, 1892, with interest annually at 6%, which has the following endorsements: Aug. 12, 1893, $300; Dec. 23, 1893, $200; Jan. 15, 1895, $50; Apr. 23, 1896, $800.

2. Find, by the Connecticut Rule, the amount due Sept. 22, 1896, on a note for $1500, dated Aug. 9, 1892, with interest annually at 6%, which has the following endorsements: Mar. 17, 1893, $250; Apr. 19, 1894, $50; Sept. 21, 1895, $500; June 26, 1896, $600.

CHAPTER XIV.

STOCKS AND BONDS, EXCHANGE, ACCOUNTS.

Stocks and Bonds.

508. Stock Companies. A stock company is an association of persons under the laws of the state for the purpose of carrying on a specified business.

509. Stock. The stock represents the capital invested in the business, and is issued in the form of *certificates*, each certifying that the person named in the certificate owns a stated number of shares of stock.

NOTE. One of the special advantages of a number of persons doing business in the form of a company is that the liability of each stockholder is usually limited to the amount of his stock.

510. Bonds are written obligations under seal given by a company; or by a municipal or state government; or by the national government; in which an agreement is made to pay a specified amount on or before a specified date, with interest payable annually, semi-annually, or quarterly.

511. Mortgage bonds are bonds secured by mortgage, but **debenture bonds** are simply notes under seal.

512. Registered bonds are bonds recorded by their numbers in a book called a register. The register contains the names of the owners of the bonds.

NOTE. Registered bonds are transferred from one person to another by giving proper notice to the registrar of the company.

513. **Coupon bonds** are bonds that have *coupons* or certificates of interest attached. These coupons are cut off as they become due, and are given up on receipt of the interest represented by them.

514. Bonds are **named** by giving the name of the corporation issuing them, and the rate of interest they bear. The date on which they become due is usually given, and the kind of bond, registered or coupon.

Thus, U. S. coupon 4's, 1907 means United States coupon bonds bearing 4 per cent interest, the principal due in 1907. U. S. registered 4's, 1925 means United States registered bonds bearing 4% interest, the principal due in 1925.

515. Persons who buy and sell stocks and bonds are called **stock brokers**, and their commission is called **brokerage.** On lots of 100 shares or more, brokerage is $\frac{1}{8}$% per share, reckoned at $100; and on lots of less than 100 shares brokerage is $\frac{1}{4}$% per share. Brokerage on bonds is reckoned at the same rate per $100 as on stocks.

516. Stocks and bonds are said to be **at par** when they sell for their *face value;*

At a premium, or **above par,** when they sell for *more than their face value;*

At a discount, or **below par,** when they sell for *less than their face value.*

The price for which stocks and bonds may be sold is called their *market value.*

517. A company uses its receipts first to pay its expenses; second, to pay the interest on its bonds. If there is a surplus, it is usually divided among the stockholders, according to the number of shares held by each. The amount allotted to each share is called the **dividend** per share.

Exercise 132.

1. What is the cost of 25 shares of Boston and Maine R.R. stock at 167, brokerage $\frac{1}{4}$?

Solution. Each share of stock costs \$167, with \$0.25 for brokerage, or \$167.25 in all. Hence, 25 shares cost 25 × \$167.25, or \$4181.25.

2. How many shares of Illinois Central R.R. stock at $101\frac{7}{8}$ can be bought for \$20,400, brokerage $\frac{1}{8}$?

Solution. Since the total cost of each share is $\$101\frac{7}{8} + \$\frac{1}{8}$, or \$102, \$20,400 will buy $\frac{20400}{102}$ shares, or 200 shares.

3. What is the annual income from 150 shares of Lake Shore and Michigan Ry. stock that pays an annual dividend of 6%?

Solution. 1 share at 6% yields \$6. Hence, 150 shares at 6% yield 150 × \$6 = \$900.

4. How much must be invested in 6% stock at 107 to yield an annual income of \$240, brokerage $\frac{1}{4}$?

Solution. \$6 is the dividend from 1 share.

\$240 dividend requires $\frac{240}{6}$, or 40 shares.

Price of 40 shares is 40 × \$107	= \$4280
Brokerage on 40 shares is 40 × \$0.25	= 10
Total cost	= \$4290

5. What per cent does the investment yield, if Lake Shore and Michigan Southern Ry. stock is bought at 170? The stock pays 6% dividend; no brokerage reckoned.

Solution. Each \$170 invested yields \$6 dividend. Hence, the income on the investment is $\frac{6}{170}$ of 100%, or $3\frac{9}{17}$%.

6. Find the cost of 350 shares of Chicago, Milwaukee and St. Paul Ry. stock at $93\frac{5}{8}$, brokerage $\frac{1}{8}$.

7. Find the cost of 165 shares of Michigan Central R.R. stock at $105\frac{3}{4}$, brokerage $\frac{1}{8}$.

8. Find the cost of 35 shares of Reading R.R. stock at $23\frac{3}{8}$, brokerage $\frac{1}{4}$.

9. What is the cost of 25 U. S. 4% registered 1925 bonds of \$1000 each, at 127⅛, brokerage ⅛?

10. What is the cost of 40 Northern Pacific R.R. 1st mortgage 6% registered bonds of \$1000 each, at 119⅞, brokerage ⅛?

11. What per cent income does the investment of Example 10 yield?

12. What is the annual income received from the investment of Example 10?

13. What is the annual income from 200 shares of Chicago and Northwestern Ry. stock that pays an annual dividend of 5%?

14. What is the cost of the investment of Example 13 at 122⅞, brokerage ⅛?

15. What per cent income does the investment of Example 13 yield?

16. How many shares of New York Central stock can be bought for \$4757.50 at 107⅞, brokerage ¼?

17. How many Chicago, Burlington and Quincy 7% bonds, \$500 each, will \$6885 buy at 114½, brokerage ¼?

18. What is the annual income from the investment of Example 17?

19. What sum of money must be invested in Northern Pacific R.R. 1st mortgage 6's at 119½ to produce an annual income of \$2400, brokerage ⅛?

20. What sum of money must be invested in Wabash R.R. 1st gold 5% bonds at 107¼ to produce an annual income of \$2000, brokerage ⅛?

21. What sum of money must be invested in Louisville and Nashville R.R. unified gold 4% bonds at 84¼ to produce an annual income of \$320, brokerage ¼?

22. What sum of money must be invested in St. Louis and San Francisco Ry. general mortgage 5% bonds at 100½ to produce an annual income of \$600, brokerage ⅛?

23. How many shares of Chicago and Northwestern Ry. stock can be bought for \$14,670 at $122\frac{1}{8}$, brokerage $\frac{1}{8}$? What is the brokerage? If 5% dividends are paid, what per cent on his investment does the purchaser receive?

24. How many shares of Michigan Central R.R. stock can be bought for \$16,940 at $105\frac{3}{4}$, brokerage $\frac{1}{8}$? What is the brokerage? If 4% dividends are paid, what per cent on his investment does the purchaser receive?

25. What is the cost of 40 shares of Central R.R. of New Jersey stock at $92\frac{7}{8}$, brokerage $\frac{1}{4}$? What is the brokerage? If 6% dividends are paid, what per cent on his investment does the purchaser receive?

26. What is the cost of 250 shares of Pullman Palace Car Co. stock at $171\frac{1}{4}$, brokerage $\frac{1}{8}$? What is the brokerage? If 8% dividends are paid, what per cent on his investment does the purchaser receive?

27. What per cent on his investment does a purchaser receive who buys New York, New Haven and Hartford R.R. stock at $180\frac{1}{2}$, if annual dividends of 8% are declared?

28. When West End Co. $4\frac{1}{2}$% bonds are selling at $107\frac{1}{8}$, how much must be invested to produce an annual income of \$900, brokerage $\frac{1}{8}$? What per cent on his investment does a purchaser receive?

29. When Mexican Central Ry. 1st mortgage 4% bonds are selling at $62\frac{1}{2}$, how much must be invested to produce an annual income of \$200, brokerage $\frac{1}{4}$? What per cent on his investment does a purchaser receive?

30. When West Shore R.R. 1st mortgage 4% bonds are selling at $108\frac{3}{4}$, how much must be invested to produce an annual income of \$800, brokerage $\frac{1}{8}$?

31. When New England Tel. and Tel. Co. 6% bonds are selling at $101\frac{1}{8}$, how much must be invested to produce an annual income of \$900, brokerage $\frac{1}{8}$?

32. If a man buys a 6% bond at 120, what rate of interest does he receive on the money invested?

33. If 3% bonds are at $88\frac{1}{8}$, what rate per cent interest will a purchaser receive on his money?

34. If an 8% stock is at 150, what rate per cent interest will a purchaser receive on his money?

35. If a 10% stock is at 175, what rate per cent interest will an investor receive on his money?

36. If a $4\frac{1}{2}$% stock is at 85, what rate per cent interest will a purchaser receive on his money?

37. If 7% bonds are at 114, what rate per cent interest will a purchaser receive on his money?

38. If 6% bonds are at 130, what rate per cent interest will a purchaser receive on his money?

39. If $8000 5% stocks are sold at 90, and the proceeds invested in $3\frac{1}{2}$% stocks at 60, find the increase or decrease in income.

40. If $10,000 $3\frac{1}{2}$% bonds are sold at 65, and the proceeds invested in 8% bonds at 130, find the increase or decrease in income.

41. If $8000 $4\frac{1}{2}$% stocks are sold at 70, and the proceeds invested in 10% stocks at 160, find the increase or decrease in income.

42. If $6000 6% bonds are sold at 90, and the proceeds invested in 10% bonds at 135, find the increase or decrease in income.

43. Find the rate of interest obtained by investing in a 5% bond at 124.

44. What is the price of stock if $7000 stock can be bought for $5880?

45. Find the amount received for 100 mining shares issued at $15 a share and sold at $2\frac{1}{4}$% discount.

46. How much $3\frac{1}{2}$% stock must be sold at $75\frac{1}{8}$ to buy $5000 4% stock at $94\frac{3}{8}$, brokerage $\frac{1}{8}$ on each transaction?

47. How much stock must be sold at $76\frac{1}{8}$ to raise a sum sufficient to discount a note for \$1075, due in 53 days, with grace, and discounted at $5\frac{1}{2}\%$?

48. A broker bought five \$1000 bonds at $88\frac{1}{2}$. At what price must he sell them to gain \$100, brokerage $\frac{1}{8}$ on each transaction?

49. If a broker buys bonds at $87\frac{7}{8}$, at what price must he sell them to make $12\frac{1}{2}\%$ profit, brokerage $\frac{1}{8}$ on each transaction?

50. Which is the more profitable stock for investment, a 4% at 85 or a 3% at 63? a $3\frac{1}{2}\%$ at $67\frac{1}{4}$ or a 4% at $81\frac{1}{2}$?

51. Find the price of a $4\frac{1}{2}\%$ bond to be as profitable an investment as a $3\frac{1}{2}\%$ bond at $88\frac{1}{2}$.

52. Find the price of a 5% bond to be as profitable an investment as a 3% bond at $89\frac{1}{2}$.

53. Find the price of a $3\frac{1}{2}\%$ bond to be as profitable an investment as a 6% bond at par.

54. Find the loss in buying \$80,000 worth of bonds at $91\frac{5}{8}$ and selling at 90, brokerage $\frac{1}{8}$ on each transaction.

55. Which is the better investment, a 5% stock at $137\frac{1}{4}$ or a $3\frac{1}{2}\%$ stock at $91\frac{1}{2}$? What rate of interest will be received from each investment?

56. A person invests \$7370 in the purchase of a stock at 92. What will be his loss if he sells at 90, brokerage $\frac{1}{8}$ on each transaction?

57. How much stock must be sold at $90\frac{5}{8}$ so that when the seller invests the proceeds in a mortgage at 6% he will receive \$543.75 annual income?

58. A person invests $\frac{2}{7}$ of his money at 6%, $\frac{3}{8}$ at $4\frac{1}{2}\%$, and the rest at $3\frac{1}{2}\%$. What per cent does he receive on the whole amount?

59. How many shares of stock must a man sell at $107\frac{1}{4}$, that when he invests the proceeds in 3% stock at $71\frac{1}{2}$ he may receive an annual income of \$900?

Exchange.

518. Exchange. The system of paying debts due persons living at a distance by transmitting drafts instead of money is called *exchange.*

519. Exchange between cities of the same country is called *Domestic Exchange.* Exchange between cities of different countries is called *Foreign Exchange.*

520. Examples. 1. Find the cost of the following draft with grace, interest 6%, exchange $\frac{1}{2}$% premium:

$800. CINCINNATI, O., Nov. 1, 1897.

Thirty days after sight, pay to the order of S. G. Clark Eight Hundred Dollars, and charge to the account of

To H. S. WRIGHT, PHILADELPHIA. P. M. CLEMENT.

SOLUTION. The discount on $800 at 6% for 33 days is $4.40.
The proceeds of the draft is $800 − $4.40 = $795.60.
The exchange is $\frac{1}{2}$% of $800 = $4.
Hence, the cost of the draft is $795.60 + $4 = $799.60.

2. Find the cost of a draft of $400, payable 60 days after sight with grace, interest 7%, exchange $\frac{1}{4}$% discount.

SOLUTION. The discount on $400 at 7% for 63 days is $4.90.
The exchange is $\frac{1}{4}$% of $400 = $1.
The total discount is $4.90 + $1 = $5.90.
Hence, the cost of the draft is $400 − $5.90 = $394.10.

3. Find the face of a draft, payable 30 days after sight with grace, that can be bought for $1000, interest 6%, exchange $\frac{1}{4}$% premium.

SOLUTION. The discount on $1 for 33 days at 6% is $0.0055; and the proceeds of $1 is $1 − $0.0055 = $0.9945.

The exchange on $1 is $0.0025; and the cost of $1 is $0.9945 + $0.0025 = $0.997.

Hence, the face of the draft is $1000 ÷ 0.997 = $1003.01.

Exercise 133.

1. Find the cost of a sight draft on New York of \$1100, exchange $\frac{1}{4}\%$ premium.

2. Find the cost of a sight draft on New Orleans of \$1350, exchange $\frac{1}{4}\%$ discount.

3. Find the cost of a draft on Boston of \$1600, payable 30 days after sight with grace, interest 6%, exchange $\frac{1}{4}\%$ premium.

4. Find the cost of a draft of \$500, payable 60 days after sight with grace, interest 7%, exchange $\frac{1}{2}\%$ discount.

5. Find the cost of a draft of \$1200, payable 90 days after sight with grace, interest 7%, exchange $\frac{1}{2}\%$ premium.

6. Find the cost of a draft of \$950, payable in 30 days with grace, interest $4\frac{1}{2}\%$, exchange at par.

7. Find the cost of a draft of \$725, payable in 60 days with grace, interest 5%, exchange $\frac{1}{4}\%$ discount.

8. Find the cost of a draft of \$810, payable in 90 days with grace, interest $5\frac{1}{2}\%$, exchange $\frac{1}{4}\%$ premium.

9. Find the face of a draft, payable 30 days after sight with grace, that can be bought for \$274, interest 6%, exchange at par.

10. Find the face of a draft, payable 60 days after sight with grace, that can be bought for \$1250, interest 7%, exchange $\frac{1}{4}\%$ premium.

11. Find the face of a draft, payable 60 days after date with grace, that can be bought for \$1125, interest $5\frac{1}{2}\%$, exchange $\frac{1}{4}\%$ discount.

12. Find the face of a draft, payable 30 days after date with grace, that can be bought for \$520, interest 4%, exchange $\frac{1}{2}\%$ premium.

13. Find the face of a draft, payable 90 days after date with grace, that can be bought for \$10,000, interest $4\frac{1}{2}\%$, exchange at par.

521. Foreign Exchange. Foreign Bills of Exchange are usually drawn in sets of three, of the same tenor and date, called *First, Second,* and *Third of Exchange.* These are sent by different mails to avoid loss or delay. When one is accepted or paid the others are void. Thus,

£500. *New York, Nov. 1, 1897.*

At Sight of this First of Exchange (Second and Third of the same tenor and date unpaid), pay to the order of James Patterson Five Hundred Pounds, value received, and charge same to account of

To Baring Brothers,
London, England. *Bliss and Morton.*

522. Exchange for sight drafts Nov. 13, 1897, was quoted in New York as follows:

On London at \$4.865 for 1 pound sterling.
On Paris at $5.18\frac{1}{8}$ francs for \$1.
On cities in Germany at 4 reichsmarks for \$0.955.
On Amsterdam at 1 guilder for $\$0.40\frac{3}{8}$.

523. Examples. 1. Find the cost in New York of a sight draft on London for £502 12s.

SOLUTION. £502 12s. = £502.6.
502.6 × \$4.865 = \$2445.15.

2. Find the face of a sight draft on London that can be bought for \$2433.47.

SOLUTION. \$2433.47 ÷ \$4.865 = 500.2.
£500.2 = £500 4s.

3. Find the cost in New York of a sight draft on Paris for 2400 francs.

SOLUTION. $1 \text{ franc} = \$\frac{1}{5.18\frac{1}{8}}.$

$$2400 \text{ francs} = 2400 \times \$\frac{1}{5.18\frac{1}{8}} = \$463.21.$$

Exercise 134.

Find the cost of a sight draft on:

1. London for £320 10s. 6d.
2. Paris for 8000 francs.
3. Hamburg for 2876 reichsmarks.
4. Amsterdam for 6486 guilders.
5. Glasgow for £5876 10s.
6. Paris for 12,842 francs.
7. Berlin for 4885 reichsmarks.
8. Rotterdam for 8282 guilders.
9. Liverpool for £1242 12s. 6d.
10. Paris for 2685 francs.

11. Find the face of a sight draft on Glasgow that can be bought for $2000.

12. Find the face of a sight draft on London that can be bought for $4000.

13. Find the cost of a sixty-day draft on London for £150, when sixty-day bills are quoted at 4.81$\frac{1}{4}$, and the broker's commission is $\frac{1}{8}\%$ of the cost of the draft.

14. How large a sight draft on Paris can be bought for $2840?

15. How large a sixty-day draft on Paris can be bought for $1500, when sixty-day bills are quoted at 5.17$\frac{5}{8}$?

16. How large a sight draft on Berlin can be bought for $8000?

17. How large a sixty-day draft on Hamburg can be bought for $2500, when German sixty-day drafts are quoted at 0.95?

18. How large a sight draft on Amsterdam can be bought for $2200?

19. How large a sixty-day draft on Rotterdam can be bought for $1200, when a sixty-day draft on Holland is quoted at 0.40$\frac{1}{8}$?

Average of Payments.

524. Example. John Smith has given to William Jones notes as follows: \$150, due May 15; \$200, due June 30; \$500, due July 21; \$400 due July 29. He wishes to pay them all at one time. When shall they be considered due?

SOLUTION. If all the notes were paid May 15, Smith would lose the use of \$200 for 46 days, of \$500 for 67 days, and of \$400 for 75 days.

The use of \$200 for 46 days = the use of \$200 × 46 for 1 day; the use of \$500 for 67 days = the use of \$500 × 67 for 1 day; and the use of \$400 for 75 days = the use of \$400 × 75 for 1 day. Smith would, therefore, lose the equivalent of \$9200 + \$33,500 + \$30,000 = \$72,700 for 1 day, and is entitled to keep the \$150 + \$200 + \$500 + \$400 = \$1250 as many days after May 15 as are required for the use of \$1250 to equal the use of \$72,700 for 1 day; that is, $\frac{72700}{1250}$ days = 58.2 days.

Hence, the *equated time* for paying the four notes is 58 days after May 15; that is, July 12.

The work may be arranged as follows:

\$150 × 00 =		
\$200 × 46 =	\$9,200	
\$500 × 67 =	33,500	
\$400 × 75 =	30,000	
\$1250	)\$72,700	
	58.2.	Hence,

525. To Find the Equated Time for the Payment of Several Debts Due at Different Dates,

Select the earliest date that any debt becomes due as the standard date.

Multiply each of the debts by the number of days from the standard date to the date that it becomes due, and divide the sum of the products by the sum of the debts.

The quotient is the number of days that must be added to the standard date to find the average time of the payments.

Exercise 135.

1. Find the equated time for the payment of $250 due in 3 mo., $400 due in 6 mo., $700 due in 8 mo.

2. Find the equated time for the payment of $300 due in 30 days, $500 due in 60 days, and $200 due in 90 days.

3. Find the equated time for the payment of $325 due now, $200 due in 30 days, $460 due in 60 days, and $150 due in 90 days.

4. Find the equated time for the payment of $240 due May 10, $420 due July 2, $310 due Sept. 14, and $600 due Oct. 1.

5. Find the equated time for the payment of $275 due June 21, $175 due July 16, $200 due Aug. 6, and $150 due Sept. 3.

6. Find the equated time for the payment of $112.30 due July 6, $115.25 due July 30, $232.15 due Sept. 4, and $102.36 due Oct. 1.

7. A owes B $200 due in 10 mo. If he pays $120 in 4 mo., when should he pay the balance?

Note. By paying $120 in 4 mo. A loses the use of $120 for 6 mo., which is equal to the use of $720 for 1 mo. Therefore, he is entitled to keep the balance ($80) $\frac{720}{80}$ mo. = 9 mo. after its maturity.

8. A owed B $2000 payable in 4 mo., but at the end of 1 mo. he paid him $500, at the end of 2 mo. $500, and at the end of 3 mo. $500. In how many months is the balance due?

9. A man, Feb. 11, 1898, gave a note for $1700 payable in 4 mo.; but he paid Mar. 22, $400, Apr. 20, $220, May 10, $300. When was the balance due?

10. A man, Jan. 4, 1898, gave a note for $2500 payable in 6 mo.; but he paid Feb. 4, $200, Mar. 4, $400, Apr. 4, $600, May 4, $500, and June 4, $300. When was the balance due?

Settlement of Accounts.

526. Examples. 1. Find the time for the payment of the balance of the following account:

ADAMS & Co. *in account with* BACON & Co.

DR. **CR.**

1898.					
Jan. 3.	To mdse. 90 dy.	$250	Apr. 11.	By cash,	$200
Mar. 7.	" 60 "	150	Apr. 30.	"	100
May 3.	" 60 "	325	May 30.	"	125
June 7.	" 30 "	175	July 2.	"	400

SOLUTION. We first find the equated time of the items on the Debit side to be May 29, 1898; and the equated time of the items on the Credit side to be May 30, 1898.

The total of the Debit side is $900, and the total of the Credit side is $825. Hence, the balance is $900 − $825 = $75.

The difference between the equated times is 1 day.

If the account were settled at the *later* date, May 30, the $900 on the Debit side would have been on interest 1 day, and this is equivalent to having the balance, $75, on interest $\frac{900}{75}$ of 1 day = 12 days. Hence, the balance should begin to draw interest 12 days *before* May 30; that is, May 18, 1898.

2. Find the time for the payment of the balance of an account if the debit and credit sides, when equated, stand as follows:

DR.	CR.
Due Feb. 18, 1898, $950	Due Jan. 20, 1898, $850

SOLUTION. The difference between the equated times is 29 days.
The balance of account is $950 − $850 = $100.

If the account were settled at the *later* date, Feb. 18, the $850 would have been on interest 29 days, which is equivalent to having the balance, $100, on interest $\frac{850}{100}$ of 29 days = $246\frac{1}{2}$ days. Hence, to increase the Debit side by *an equal amount of interest*, the balance should remain unpaid 247 days; that is, the balance is due Oct. 23, 1898. Hence,

527. To Find the Time the Balance of an Account Falls Due,

Find the equated time for each side of the account.

Multiply the side of the account that falls due first by the number of days between the dates of the equated times of the two sides, and divide the product by the balance of the account.

The quotient will give the number of days to the maturity of the balance, to be counted forward from the later date when the smaller side falls due first, and backward when the larger side falls due first.

NOTE 1. In finding the equated time of accounts it is customary to neglect cents if less than 50, and if 50 or more to consider them as $1. A fraction of a day in the result is rejected if less than ½; if ½ or more it is called a day.

NOTE 2. When an account is settled by cash at any other date than that on which the balance becomes due, the interest is found on the balance for the interval between the day of settlement and the day the balance is due, and is added to or deducted from the balance, according as the settlement is made *after* or *before* the balance is due.

EXERCISE 136.

Find the time for paying the balance in the following equated bills:

	Average due.	DR.	*Average due.*	CR.
1.	May 17, 1897 . . .	$950	Apr. 12, 1897	$1000
2.	Apr. 12, 1897 . . .	$950	May 17, 1897	$1000
3.	May 30, 1898 . .	$1000	June 23, 1898	$920
4.	July 6, 1897 . . .	$500	Apr. 14, 1897	$480
5.	Aug. 13, 1897 . .	$875	Sept. 13, 1897	$600
6.	May 28, 1898 . .	$500	June 4, 1898	$550
7.	Apr. 4, 1898 . . .	$400	June 6, 1898	$300
8.	Mar. 12, 1898 . .	$750	Feb. 4, 1898	$500
9.	Feb. 4, 1898 . . .	$750	Mar. 12, 1898	$500

528. The common method of finding the balance of an account is to compute the interest on each item, from its date to the day of settlement, reckoning the time in days.

1897.		Dr.	Int.	1897.		Cr.	Int.
Apr. 8.	To cash,	$250	$7.71	Apr. 4.	By mdse.,	$300	$9.45
May 31.	"	380	8.36	May 19.	"	350	8.40
July 20.	"	420	5.74	June 9.	"	610	12.50
Oct. 10.	To bal. acct.	210					
" 10.	" int.		8.54	Settled Oct. 10, 1897.			
		$1260	$30.35			$1260	$30.35

Hence, the cash balance = $210 + $8.54, or $218.54.

Note. When the balance of account and the balance of interest fall on *opposite* sides, the cash balance is their *difference*.

Exercise 137.

Find the cash balance of the following accounts, reckoning interest at 6%:

1.

1897.		Dr.	1897.		Cr.
Apr. 5.	To mdse.,	$250.00	Apr. 20.	By cash,	$200.00
" 27.	"	610.00	" 30.	"	500.00
June 1.	"	200.00	June 4.	"	400.00

Settled June 19, 1897.

2.

1897.		Dr.	1897.		Cr.
Jan. 15.	To mdse. 3 mo.,	$250.00	Apr. 26.	By cash,	$150.00
Feb. 25.	" "	98.50	May 17.	"	150.00
Mar. 8.	" "	300.00	July 7.	"	200.00

Settled Oct. 11, 1897.

3.

1897.		Dr.	1897.		Cr.
Jan. 2.	To mdse. 60 dy.,	$100.00	Feb. 25.	By cash,	$100.00
Mar. 8.	" "	200.00	Mar. 22.	"	150.00
May 10.	" 30 dy.,	150.00	June 21.	"	200.00
June 2.	"	95.00			

Settled Aug. 2, 1897.

Savings Banks Accounts.

529. Savings banks receive money on deposit, and pay depositors compound interest, adding the interest to the principal every three months, six months, or twelve months.

530. The interval between the dates at which interest is computed is called an **Interest Term.**

Interest is added at the end of every interest term, computed on the smallest balance on deposit at any time during the whole interest term.

Each depositor has a bank book, in which is recorded every sum deposited, every sum withdrawn, and the interest due at the end of each interest term.

531. Example. Find the balance on deposit Oct. 1, 1897, on the following account, interest 4%, reckoned quarterly:

Deposited Jan. 1, 1897, $50; Feb. 4, 1897, $40; May 6, 1897, $60; Aug. 3, 1897, $40. Withdrawn Mar. 3, 1897, $20; Apr. 22, 1897, $30; June 19, 1897, $25; Sept. 20, 1897, $40.

STATEMENT.

DATE.	DEPOSITED.		WITHDRAWN.		INTEREST.		BALANCE.	
1897.								
Jan. 1,	$50	00					$50	00
Feb. 4,	40	00					90	00
Mar. 3.			$20	00			70	00
Apr. 1,					$0	50	70	50
Apr. 22,			30	00			40	50
May 6,	60	00					100	50
June 19,			25	00			75	50
July 1,					0	40	75	90
Aug. 3,	40	00					115	90
Sept. 20,			40	00			75	90
Oct. 1,					0	76	76	66

The smallest sum on deposit during the first interest term was $50. The interest on $50 for 3 mo. at 4% is $0.50, which, added to the balance on deposit, makes $70.50.

The smallest sum on deposit during the second interest term was $40.50. The interest on $40.50 for 3 mo. at 4% is $0.40, which, added to the balance on deposit, makes $75.90.

The smallest sum on deposit during the third interest term was $75.90. The interest on $75.90 for 3 mo. at 4% is $0.76, which, added to the balance on deposit Oct. 1, 1897, makes $76.66.

Exercise 138.

Find the balance on deposit Jan. 1, 1898, on the following account:

1. Interest being 4%, computed quarterly. Deposited Jan. 1, 1897, $125; Mar. 22, 1897, $40; June 8, 1897, $35; July 30, 1897, $85; Sept. 24, 1897, $65. Withdrawn Apr. 2, 1897, $110; June 30, 1897, $40; Oct. 22, 1897, $10; Dec. 17, 1897, $25.

2. Interest being 3%, computed quarterly. Deposited Jan. 1, 1897, $200; Feb. 14, 1897, $125; Mar. 10, 1897, $75; May 31, 1897, $50; Aug. 2, 1897, $100. Withdrawn May 7, 1897, $25; June 22, 1897, $40; Oct. 2, 1897, $50; Nov. 4, 1897, $65; Dec. 14, 1897, $75.

3. Interest being 3 %, computed semi-annually. Deposited Jan. 1, 1897, $425; May 10, 1897, $15; Sept. 24, 1897, $200; Oct. 5, 1897, $25; Nov. 15, 1897, $65. Withdrawn Feb. 1, 1897, $25; Mar. 20, 1897, $45; Aug. 2, 1897, $50; Aug. 28, 1897, $125; Dec. 10, 1897, $100.

4. Interest being 3%, computed annually. Deposited Jan. 1, 1897, $266.50; May 3, 1897, $122.50; Aug. 2, 1897, $57; Aug. 9, 1897, $108; Sept. 4, 1897, $64.50. Withdrawn June 15, 1897, $40; Oct. 8, 1897, $75; Nov. 1, 1897, $60; Dec. 4, 1897, $85; Dec. 20, 1897, $142.

CHAPTER XV.

POWERS AND ROOTS.

532. The **square** of a number is the product of *two* factors, each equal to the number (§ 69).

Thus, the squares of 1, 2, 3, 4, 5, 6, 7, 8, 9, 10,
are 1, 4, 9, 16, 25, 36, 49, 64, 81, 100.

533. The **square root** of a number is one of the *two equal factors* of the number (§ 69).

Thus, the square roots of 1, 4, 9, 16, 25, 36, 49, 64, 81, 100,
are 1, 2, 3, 4, 5, 6, 7, 8, 9, 10.

534. The square root of a number is indicated by the *radical sign* $\sqrt{\ }$, or by the fractional exponent $\frac{1}{2}$.

Thus, $\sqrt{27}$, or $27^{\frac{1}{2}}$, means the square root of 27.

535. Since $35 = 30 + 5$, the square of 35 may be obtained as follows:

$$\begin{array}{l} 30 + 5 \\ 30 + 5 \\ \hline 30^2 + (30 \times 5) \\ \quad + (30 \times 5) + 5^2 \\ \hline 30^2 + 2\,(30 \times 5) + 5^2 \end{array} \qquad \begin{array}{r} 30^2 = 900 \\ 2\,(30 \times 5) = 300 \\ 5^2 = \underline{\ \ 25} \\ 35^2 = 1225 \end{array}$$

536. Since every number of two or more figures may be regarded as composed of tens and units, if we represent the number of tens by a and the number of units by b,

$$(a + b)^2 = a^2 + 2\,ab + b^2. \quad \text{Hence,}$$

The square of a number is equal to the square of the tens, plus twice the tens multiplied by the units, plus the square of the units.

537. The first step in extracting the square root of a number is to separate the figures of the number into groups.

Since $1 = 1^2$, $100 = 10^2$, $10{,}000 = 100^2$, and so on, it is evident that the square root of any number between 1 and 100 lies between 1 and 10; of any number between 100 and 10,000 lies between 10 and 100. In other words, the square root of any integral number expressed by *one* or *two* figures is a number of *one* figure; expressed by *three* or *four* figures is a number of *two* figures, and so on.

If, therefore, an integral number is divided into groups of two figures each, from the right to the left, the number of figures in the root will be equal to the number of groups of figures. The last group to the left may have one or two figures.

Example. Find the square root of 1225.

SOLUTION. The first group, 12, contains the square of the tens' number of the root.

The greatest square in 12 is 9, and the square root of 9 is 3. Hence, 3 is the tens' figure of the root.

```
   12 25(35
    9
65) 3 25
    3 25
```

The square of the tens is subtracted, and the remainder contains twice the tens × the units + the square of the units. Twice the 3 tens is 6 tens, and 6 tens is contained in the 32 tens of the remainder 5 times. Hence, 5 is the units' figure of the root. Since twice the tens × the units + the square of the units is equal to (twice the tens + the units) × the units, the five units are annexed to the 6 tens, and the result, 65, is multiplied by 5.

538. The same method will apply to numbers of more than two groups of figures, by considering *the part of the root already found as so many tens* with respect to the next figure of the root.

Example. Extract the square root of 7,890,481.

```
      7 89 04 81(2809
      4
  48) 3 89
      3 84
5609) 5 04 81
      5 04 81
```

SOLUTION. When the third group, 04, is brought down, and the divisor, 56, formed, the next figure of the root is 0, because 56 is not contained in 50. Therefore, 0 is placed both in the root and the divisor, and the next group, 81, is brought down.

539. If the square root of a number has decimal places, the number itself will have *twice* as many.

Thus, if 0.11 is the square root of some number, the number will be $(0.11)^2 = 0.11 \times 0.11 = 0.0121$. Hence, if a given number contains a decimal, we divide it into groups of two figures each, beginning at the decimal point and marking toward the left for the integral number, and toward the right for the decimal. The last group of the decimal must have *two* figures, a cipher being annexed if necessary.

Example. Extract the square root of 52.2729.

```
      52.27 29(7.23
      49
 142) 3 27
      2 84
1443)  43 29
       43 29
```

SOLUTION. It will be seen from the groups of figures that the root will have one integral and two decimal places.

540. If a number is not a perfect square, ciphers may be annexed, and an *approximate* value of the root found.

Example. Extract the square root of 17 to six places.

```
     17.00 00 00(4.123106
     16
  81)1 00
       81
 822) 19 00
      16 44
 8243)2 56 00
       2 47 29
  8246)  8 710
         8 246
          46400
```

SOLUTION. In this example, after finding four figures of the root, the other three are found by common division.

The rule in such cases is that *one less than the number of figures already obtained may be found without error by division, the divisor to be employed being twice the part of the root already found.*

541. The square root of a common fraction is found by extracting the square root of the numerator and of the denominator. If the denominator is not a perfect square, multiply both terms of the fraction by a number that will make the denominator a perfect square, or reduce the fraction to a decimal and extract the root of the decimal.

542. Rule for Square Root. *Separate the number into groups of two figures each, beginning at the units.*

Find the greatest square in the left-hand group and write its root for the first figure of the required root.

Square this root, subtract the result from the left-hand group, and to the remainder annex the next group for a dividend.

For a partial divisor, double the root already found, considered as tens, and divide the dividend by it. The quotient (or the quotient diminished) will be the next figure of the root.

To this partial divisor add the last figure of the root for a complete divisor. Multiply this complete divisor by the last figure of the root, subtract the product from the dividend, and to the remainder annex the next group for a new dividend.

Proceed in this manner until all the groups have been thus annexed. The result will be the square root required.

Note 1. If the number is not a perfect square, annex groups of zeros and continue the process.

Note 2. If the given number contains a decimal, divide it into groups of two figures each, beginning at the decimal point and marking toward the left for the integral number and toward the right for the decimal number. The last group on the right of the decimal must contain *two* figures, a zero being annexed if necessary.

Exercise 139.

Find the square root of:

1. 2916.	**9.** 53.7289.	**17.** $8\frac{5}{8}$.
2. 7921.	**10.** 883.2784.	**18.** 0.9.
3. 494,209.	**11.** 1.97262025.	**19.** $\frac{4}{9}$.
4. 20,164.	**12.** 0.0002090916.	**20.** $\frac{5}{8}$.
5. 3,345,241.	**13.** 2.	**21.** $\frac{1}{2}$.
6. 125,457.64.	**14.** 5.	**22.** $\frac{3}{5}$.
7. 47,320,641.	**15.** 0.3.	**23.** $\frac{3}{4}$.
8. 21,609.	**16.** $3\frac{1}{4}$.	**24.** $\frac{2}{3}$.

Cube Root.

543. The **cube** of a number is the product of *three* factors, each equal to the number (§ 69).

Thus, the cubes of 1, 2, 3, 4, 5, 6, 7, 8, 9, 10,
are 1, 8, 27, 64, 125, 216, 343, 512, 729, 1000.

544. The **cube root** of a number is one of the *three equal factors* of the number (§ 69).

Thus, the cube roots of 1, 8, 27, 64, 125, 216, 343, 512, 729, 1000,
are 1, 2, 3, 4, 5, 6, 7, 8, 9, 10.

545. The cube root of a number is indicated by $\sqrt[3]{\ }$, a small figure 3 being written above the radical sign, or by the fractional exponent $\frac{1}{3}$.

Thus, $\sqrt[3]{343}$, or $343^{\frac{1}{3}}$, means the cube root of 343.

546. Since $35 = 30 + 5$, the cube of 35 may be obtained as follows:

$$\begin{array}{l}
30 + 5 \\
30 + 5 \\
\hline
30^2 + (30 \times 5) \\
\quad + (30 \times 5) + 5^2 \\
\hline
30^2 + 2\,(30 \times 5) + 5^2 \\
30 + 5 \\
\hline
30^3 + 2\,(30^2 \times 5) + (30 \times 5^2) \\
\quad + (30^2 \times 5) + 2\,(30 \times 5^2) + 5^3 \\
\hline
30^3 + 3\,(30^2 \times 5) + 3\,(30 \times 5^2) + 5^3
\end{array}
\qquad
\begin{array}{rr}
30^3 = & 27{,}000 \\
3\,(30^2 \times 5) = & 13{,}500 \\
3\,(30 \times 5^2) = & 2{,}250 \\
5^3 = & 125 \\
\hline
35^3 = & 42{,}875
\end{array}$$

If we represent the number of tens by a and of units by b

$$(a + b)^3 = a^3 + 3\,a^2b + 3\,ab^2 + b^3. \quad \text{Hence,}$$

The cube of a number is equal to the cube of the tens, plus three times the product of the square of the tens by the units, plus three times the product of the tens by the square of the units, plus the cube of the units.

547. In extracting the cube root of a number, the first step is to separate the figures of the number into groups.

Since $1 = 1^3$, $1000 = 10^3$, $1,000,000 = 100^3$, and so on, it follows that the cube root of any integral number between 1 and 1000, that is, of any integral number that has *one*, *two*, or *three* figures, is a number of *one* figure; that the cube root of any integral number between 1000 and 1,000,000, that is, of any integral number that has *four*, *five*, or *six* figures, is a number of *two* figures, and so on.

If, therefore, an integral number is divided into groups of three figures each, from right to left, the number of figures in the root will be equal to the number of groups. The last group to the left may consist of one, two, or three figures.

Example. Extract the cube root of 42,875.

Solution. Since 42,875 consists of two groups, the cube root will consist of two figures.

The first group, 42, contains the cube of the tens' number of the root.

The greatest cube in 42 is 27, and the cube root of 27 is 3. Hence, 3 is the tens' figure of the root.

$$
\begin{array}{r|l}
 & 42\ 875\ (35 \\
 & 27 \\
\cline{2-2}
3 \times 30^2 = 2700 & 15\ 875 \\
3 \times (30 \times 5) = \ \ 450 & \\
5^2 = \ \ \ \ 25 & \\
\cline{1-1}
3175 & 15\ 875 \\
\cline{2-2}
\end{array}
$$

The remainder, 15,875, resulting from subtracting the cube of the tens, will contain three times the product of the square of the tens by the units + three times the product of the tens by the square of the units + the cube of the units.

Each of these three parts contains the units' number as a factor.

Hence, the 15,875 consists of two factors, one of which is the units' number of the root; and the other factor is three times the square of the tens + three times the product of the tens by the units + the square of the units. The largest part of this second factor is three times the square of the tens.

If the 158 hundreds of the remainder is divided by the $3 \times 30^2 = 27$ hundreds, the quotient will be the units' number of the root.

The second factor can now be completed by adding to the 2700 $3 \times (30 \times 5) = 450$ and $5^2 = 25$

548. The same method will apply to numbers of more than two groups of figures, by considering the part of the root already found as *so many tens with respect to the next figure of the root.*

Example. Extract the cube root of 57,512,456.

		57 512 456 (386
		27
$3 \times 30^2 =$	2700	30 512
$3 \times (30 \times 8) =$	720	
$8^2 =$	64	
	3484	27 872
		2 640 456
$3 \times 380^2 =$	433200	
$3 \times (380 \times 6) =$	6840	
$6^2 =$	36	
	440076	2 640 456

549. If the cube root of a number has decimal places, the number itself will have *three times* as many.

Thus, if 0.11 is the cube root of a number, the number is $0.11 \times 0.11 \times 0.11 = 0.001331$. Hence, if a given number contains a decimal, we divide the figures of the number into groups of three figures each, by beginning at the decimal point and marking toward the left for the integral number, and toward the right for the decimal. We must be careful to have the last group on the right of the decimal point contain *three* figures, annexing ciphers when necessary.

Example. Extract the cube root of 187.149248.

		187.149 248 (5.72
		125
$3 \times 50^2 =$	7500	62 149
$3 \times (50 \times 7) =$	1050	
$7^2 =$	49	
	8599	60 193
		1 956 248
$3 \times 570^2 =$	974700	
$3 \times (570 \times 2) =$	3420	
$2^2 =$	4	
	978124	1 956 248

It will be seen from the groups of figures that the root will have one integral and two decimal places, and therefore the decimal point must be placed in the root as soon as one figure of the root is obtained.

550. If the given number is not a perfect cube, ciphers may be annexed, and a value of the root may be found as near to the *true* value as we please.

Example. Extract the cube root of 5 to five places.

	5.000 (1.70997
	1
$3 \times 10^2 = 300$	4 000
$3 \times (10 \times 7) = 210$	
$7^2 = 49$	
559	3 913
259	87 000 000
$3 \times 1700^2 = 8670000$	
$3 \times (1700 \times 9) = 45900$	
$9^2 = 81$	
8715981	78 443 829
45981	8 556 1710
$3 \times 1709^2 = 8762043$	7 885 8387
	670 33230
	613 34301

After the first two figures of the root are found, the next trial divisor is obtained by bringing down the sum of the 210 and 49 obtained in completing the preceding divisor, then adding the three numbers connected by the brace, and annexing two ciphers to the result.

It is seen at a glance that, when the trial divisor is increased by 3 times the 17 tens of the root, it will be greater than 87,000; so that 0 is placed in the root, and 3×1700^2 is obtained by annexing two ciphers to the 86,700. Again: the last divisor is obtained by bringing down the sum of the 45,900 and 81, which were obtained in completing the preceding divisor, then adding the three numbers connected by the brace.

The last two figures of the root are found by division.

The rule in such cases is that *two less than the number of figures already obtained may be found without error by division, the divisor to be employed being three times the square of the part of the root already found.*

551. The cube root of a common fraction is found by taking the cube root of the numerator and of the denominator. If the denominator is not a perfect cube, multiply both terms of the fraction by a number that will make the denominator a perfect cube, or reduce the fraction to a decimal, and then extract the cube root of the decimal.

552. RULE FOR CUBE ROOT. *Separate the number into groups of three figures each, beginning at the units.*

Find the greatest cube in the left-hand group and write its root for the first figure of the required root.

Cube this root, subtract the result from the left-hand group, and to the remainder annex the next group for a dividend.

For a partial divisor, take three times the square of the root already found, considered as tens, and divide the dividend by it. The quotient (or the quotient diminished) will be the second figure of the root.

To this partial divisor add three times the product of the first figure of the root considered as tens by the second figure, and also the square of the second figure. This sum will be the complete divisor.

Multiply the complete divisor by the second figure of the root, subtract the product from the dividend, and to the remainder annex the next group for a new dividend.

Proceed in this manner until all the groups have been annexed. The result will be the cube root required.

NOTE 1. If the number is not a perfect cube, annex groups of zeros and continue the process.

NOTE 2. If the given number contains a decimal, divide it into groups of three figures each, beginning at the decimal point and marking toward the left for the integral number and toward the right for the decimal number. The last group on the right of the decimal must contain *three* figures, zeros being annexed if necessary.

Exercise 140.

Find the cube root of:

1.	1331.	**8.**	304,957.115891.	**15.**	10.
2.	1728.	**9.**	0.007821346625.	**16.**	$3\frac{5}{8}$.
3.	12.167.	**10.**	104.600290750613.	**17.**	$8\frac{1}{3}$.
4.	300.763.	**11.**	17,183,498,535,125.	**18.**	5.
5.	148,877.	**12.**	122,615.327232.	**19.**	$\frac{5}{9}$.
6.	2,048,383.	**13.**	116,400.	**20.**	$7\frac{3}{5}$.
7.	59.776471.	**14.**	22,406,807.	**21.**	$\frac{3}{4}$.

Geometrical Representation of Square and Cube Roots.

553. We will illustrate square root by giving a Geometrical representation of the square root of 1225.

The square root of 1225 is 35. (§ 537)

The square of $(30 + 5) = 30^2 + 2(30 \times 5) + 5^2$. (§ 536)

The 30^2 may be represented by a square (Fig. 1) 30 in. on a side.

The $2(30 \times 5)$ may be represented by two strips 30 in. long and 5 in. wide of Fig. 2, which are added to two adjacent sides of Fig. 1.

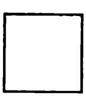


Fig. 1.

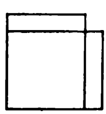


Fig. 2.

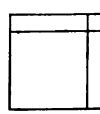


Fig. 3.

The 5^2 may be represented by the small square of Fig. 3 required to make Fig. 2 a complete square.

In extracting the square root of 1225, the large square, which is 30 in. on a side, is first removed, and a surface of 325 sq. in. remains.

This surface consists of two equal rectangles, each 30 in. long, and a small square whose side is equal to the width of the rectangles.

The width of the rectangles is found by dividing the 325 sq. in. by the sum of their lengths; that is, by 60 in., which gives 5 in.

Hence, the entire length of the surfaces added is 30 in. + 30 in. + 5 in. = 65 in., and the width is 5 in.

Therefore, the total area is (65×5) sq. in. = 325 sq. in.

554. We will illustrate cube root by giving a Geometrical representation of the cube root of 42,875.

The cube root of 42,875 is 35. (§ 547)

The cube of $(30 + 5) = 30^3 + 3(30^2 \times 5) + 3(30 \times 5^2) + 5^3$. (§ 546)

The 30^3 may be represented by a cube whose edge is 30 in. (Fig. 1).

The $3(30^2 \times 5)$ may be represented by three rectangular solids, each 30 in. long, 30 in. wide, and 5 in. thick, to be added to three adjacent faces of Fig. 1.

The $3(30 \times 5^2)$ may be represented by three equal rectangular solids, 30 in. long, 5 in. wide, and 5 in. thick, to be added to Fig. 2.

The 5^3 may be represented by the small cube required to complete the cube of Fig. 3.

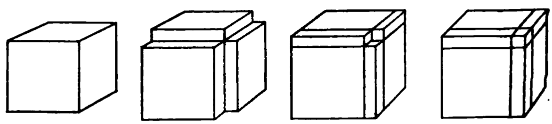


FIG. 1. FIG. 2. FIG. 3. FIG. 4.

In extracting the cube root of 42,875, the large cube (Fig. 1), whose edge is 30 in., is first removed.

There remain (42,875 − 27,000) cu. in. = 15,875 cu. in.

The greatest part of this is contained in the three rectangular solids which are added to Fig. 1, and are each 30 in. long and 30 in. wide.

The thickness of these solids is found by dividing the 15,875 cu. in. by the sum of the three faces, each of which is 30 in. square; that is, by 2700 sq. in. The result is 5 in.

There are also the three rectangular solids which are added to Fig. 2, and which are 30 in. long and 5 in. wide; and a cube which is added to Fig. 3, and which is 5 in. long and 5 in. wide.

Hence, the sum of the products of two dimensions of all these solids is

For the larger rectangular solids,	$3(30 \times 30)$ sq. in. =	2700 sq. in.
For the smaller rectangular solids,	$3(30 \times 5)$ sq. in. =	450 sq. in.
For the small cube,	(5×5) sq. in. =	25 sq. in.
		3175 sq. in.

This number multiplied by the third dimension gives (5×3175) cu. in. = 15,875 cu. in.

CHAPTER XVI.

MENSURATION.

555. We have already considered Areas of Rectangles and Circles; and Volumes and Surfaces of Rectangular Solids, Spheres, and Right Cylinders.

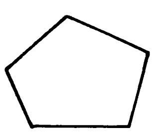
556. A **polygon** is a plane figure bounded by straight lines.

A polygon of *three* sides is a *triangle;* of *four* sides, a *quadrilateral;* of *five* sides, a *pentagon;* of *six* sides, a *hexagon;* of *eight* sides, an *octagon;* of *ten* sides, a *decagon;* of *twelve* sides, a *dodecagon;* and so on.

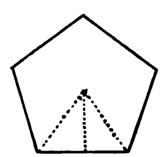
557. The area of any polygon may be found by dividing it into triangles and finding the sum of their areas.

558. A **vertex** of a polygon is the point of intersection of two adjacent sides.

559. A **diagonal** of a polygon is a straight line joining any two vertices not adjacent.



Polygon.



Regular Polygon.

560. A **regular polygon** is a polygon with all its sides equal and all its angles equal. The *centre* of a regular polygon is a point equidistant from the vertices and also equidistant from the sides. The *radius* of a regular polygon is the distance from the centre to any vertex.

The radii of a regular polygon divide the polygon into equal *isosceles* triangles; that is, into triangles having *two sides equal.* The *apothem* of a regular polygon is the distance from the centre to any side.

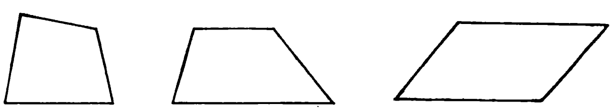
561. *The area of a regular polygon* $= \frac{1}{2}$(*perimeter* × *apothem*).

562. The apothem of a regular polygon bears a constant ratio to one side.

The following table shows the ratio of the apothem to one side in the most common regular polygons:

Triangle	0.2887 : 1.	Heptagon	1.0382 : 1.
Quadrilateral	0.5000 : 1.	Octagon	1.2071 : 1.
Pentagon	0.6882 : 1.	Decagon	1.5388 : 1.
Hexagon	0.8660 : 1.	Dodecagon	1.8660 : 1.

Quadrilaterals.



Trapezium. Trapezoid. Parallelogram.

563. A **trapezium** is a quadrilateral with no two of its sides parallel.

NOTE. Two lines are parallel if all points of one are equally distant from the other.

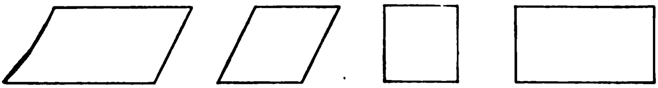
564. A **trapezoid** is a quadrilateral with two of its sides parallel, but the other two sides not parallel.

565. A **parallelogram** is a quadrilateral with its opposite sides parallel.

566. A **rhomboid** is a parallelogram with its angles not right angles.

567. A **rhombus** is a parallelogram with its angles not right angles, but with all its sides equal.

Parallelograms.



Rhomboid. Rhombus. Square. Rectangle.

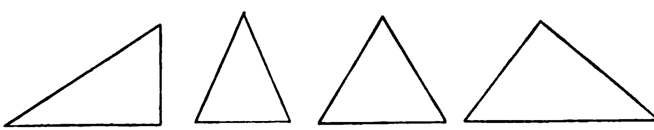
568. The **altitude** of a parallelogram or of a trapezoid is the shortest distance between its parallel sides regarded as *bases*.

569. *The area of any parallelogram = base × altitude.*

570. *The area of a rhombus also = half the product of its diagonals.*

571. *The area of a trapezoid* = ½ (*sum of bases × altitude*).

Triangles.



Right. Isosceles. Equilateral. Scalene.

572. A **right triangle** is a triangle one of whose angles is a right angle. The *hypotenuse* is the side opposite the right angle, and the other two sides, called *legs*, are the *base* and the *perpendicular*.

573. Other kinds of triangles are, *isosceles*, with two sides equal; *equilateral*, with three sides equal; *scalene*, with no two sides equal. The *altitude* of a triangle is the shortest distance from the vertex to the base or the base produced.

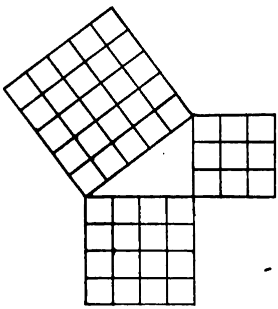
574. When the *base* and *altitude* are given,

The area of the triangle = ½ (*base × altitude*).

575. When the *sides* of a triangle are given,

The area of the triangle is the square root of the product of half the sum of the sides multiplied in succession by the three remainders obtained by subtracting each side separately from the half sum of the sides.

576. In a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.



The hypotenuse is, therefore, equal to the square root of the sum of the squares of the other two sides; and either leg is equal to the square root of the difference of the squares of the hypotenuse and the other leg.

577. Examples. Find the area of:

1. A regular hexagon, each side of which is 3 in.

SOLUTION. The apothem = 0.8660×3 in. = 2.598 in.;
and the perimeter = 6×3 in. = 18 in.
Therefore, the area = $\frac{1}{2}(18 \times 2.598)$ sq. in. = 23.382 sq. in.

2. A parallelogram, base 12 in., altitude 7 in.

SOLUTION. The area = (12×7) sq. in. = 84 sq. in.

3. A trapezoid, if its altitude is 10 in., and its parallel sides are 16 in. and 12 in., respectively.

SOLUTION. The sum of the bases is 16 in. + 12 in. = 28 in.
Therefore, the area = $\frac{1}{2}(28 \times 10)$ sq. in. = 140 sq. in.

4. A triangle, base 12 in., altitude 8 in.

SOLUTION. The area = $\frac{1}{2}(12 \times 8)$ sq. in. = 48 sq. in.

5. A triangle, sides 5 in., 6 in., 7 in.

SOLUTION. The half sum of the sides is $\frac{1}{2}(5 + 6 + 7)$ in., or 9 in.
Hence, the area = $\sqrt{9 \times 4 \times 3 \times 2}$ sq. in. = $\sqrt{216}$ sq. in. = 14.696 sq. in.

6. The base of a right triangle is 20 ft., and the perpendicular is 15 ft. Find the hypotenuse.

SOLUTION. $\sqrt{20^2 + 15^2} = \sqrt{400 + 225} = \sqrt{625} = 25.$

Therefore, the length of the hypotenuse is 25 ft.

7. The base of a right triangle is 16 ft., and the hypotenuse is 20 ft. Find the perpendicular.

SOLUTION. $\sqrt{20^2 - 16^2} = \sqrt{400 - 256} = \sqrt{144} = 12.$

Therefore, the length of the perpendicular is 12 ft.

EXERCISE 141.

Find the area of:

1. A parallelogram, base 18 in., altitude 11 in.
2. A triangle, base 16 in., altitude 12 in.
3. A rectangle, base 24 in., altitude 18 in.
4. A square, side 18 in.
5. A rhombus, diagonals 8 in. and 10 in.
6. A triangle, sides 12 in., 11 in., and 10 in., respectively.
7. A regular hexagon, side 4 in.
8. A regular octagon, side 2 in.
9. A triangle, base 185 yd., altitude 154 yd.
10. A square, side 212 yd.
11. A rectangle, base 106 yd., altitude 66 yd.
12. A parallelogram, base 24 ft., altitude 18 ft.
13. An equilateral triangle, side 132 yd.
14. A right triangle, base 164 ft., perpendicular 150 ft.
15. A regular pentagon, side $5\frac{1}{2}$ in.
16. A parallelogram, base 122 yd., altitude 76 yd.
17. A regular decagon, side $2\frac{1}{2}$ in.
18. A triangle, base 82^{cm}, altitude 51^{cm}.
19. A rhombus, diagonals 16 ft. and 12 ft.
20. A circle, diameter 72 ft.
21. A trapezoid, parallel sides 106 ft. and 56 ft., respectively, altitude 48 ft.

22. Find the number of hektars in a triangular field, one side of which is 82.1^m, and the distance to this side from the opposite corner 47.3^m.

23. Find the number of acres in a triangular field, one side of which is 343.6 ft., and the distance to this side from the opposite corner 163.2 ft.

24. Find the area of a circle that has a radius of 10 in.; of a circle that has a diameter of 10 ft.; of a circle that has a circumference of 30 in.

25. A horse is tied by a rope 27.8^m long; over what part of a hektar can he graze?

26. How many square feet in a circle that has a diameter of $17\frac{2}{3}$ yd.?

27. How many square feet in a circle that has a circumference of 117 yd.?

28. Find the area of a triangle whose sides are 73 ft., 57 ft., and 48 ft.

29. Find the number of hektars in a triangular field whose sides are 37.5^m, 91.7^m, and 78.9^m.

30. Find the number of hektars in a triangular field whose sides are 67.5^m, 81.2^m, and 102.7^m.

31. Find the number of acres in a triangular field whose sides are 227 ft., 342 ft., and 416 ft.

32. Find the number of acres in a triangular field whose sides are 79.08 ch., 57.03 ch., and 102.19 ch.

33. Find the number of square rods in a triangle whose sides are 7 rd. 2 yd.; 6 rd. 5 yd.; and 9 rd. $4\frac{1}{2}$ ft.

34. One diagonal of a trapezium is 10 rd., and the perpendiculars upon it from the opposite corners are 6 rd. and 8 rd. Find the area.

35. Find the area of a lot of land in the shape of a trapezium, if one diagonal is 108 ft., and the perpendiculars upon it from the opposite corners are 55 ft. and 60 ft.

36. What is the area of the ground covered by a tent, the base of which is a regular heptagon 25 ft. on a side?

37. How many paving stones will be required to pave a rectangular court 60 ft. long and 40 ft. wide, if each stone is in the shape of a regular hexagon 5 in. on a side?

38. At $225 an acre, what is the value of a field in the shape of a regular pentagon 250 yd. on a side?

39. A rectangular field 100 yd. wide contains $3\frac{1}{8}$ A. What is its length?

40. The dimensions of a rectangle are 45 yd. and 28 yd. What is the length of its diagonal?

41. A field has the shape of a right triangle, and the two legs are 75 yd. and 60 yd., respectively. What decimal of an acre does the field contain?

42. Compare the areas of a square and an equilateral triangle, if the perimeter of each is 60 ft.

43. Find the area of a field in the shape of a trapezoid, if the altitude is 240 yd., and the parallel sides are 510 yd. and 725 yd., respectively.

44. The legs of a right triangle are each equal to 12 ft. Find the hypotenuse.

45. A city lot in the shape of a right triangle has for its base 119 ft., and for its perpendicular 120 ft. Find the area and the hypotenuse of the lot.

46. Find the base and the area of a right triangle, hypotenuse 130 yd., and perpendicular 112 yd.

47. Find the perpendicular and the area of a right triangle, hypotenuse 164 ft., and base 160 ft.

48. Find the hypotenuse and the area of a right triangle, base 100 yd., and perpendicular 105 yd.

49. Find the hypotenuse and the area of a right triangle, base 96 ft., and perpendicular 110 ft.

50. Find the area of a field in the shape of a right triangle, if the hypotenuse is 709 yd., and one leg 660 yd.

51. A rectangular field is 345 yd. long and 152 yd. wide. What is the length of its diagonal?

52. The legs of a right triangle are 44 ft. 4 in. and 13 ft. 9 in., respectively. Find the length of its hypotenuse.

53. The hypotenuse of a right triangle is 7 ft. 1 in., and one leg is 6 ft. 5 in. Find the other leg and the area.

54. The hypotenuse of a right triangle is 3 ft. 1 in., and one leg is 2 ft. 11 in. Find the other leg and the area.

55. The area of a lot in the shape of a right triangle is 1560 sq. yd., and the base is 80 yd. Find the perpendicular and the hypotenuse.

56. The area of a right triangle is 60 sq. in., and one leg is 8 in. Find the hypotenuse and the other leg.

57. The length and diagonal of a rectangular field are 60 rd. and 65 rd., respectively. What is its area?

58. What is the length of a side of a square that contains 390,625 sq. ft.?

59. Express to six places of decimals the length of the diagonal of a square in terms of a side.

60. The hypotenuse of a right triangle is 95 ft., and the two legs are as 3 to 4. Find the legs and the area.

61. St. Mark's Square in Venice has the shape of a trapezoid. The parallel sides are 61 yd. and 90 yd., respectively, and the altitude is 192 yd. What is its area?

62. The perimeter of a regular hexagon is 45 in. Find its area.

63. A circular pond contains 12 acres. Express its diameter in feet.

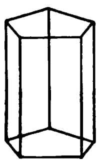
NOTE. Multiply the area in square feet by 0.31831, and take the square root for the radius.

64. What is the diameter of a circle whose area is 1262 sq. ft.?

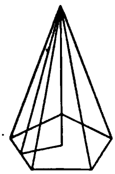
65. What is the diameter of a circle whose area is 2206 sq. ft.?

Solids.

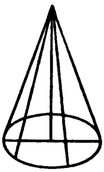
578. A **right prism** is a solid bounded by two equal parallel polygons, called the *bases*, and by rectangles, called the *lateral faces*. The *altitude* of a right prism is the shortest distance between its bases.



Right Prism.



Regular Pyramid.



Right Cone.

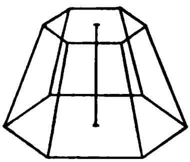
579. A **regular pyramid** is a solid bounded by a regular polygon, called the *base*, and by isosceles triangles, called the *lateral faces*. These triangles all terminate in a point called the *vertex* of the pyramid. The *altitude* of a regular pyramid is the shortest distance from its vertex to its base. The *slant height* is the altitude of the lateral faces.

580. A **cone** is a solid bounded by a circle, called the *base*, and by a curved surface, called the *lateral surface*, which terminates in a point called the *vertex*.

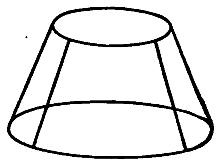
581. A **right cone** is a cone whose vertex is in the perpendicular erected at the centre of the base. The *altitude* of a right cone is the shortest distance from its vertex to its base. The *slant height* of a right cone is the distance from its vertex to the *circumference* of its base.

582. A **frustum of a regular pyramid** or a **frustum of a right cone** is the part of the pyramid or of the cone left after its top has been cut off by a plane parallel to its base. The *lateral faces* of a frustum of a regular pyramid are trapezoids.

583. The *bases* of a frustum of a regular pyramid or of a frustum of a right cone are the base of the pyramid or cone and the section made by the cutting plane. The *altitude* of the frustum of a regular pyramid or of the frustum of a right cone is the shortest distance between its bases.



Frustum of a Regular Pyramid.



Frustum of a Right Cone.

584. The **area of the lateral surface** of a right prism, of a regular pyramid, or of the frustum of a regular pyramid, is the sum of the areas of its lateral faces.

585. *The lateral surface of a right prism = perimeter of base × altitude.*

586. *The volume of a right prism = base × altitude.*

587. *The lateral surface of a regular pyramid =* $\frac{1}{2}$ *(perimeter of base × slant height).*

588. *The volume of a regular pyramid =* $\frac{1}{3}$ *(base × altitude).*

589. *The lateral surface of a right cone =* $\frac{1}{2}$ *(circumference of base × slant height).*

590. *The volume of a right cone =* $\frac{1}{3}$ *(base × altitude).*

591. *The lateral surface of a frustum of a regular pyramid or of a frustum of a right cone =* $\frac{1}{2}$ *(sum of perimeters of bases × slant height).*

592. *To find the volume of a frustum of a regular pyramid or of a frustum of a right cone, we take the sum of the areas of its bases and the square root of their product; and multiply this sum by one third the altitude.*

593. Examples. 1. Find the lateral surface and the volume of a right prism, base a square 3 ft. on a side, and altitude 5 ft.

SOLUTION. The perimeter of base $= 4 \times 3$ ft. $= 12$ ft.
Hence, the lateral surface $= (12 \times 5)$ sq. ft. $= 60$ sq. ft.
The area of base $= (3 \times 3)$ sq. ft. $= 9$ sq. ft.
Hence, the volume $= (9 \times 5)$ cu. ft. $= 45$ cu. ft.

2. Find the lateral surface and the volume of a regular pyramid, base a square 6 ft. on a side, and altitude 4 ft.

SOLUTION. The perimeter of base $= 4 \times 6$ ft. $= 24$ ft.
The slant height $= \sqrt{3^2 + 4^2}$ ft. $= \sqrt{9 + 16}$ ft $= \sqrt{25}$ ft. $= 5$ ft.
Hence, the lateral surface $= \frac{1}{2}(24 \times 5)$ sq. ft. $= 60$ sq. ft.
The area of base $= (6 \times 6)$ sq. ft. $= 36$ sq. ft.
Hence, the volume $= \frac{1}{3}(36 \times 4)$ cu. ft. $= 48$ cu. ft.

3. Find the volume of a frustum of a regular pyramid, bases squares, 5 ft. and 3 ft., respectively, on a side, and altitude 6 ft.

SOLUTION. The areas of the bases are 25 sq. ft. and 9 sq. ft., respectively; and the square root of their product is 15 sq. ft.
Hence, the volume $= \frac{1}{3} \times 6(25 + 9 + 15)$ cu. ft. $= 98$ cu. ft.

EXERCISE 142.

1. Find the volume of a triangular prism, height 11 in., and sides of the ends 2 in., 3 in., and 4 in., respectively.

2. Find the capacity in bushels of a bin 6 ft. long, the end of which is a square 3 ft. 3 in. on a side.

3. Find the lateral surface and the volume of a regular pyramid, base a regular hexagon 9 in. on a side, altitude 40 in., and slant height 40.75 in.

4. Find the number of cubic yards in a prism, base a square 200 ft. on a side, height 40 ft.

5. How many square yards of canvas are required for a conical tent 9 ft. 11 in. high, diameter of base 20 ft.?

6. Find the volume and the lateral surface of a frustum of a regular pyramid, bases squares, 24 in. and 12 in. on a side, respectively, altitude $17\frac{1}{2}$ in., slant height $18\frac{1}{2}$ in.

7. Find the volume and the lateral surface of a frustum of a right cone, radii of bases 50^{cm} and 30^{cm}, respectively, altitude 48^{cm}, and slant height 52^{cm}.

8. Find the volume and the surface of a sphere whose diameter is 17.2^{cm}.

9. A right cylinder is 3 ft. 2 in. in diameter and 4 ft. 6 in. high. Find its volume and its lateral surface.

10. Find the length of an edge of a cubical vessel that will hold a ton of water.

11. A rectangular tank 6 ft. long and $4\frac{1}{2}$ ft. wide holds 108 cu. ft. of water. What is the height of the tank?

12. Find the total surface of a regular pyramid, base a square 5 ft. on a side, and slant height 20 ft.

13. The circumference of the base of a right cone is 12 ft., and the height of the cone is 12 ft. Find the volume.

14. Find the surface of a megaphone in the shape of a frustum of a right cone, diameters of the upper and lower bases 24 in. and 3 in., respectively, slant height 30 in.

15. Find the difference between the volume of a frustum of a regular pyramid, bases squares, 8 ft. and 6 ft., respectively, on a side and altitude 9 ft., and the volume of a right prism, base a square 7 ft. on a side, altitude 9 ft.

16. Find the surface and the volume of a sphere whose diameter is 28 in.

17. Find the ratio of the volume of a cube of wood 15 in. on an edge to the volume of the largest sphere that can be turned from it. Find the ratio of their surfaces.

18. Find the ratio of the volume of a cube of wood to the volume of the largest right cylinder that can be turned from it. Find the ratio of their surfaces.

19. Find the ratio of the volume of a right cylinder of wood to the volume of the largest right cone that can be turned from it. Find the ratio of their lateral surfaces.

20. Find the length of an edge of a cube that contains 100 cu. in.

21. The Great Pyramid of Egypt was originally made in the form of a regular pyramid, altitude $480\frac{3}{4}$ ft., and base a square 764 ft. on a side. Find in acres the area of the ground covered by the pyramid. Find in cubic yards the volume, and in square yards the lateral surface of the pyramid.

22. The mast of a ship is 80 ft. high, and the diameters of its ends are 4 ft. 6 in. and 2 ft., respectively. Find its value at 75 cents a cubic foot.

23. A spherical shot 6 in. in diameter is melted and cast into a cylinder 3 in. in diameter. What is the height of this cylinder?

24. A cylindrical pail 14 in. high holds 2 cu. ft. of water. What is the diameter of its base?

25. A regular pyramid 14 in. high has for its base an equilateral triangle 6 in. on a side. What is its volume?

26. A right prism 8 in. high has for its base a trapezoid whose altitude is 4 in., and whose parallel sides are 5 in. and 3 in., respectively. What is the volume and the total surface of the prism?

27. A rectangular room is 18 ft. long, 16 ft. wide, and 12 ft. high. What is the distance from the upper right-hand corner to the opposite lower left-hand corner?

28. A conical spire 40 ft. high has a base 15 ft. in diameter. Find the cost at 5 cents a square inch of gilding the spire.

594. Similar Figures. Figures that have the same shape are called *similar figures.*

595. *The corresponding lines of similar figures are proportional.*

596. *The surfaces of similar figures are to each other as the squares of their corresponding dimensions; and their volumes are to each other as the cubes of their corresponding dimensions.*

597. *The corresponding dimensions of similar figures are to each other as the square roots of their surfaces, or as the cube roots of their volumes.*

598. Examples. 1. A rectangle is 8 in. long and 6 in. broad. Find the length and the area of a similar rectangle whose breadth is 9 in.

SOLUTION. $6:9 = 8$ in. : required length.
Therefore, the required length $= 12$ in.
The area of the given rectangle $= 48$ sq. in.
Hence, 48 sq. in. : required area $= 6^2:9^2 = 4:9$.
Therefore, the required area $= 108$ sq. in.

2. The altitude of a right prism that contains 8 cu. ft. is 3 ft. Find the altitude of a similar right prism that contains 27 cu. ft.

SOLUTION. 3 ft. : required altitude $= \sqrt[3]{8} : \sqrt[3]{27} = 2:3$.
Hence, the required altitude $= 4\frac{1}{2}$ ft.

EXERCISE 143.

1. If the diameter of the moon is reckoned at 2000 mi., and that of the earth at 8000 mi., find the ratio of their surfaces and the ratio of their volumes.

2. If the diameters of two circles are 20 in. and 40 in., find the ratio of their circumferences, and of their surfaces.

3. If the areas of two circles are 8000 sq. in. and 36,000 sq. in., respectively, find the ratio of their diameters.

4. If the volumes of two spheres are 100 cu. in. and 1000 cu. in., respectively, find the ratio of their diameters.

5. If an ox 7 ft. in girth weighs 1500 lb., what will be the girth of a similar ox that weighs 2500 lb.?

6. The surface of a pyramid is 560 sq. in. What is the surface of a similar pyramid whose volume is 27 times as great?

7. The volume of a pyramid is 1331 cu. in. What is the volume of a similar pyramid whose surface is 4 times as great?

8. If a well-proportioned man 5 ft. 10 in. high weighs 160 lb., what should a man 6 ft. high weigh, to the nearest tenth of a pound? What should be the height, to the nearest tenth of an inch, of a man who weighs 210 lb.?

9. A three-gallon jug and a one-gallon jug are similar. Find to three decimals the ratio of their diameters.

10. Two hills have exactly the same shape; one is 900 ft. high, the other 1200 ft. Find the ratio of their surfaces, and also the ratio of their volumes.

11. A ball 3 in. in diameter weighs 4 lb.; another ball of the same metal weighs 9 lb. Find the diameter of the second ball to the nearest thousandth of an inch.

12. If Apollo's altar were a perfect cube 10 ft. on an edge, what would be the edge of a new cubical altar containing twice as much stone?

13. A man standing 40 ft. from a building 24 ft. wide observed that, when he closed one eye, the width of the building hid from view 90 rods of fence which was parallel to the width of the building. Find the distance from the eye of the observer to the fence.

14. A bushel measure and a peck measure are of the same shape. Find the ratio of their heights.

15. If the height and the diameter of a cylinder are both doubled, in what ratio is the volume altered?

CHAPTER XVII.

CONTINUED FRACTIONS AND SCALES OF NOTATION.

599. As in decimals we often require a result accurate to a specified number of places, so in common fractions we often require the most nearly accurate value of a ratio that can be expressed by a fraction with a denominator limited to a certain size.

600. Example. Find the most nearly accurate value of the ratio of the circumference of a circle to the diameter expressed by a fraction with a denominator less than 10; less than 100; less than 1000.

SOLUTION. The ratio 3.1416 is true to the nearest ten-thousandth. Reducing $\frac{1416}{10000}$ to its lowest terms, we have $\frac{177}{1250}$.

```
177)1250(7
    1239
      11)177(16
         176
           1)11(11
             11
```

Then, as in the margin, we divide the denominator by the numerator; the last divisor by the last remainder; and so on, as in finding the greatest common measure.

If, therefore, we divide both terms of the fraction $\frac{177}{1250}$ by the numerator, we have $\dfrac{1}{7\frac{11}{177}}$; and if we omit the fraction in the denominator, we have for the required ratio with a denominator less than 10, $3\frac{1}{7}$, or $\frac{22}{7}$.

If we put the fraction $\frac{11}{177}$ in the form of $\dfrac{1}{16\frac{1}{11}}$ and omit the fraction in the denominator, the ratio becomes

$$3\frac{1}{7\frac{1}{16}} = 3\tfrac{16}{113} = \tfrac{355}{113};$$

which shows that $\frac{22}{7}$ is the most nearly accurate value of the ratio expressed by a fraction with a denominator less than 100, and that $\frac{355}{113}$ is the most nearly accurate value of the ratio expressed by a fraction with a denominator less than 1000.

601. Continued Fractions. After the quotients have been found the results may be written in a fractional form as follows:

$$3+\cfrac{1}{7+\cfrac{1}{16+\cfrac{1}{11}}}.$$

Such a fraction is called a *continued fraction.*

602. To find the successive approximate values of a continued fraction we begin at the top and take first one, then two, then three, and so on, of its parts. Thus,

The first approximate value is 3.

The second is $3+\frac{1}{7}=\frac{22}{7}$.

The third is $3+\cfrac{1}{7+\cfrac{1}{16}}=3+\frac{16}{113}=\frac{355}{113}$.

The fourth is $3+\cfrac{1}{7+\cfrac{1}{16+\cfrac{1}{11}}}$

and this $=3+\frac{1}{7\frac{11}{177}}=3+\frac{177}{1250}=\frac{3927}{1250}$, or 3.1416.

603. In reducing the part of a continued fraction selected for an approximate value, we begin with the last fraction.

Thus, find the value of the continued fraction

$$\cfrac{1}{2+\cfrac{1}{3+\cfrac{1}{4+\cfrac{1}{5}}}}$$

$$\frac{1}{4\frac{1}{5}}=\frac{5}{21};\ \frac{1}{3\frac{5}{21}}=\frac{21}{68};\ \frac{1}{2\frac{21}{68}}=\frac{68}{157}.$$

EXERCISE 144.

1. Change $\frac{3}{11}$, $\frac{13}{75}$, $\frac{29}{127}$, $\frac{135}{64}$ to continued fractions.

2. Find the approximate values of $\frac{20}{37}$; $\frac{53}{47}$; $\frac{734}{851}$.

3. Find a series of fractions approximating to 0.236; 0.2361; 1.609.

4. Find a series of fractions approximating to 0.382; 1.732; 0.6253.

5. Find approximate values of $\frac{171}{457}$; $\frac{913}{757}$; $\frac{711}{813}$; $\frac{937}{113}$.

6. Find the proper fraction that, when changed to a continued fraction, will have 2, 3, 5, 6, 7 as quotients.

7. Find a series of fractions approximating to the ratio of the pound troy (5760 gr.) to the pound avoirdupois (7000 gr.).

8. Find a series of fractions approximating to the ratio of the side of a square to its diagonal; that ratio being 1 : 1.414214, nearly.

9. Find a series of fractions approximating to the ratio of the ar to the square chain, from the equality

$$1 \text{ ar} = 0.2471 \text{ sq. ch.}$$

10. Find a series of fractions approximating to the ratio of the weight of the 48-pound shot to the weight of the French shot of 24^{kg}.

11. If the mean diameter of the Earth is reckoned at 7912 mi., and that of Mars 4189 mi., find a series of fractions approximating to the ratio of the mean diameters of these two planets.

12. Find a series of fractions approximating to the ratio of a cubic yard to a cubic meter, from the equality

$$1 \text{ cu. yd.} = 0.76453^{\text{cbm}}.$$

13. Find a series of fractions approximating to the ratio of the kilometer to the mile, from the equality

$$1^{\text{m}} = 1.09362 \text{ yd.}$$

14. Find the proper fraction that, if changed to a continued fraction, will have as quotients 1, 7, 5, 2.

15. Find a series of fractions approximating to 0.5236; approximating to 0.7854.

16. Find a series of fractions approximating to the continued fraction that has as quotients 7, 2, 1, 2, 6, 4; that has as quotients 1, 2, 3, 4, 5, 6.

Scales of Notation.

604. The common mode of representing numbers is called the **common scale of notation,** and **10** is called its **radix** or **base.**

605. In the common or decimal scale every figure placed to the left of another represents *ten* times as much as if it were in the place of that other.

606. Instead of the radix number 10, any other integral number might be used as the base of a system of notation.

Thus, the number 6532 stands for:

In the scale of 10, $6 \times 10^3 + 5 \times 10^2 + 3 \times 10 + 2$.
In the scale of 8, $6 \times 8^3 + 5 \times 8^2 + 3 \times 8 + 2$.
In the scale of 7, $6 \times 7^3 + 5 \times 7^2 + 3 \times 7 + 2$.

607. A given number can be changed from one scale to another scale.

608. Examples. 1. Express 6532 in the scale of 6.

SOLUTION. The quotients and remainders of the successive divisions by 6 are as follows:

$$\begin{array}{r|l l}
6 & 6532 & \\
6 & 1088 & \text{remainder } 4. \\
6 & 181 & \text{remainder } 2. \\
6 & 30 & \text{remainder } 1. \\
 & 5 & \text{remainder } 0.
\end{array}$$

Therefore, 6532 expressed in the scale of 6 is 50,124.

2. Change 50,124 from the scale of 6 to the scale of 8.

SOLUTION.

```
8|50124
 8|3440  remainder 4.
  8|250  remainder 0.
   8|20  remainder 6.
      1  remainder 4.
```

Therefore, the number required is 14,604.

Since 50,124 is in the scale of 6, each figure has *six* times the value it would have one place to the right. Hence, at the beginning we have to divide $6 \times 5 + 0$ by 8, and we get 3 for the quotient and 6 for the remainder. The next partial dividend is $6 \times 6 + 1$, or 37, and this divided by 8 gives 4 for the quotient and 5 for the remainder. The next partial dividend is $6 \times 5 + 2$, or 32, and this divided by 8 gives 4 for the quotient and 0 for the remainder; and so on.

3. Change 14,604 from the scale of 8 to the scale of 10.

SOLUTION.

```
10|14604
 10|1215  remainder 2.
   10|101  remainder 3.
        6  remainder 5.
```

Therefore, the required number is 6532.

4. Add 56,432 and 15,646 (scale of 7).

```
 56432
 15646
------
105411
```

SOLUTION. The process differs from that in the decimal scale only in that when a sum greater than *seven* is reached, we divide by *seven* (not ten), set down the remainder, and add the quotient with the next column.

5. Subtract 34,561 from 61,235 (scale of 8).

```
61235
34561
-----
24454
```

SOLUTION. When the number of any order of units in the minuend is less than the number of the corresponding order in the subtrahend, we increase the number in the minuend by eight instead of ten as in the common scale.

6. Multiply 5732 by 428 (scale of 9).

```
   5732
    428
-------
  51477
 12564
25238
-------
2712127
```

SOLUTION. We divide each partial product by *nine*, set down the remainder, and add the quotient to the next partial product.

7. Divide 2,712,127 by 5732 (scale of 9).

```
           428
5732 ) 2712127
       25238
       -----
        17722
        12564
        -----
         51477
         51477
         -----
```

The operations of multiplication and subtraction involved in this problem are precisely the same as in the decimal notation. The only difference is that the radix number is 9 instead of 10.

Exercise 145.

Change 4852 of the common scale to:

1. The scale of 7.
2. The scale of 2.
3. The scale of 9.
4. The scale of 3.
5. The scale of 6.
6. The scale of 5.
7. The scale of 8.
8. The scale of 4.

Change:

9. 54,231 of the scale of 6 to the common scale.
10. 54,231 of the scale of 7 to the common scale.
11. 54,231 of the scale of 8 to the common scale.
12. 54,231 of the scale of 9 to the common scale.

Perform the following arithmetical processes:

13. Add 67,814; 76,406; 88,718 (scale of 9).
14. Add 44,231; 13,432; 12,304 (scale of 5).
15. Subtract 77,614 from 114,672 (scale of 8).
16. Subtract 52,515 from 112,252 (scale of 6).
17. Multiply 14,612 by 6502 (scale of 7).
18. Multiply 72,645 by 46,723 (scale of 8).
19. Divide 162,542 by 6522 (scale of 7).
20. Divide 468,722 by 5432 (scale of 9).

CHAPTER XVIII.

SERIES.

609. Series. A succession of numbers that proceed according to some fixed law is called a *series*. The successive numbers are called the *terms* of the series.

610. A series that ends at some particular term is called a **finite series.** A series that continues without end is called an **infinite series.**

611. The number of different kinds of series is unlimited; in this chapter we shall consider only Arithmetical Series, Geometrical Series, and Harmonical Series.

Arithmetical Progression.

612. A series of numbers that increase or decrease by a *common difference* is called an **Arithmetical Series** or an **Arithmetical Progression.**

Thus, the numbers 5, 8, 11, 14 form an arithmetical progression with a common difference 3; and the numbers 12, 10, 8, 6 form an arithmetical progression with a common difference 2.

613. In the *increasing* arithmetical progression

1st	2d	3d	4th	5th	6th
2,	5,	8,	11,	14,	17,

we find any term, as the 6th, by adding to the first term the product of the common difference by a number one less than the number of the term: $2 + (3 \times 5)$, or 17.

In the *decreasing* arithmetical progression

1st	2d	3d	4th	5th	6th	7th
50,	46,	42,	38,	34,	30,	26,

we find any term, as the 7th, by subtracting from the first term the product of the common difference by a number one less than the number of the term : 50 — (4 × 6), or 26. Hence,

614. To Find Any Term of an Arithmetical Progression,

Multiply the common difference by a number one less than the number of the required term. Add this product to the first term if the series is an increasing series; subtract this product from the first term if the series is a decreasing series.

Exercise 146.

1. Find the seventh term of the series 3, 5, 7, etc.
2. Find the fifteenth term of the series 2, 7, 12, etc.
3. Find the sixth term of the series 2, $2\frac{5}{7}$, $3\frac{3}{7}$, etc.
4. Find the twentieth term of the series 2, $3\frac{1}{4}$, $4\frac{1}{2}$, etc.
5. Find the seventh term of the series 21, 19, 17, etc.
6. Find the twelfth term of the series 18, $17\frac{1}{3}$, $16\frac{2}{3}$, etc.
7. If the first term of a series is 5, and the common difference $2\frac{1}{4}$, find the thirteenth and eighteenth terms.
8. If the fourth term of a series is 18, and the common difference 3, find the seventh and eleventh terms.
9. If the fifth term of a decreasing series is 52, and the common difference $3\frac{1}{2}$, find the twelfth and eighteenth terms.
10. If the fourth term of a series is 14, and the twelfth term 38, what is the common difference?

Hint. The difference between the fourth and twelfth terms is evidently eight times the common difference.

Find the common difference in a series :

11. If the fourth term is 12 and the seventh term 27.
12. If the first term is 20 and the fourth term 40.
13. If the first term is 2 and the eleventh term 20.
14. If the third term is 7 and the eighth term $12\frac{1}{2}$.
15. If the first term is 1 and the fourth term 19.

615. The sum of seven terms of the series 3, 5, 7, etc., is

$$\begin{array}{lc} & 3+\ 5+\ 7+\ 9+11+13+15, \\ \text{in reverse order is} & \underline{15+13+11+\ 9+\ 7+\ 5+\ 3.} \\ \text{Hence, \textit{twice the sum} is} & 18+18+18+18+18+18+18, \end{array}$$

or 7×18.

Therefore, the *sum* is $\frac{1}{2}(7 \times 18)$, or 63.

Here 7 is the number of terms, and 18 is the *sum* of the first and last terms. Hence,

616. To Find the Sum of the Terms of an Arithmetical Progression,

Multiply one half the sum of the first and last terms by the number of terms.

Thus, the sum of eight terms of the series whose first term is 3 and last term 38 is $8 \times \frac{1}{2}(3 + 38) = 164$.

Exercise 147.

1. Find the sum of 1, 5, 9, etc., to twenty terms.
2. Find the sum of 4, $5\frac{1}{2}$, 7, etc., to eight terms.
3. Find the sum of 8, $7\frac{5}{8}$, $7\frac{1}{4}$, etc., to sixteen terms.
4. Find the sum of 20, $18\frac{1}{4}$, $16\frac{1}{2}$, etc., to seven terms.
5. Find the sum of the first twenty natural numbers.
6. Find the sum of the natural numbers from 37 to 53 both inclusive.
7. Find the sum of a series of thirty terms, if the first term is 21 and the last 59.
8. Find the sum of the series whose first two terms are 3 and 9, and the last term 75.
9. Find the sum of a series of twenty terms whose third and fifth terms are 10 and 15, respectively.
10. A body falls through a space of $16\frac{1}{12}$ ft. in the first second of its fall, and in each succeeding second $32\frac{1}{6}$ ft. more than in the second just before. How far will a stone fall in the seventh second? How far in seven seconds?

11. A travels 8 miles the first day, 11 miles the second, 14 miles the third, and so on, and overtakes in 17 days B who started at the same time, and traveled at a uniform rate. What is B's rate per day?

12. In a potato race 100 potatoes are placed in a straight line 3 ft. distant from each other. A boy, starting from a basket 3 ft. from the first potato, is required to pick them up one by one and carry them to the basket. To finish the race, how far must the boy run?

13. How many times a day does a clock strike that strikes the hours only?

14. A body falls through a space of 4.9^m in the first second of its fall, and in each succeeding second 9.8^m more than in the second just before. A stone dropped from a balloon was 35 seconds in reaching the ground. How high was the balloon?

Geometrical Progression.

617. A series of numbers each term of which after the first is obtained by multiplying the preceding term by a constant multiplier is called a **Geometrical Series,** or a **Geometrical Progression.** The constant multiplier is called the *ratio.*

Thus, 3, 6, 12, 24 form a geometrical progression with a ratio 2; and 9, 3, 1, $\frac{1}{3}$ form a geometrical progression with a ratio $\frac{1}{3}$.

618. In the geometrical progression

1st	2d	3d	4th	5th	6th
2,	6,	18,	54,	162,	486,

the second term is 2×3; the third term is 2×3^2; the fourth term is 2×3^3; the fifth term is 2×3^4; and so on. Hence,

619. To Find Any Term of a Geometrical Progression,

Multiply the first term by that power of the ratio that is one less than the number of the term required.

620. If any two consecutive terms of a geometrical progression are known, the ratio may be found by dividing the second of these terms by the first.

If two terms not consecutive are known, the ratio may be found as in the following

Example. Find the ratio in a geometrical progression if the second and sixth terms are 5 and 80, respectively.

SOLUTION. $\quad$ 6th term = 2d term $\times$ (ratio)^4.

Therefore, $\quad$ $(\text{ratio})^4 = \frac{80}{5} = 16$,

and $\quad$ $\text{ratio} = \sqrt[4]{16} = 2$.

EXERCISE 148.

1. Find the eighth term of the series 2, 6, 18, etc.
2. Find the fifth term of the series 8, 4, 2, etc.
3. Find the seventh term of the series 2, 3, $4\frac{1}{2}$, etc.
4. Find the sixth term of the series 4, $2\frac{2}{3}$, $1\frac{7}{9}$, etc.
5. Find the eighth term of the series 4, 10, 25, etc.
6. Find the fifth term of the series $\frac{1}{4}$, $\frac{1}{16}$, $\frac{1}{64}$, etc.
7. Find the ninth term of the series 4, 2, 1, etc.
8. Find the sixth term of the series 6, 9, $13\frac{1}{2}$, etc.
9. Write the first six terms of the geometrical series whose fifth and sixth terms are 112 and 224, respectively.
10. The seventh and ninth terms of a geometrical series are 100 and 144, respectively. Find the twelfth term.
11. A capital of $1000 is increased by $\frac{1}{10}$ of itself each year. What will it be at the beginning of the fifth year?
12. A capital of $1000 is increased by $\frac{8}{100}$ of itself each year. What will it be at the beginning of the sixth year?

621. In the geometrical progression 4, 12, 36, 108, 324, etc., the sum of five terms is $4 + 12 + 36 + 108 + 324$.

If we multiply this sum by the ratio 3, and from the product subtract the sum of the five terms, we shall have *twice the sum of the terms.* That is,

$$\begin{array}{r} 12+36+108+324+972 \\ \underline{4+12+36+108+324} \\ 972-4. \end{array}$$

Therefore, the sum $= \dfrac{972-4}{2}$.

The numerator is the difference between the product of the last term by the ratio and the first term; and the denominator is the ratio minus 1. Hence,

622. To Find the Sum of the Terms of a Geometrical Progression,

Multiply the last term by the ratio, and subtract the first term from the product. Divide the remainder by the ratio minus 1.

If the ratio is less than 1,

Multiply the last term by the ratio, and subtract the product from the first term. Divide the remainder by 1 minus the ratio.

EXERCISE 149.

1. Find the sum of 2, 6, 18, etc., to six terms.
2. Find the sum of 1, 2, 4, etc., to nine terms.
3. Find the sum of 3, 9, 27, etc., to five terms.
4. Find the sum of 2, 3, $4\frac{1}{2}$, etc., to eight terms.
5. Find the sum of 1, $\frac{1}{3}$, $\frac{1}{9}$, etc., to eight terms.
6. Find the sum of 1, $\frac{1}{2}$, $\frac{1}{4}$, etc., to ten terms.
7. Find the sum of $\frac{1}{2}$, $\frac{1}{3}$, $\frac{2}{9}$, etc., to eight terms.
8. Find the sum of the first six terms of the series whose first term is 3 and ratio 5.
9. Find the sum of the first eight terms of the series whose first term is 3 and ratio $\frac{1}{3}$.
10. A man saved in one year \$64, and in each succeeding year, for 9 years more, $1\frac{1}{2}$ times as much as in the preceding year. Find the whole amount he saved.

623. If we represent the first term by a, the last term by l, the ratio by r, the number of terms by n, and the sum of the terms by s, we have, if the ratio is less than 1 (§ 622),

$$s=\frac{a-r\times l}{1-r}.$$

Now, since $l=a\times r^{n-1}$, $rl=ar^n$; and we have

$$s=\frac{a-ar^n}{1-r}=\frac{a\times(1-r^n)}{1-r}.$$

Since r is less than 1, r^n becomes smaller as n becomes larger; and when n is too large to be counted, r^n becomes too small to be considered. We then have

$$s=\frac{a}{1-r}.$$ Hence,

624. To Find the Sum of the Terms of an Infinite, Decreasing Geometrical Progression,

Divide the first term by 1 minus the ratio.

EXERCISE 150.

Find the sum of the infinite series :

1. $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \cdots\cdots$.	**6.** 0.212121 ·····.
2. $\frac{2}{3}, \frac{4}{9}, \frac{8}{27}, \cdots\cdots$.	**7.** 0.9999 ·····.
3. $\frac{1}{4}, \frac{1}{16}, \frac{1}{64}, \cdots\cdots$.	**8.** 0.232323 ·····.
4. $\frac{1}{5}, \frac{1}{25}, \frac{1}{125}, \cdots\cdots$.	**9.** 0.36848484 ·····.
5. 0.171717 ·····.	**10.** 0.15272727 ·····.

625. A series is called a **Harmonical Series, or a Harmonical Progression,** if the *reciprocals* of its terms form an *arithmetical series.*

Thus, the numbers 1, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$ form a harmonical progression, since 1, 2, 3, 4, the reciprocals of the terms, are in arithmetical progression.

626. Questions involving a harmonical progression may be solved by writing the reciprocals of the terms so as to form an arithmetical progression.

CHAPTER XIX.

COMMON LOGARITHMS.

627. In the common system of notation the expression of numbers is founded on their relation to 10.

Thus, 5434 indicates that this number consists of 10^3 five times, 10^2 four times, 10 three times, and four units.

628. But any number may be expressed, exactly or approximately, as a power of 10.

Thus, 5434 is greater than 10^3 and less than 10^4, and is expressed approximately by $10^{3.7351}$.

629. When a number is expressed as a power of 10, the *exponent* of 10 is called the **logarithm** of that number.

Thus, if $10^{3.7351} = 5434$, the logarithm of $5434 = 3.7351$, a statement that is generally written $\log 5434 = 3.7351$.

630. By division $\dfrac{10^2}{10^2} = 1$; and by the rule for exponents (§ 108) $\dfrac{10^2}{10^2} = 10^{2-2} = 10^0$. Therefore, $10^0 = 1$.

$$
\begin{array}{llll}
\text{Since} & 10^0 = 1, & & \log 1 = 0, \\
 & 10^1 = 10, & & \log 10 = 1, \\
 & 10^2 = 100, & & \log 100 = 2, \\
 & 10^3 = 1000, & & \log 1000 = 3, \text{ and so on.}
\end{array}
$$

631. It is often convenient to write the reciprocals of powers of 10 in an integral form; $\dfrac{1}{10}$ is written 10^{-1}; $\dfrac{1}{10^2}$ is written 10^{-2}; $\dfrac{1}{10^3}$ is written 10^{-3}; and so on.

NOTE. An exponent with the *minus sign* prefixed is called a *negative* exponent; and an exponent with the *plus sign* prefixed, or without any sign, is called a *positive* exponent.

632. Since $10^{-1} = 0.1$, $\log 0.1 = -1$,
$10^{-2} = 0.01$, $\log 0.01 = -2$,
$10^{-3} = 0.001$, $\log 0.001 = -3$, and so on.

633. It is evident, therefore, that the logarithms of all numbers between

1 and 10	will lie between	0 and	1,
10 and 100	will lie between	1 and	2,
100 and 1000	will lie between	2 and	3,
1 and 0.1	will lie between	0 and	-1,
0.1 and 0.01	will lie between	-1 and	-2,
0.01 and 0.001	will lie between	-2 and	-3, and so on.

634. If a number is less than 1, its logarithm is negative (§ 633), but is written in such a form that its *decimal part* is always *positive.*

Thus, $\log 0.0284 = -(1.5467) = (-1) + (-.5467) = -2 + .4533$.

If the integral part of a logarithm is negative, the minus sign is written over the integral part.

Thus, $\log 0.0284$ is written $\bar{2}.4533$.

635. Every logarithm, therefore, consists of two parts: a *positive* or *negative* integral number called the **characteristic,** and a *positive* decimal called the **mantissa.**

Thus, in the logarithm 2.4533, the integral number 2 is the characteristic, and the decimal .4533 is the mantissa. In the logarithm $\bar{2}.4533$, the negative integral number -2 is the characteristic and the positive decimal .4533 is the mantissa.

636. If a logarithm has a negative characteristic, it is customary to change its form by *adding* 10 or a multiple of 10 to the characteristic, and *indicating* the subtraction of the same number from the result.

Thus, the logarithm $\bar{2}.4533$ is written $8.4533 - 10$, and the logarithm $\overline{13}.4533$ is written $7.4533 - 20$.

637. From an inspection of § 633, we deduce the following rules for writing the characteristic of a logarithm :

RULE 1. *If the given number is greater than 1, make the characteristic of its logarithm one less than the number of figures to the left of the decimal point in the number.*

RULE 2. *If the given number is less than 1, make the characteristic of its logarithm negative, and one more than the number of zeros between the decimal point and the first significant figure of the given number.*

Thus, the characteristic of log 7849.27 is 3; the characteristic of log 0.037 is -2, or $8.0000-10$.

638. The *mantissa* of the logarithm of any number depends only upon the sequence of the digits of the number, and is unchanged so long as the *sequence of the digits* remains the same.

For changing the position of the decimal point in a number is equivalent to multiplying or dividing the number by a power of 10. Its logarithm, therefore, will be increased by or diminished by the *exponent* of that power of 10; and since this exponent is integral, the *mantissa*, or decimal part of the logarithm, will be unchanged.

Thus,

$$27{,}196 = 10^{4.4345}, \qquad 2.7196 = 10^{0.4345},$$
$$2719.6 = 10^{3.4345}, \qquad 0.27196 = 10^{9.4345-10},$$
$$27.196 = 10^{1.4345}, \qquad 0.0027196 = 10^{7.4345-10}.$$

639. A Four-place Table of Logarithms. A four-place table of logarithms is given on pages 350 and 351. This table contains the *mantissas* of the logarithms of all integral numbers under 1000, *the decimal point being omitted.*

NOTE. Tables containing logarithms to more decimal places can be procured, but this table will serve for many practical uses, and will enable the student to understand the use of five-place, seven-place, and ten-place logarithms in work that requires greater accuracy.

640. In working with a four-place table, numbers corresponding to logarithms will be correct to *four significant digits.*

To Find the Logarithm of a Given Number.

641. The characteristic of the logarithm is determined by the rules of § 637.

642. If the given number consists of a *single digit*, as 4, 8, etc., the mantissa of its logarithm is the same as the mantissa of the logarithm of 40, 80, etc.

643. If the given number contains *two digits*, it is in the column headed **N**, and the mantissa of its logarithm is on the same line, and in the column headed **0**.

Thus, $\log 28 = 1.4472$, $\log 0.086 = 8.9345 - 10$,
$\log 40 = 1.6021$, $\log 7 = 0.8451$.

644. If the given number contains *three digits*, or three significant digits followed by one or more zeros, the first two digits of the number are in the column headed **N**, and the third at the top of the page in the line containing the figures **0**, **1**, **2**, etc. The mantissa of its logarithm is in the column headed by the third figure and on the same line with the first two figures.

Thus, $\log 742 = 2.8704$, $\log 84{,}100 = 4.9248$,
$\log 6090 = 3.7846$, $\log 0.00261 = 7.4166 - 10$.

645. If the given number contains *four or more digits*, the mantissa of its logarithm is found as in the following

Examples. 1. Find the logarithm of 2034.

SOLUTION. The required mantissa is (§ 638) the same as the mantissa for 203.4; hence, it is found by adding to the mantissa for 203 four tenths of the difference between the mantissas for 203 and 204.

The mantissa for 203 is 3075; and for 204 is 3096.

The difference between the mantissas for 203 and 204 is 21, and 0.4 of $21 = 8$. Hence, 8 must be added to 3075.

Therefore, the mantissa for 203.4 is $3075 + 8 = 3083$.

Therefore, $\log 2034 = 3.3083$.

2. Find the logarithm of 0.0015764.

SOLUTION. The required mantissa is (§ 638) the same as the mantissa for 157.64; hence, we add to the mantissa for 157 sixty-four

hundredths of the difference between the mantissas for 157 and 158.

The mantissa for 157 is 1959; and for 158 is 1987.

The difference between the mantissas for 157 and 158 is 28, and 0.64 of 28 = 18. Hence, 18 must be added to 1959.

Therefore, the mantissa for 157.64 is 1959 + 18 = 1977.

Therefore, log 0.0015764 = 7.1977 − 10.

NOTE. When the fraction of a unit in the part to be added to the mantissa for three figures is less than 0.5, it is neglected; when it is 0.5 or more, it is taken as one unit.

EXERCISE 151.

Find the logarithm of:

1. 70.	6. 6897.	11. 77,860.	16. 5.0009.
2. 101.	7. 9901.	12. 30,127.	17. 0.3769.
3. 333.	8. 4389.	13. 730.84.	18. 0.070707.
4. 3491.	9. 1111.	14. 0.008765.	19. 0.03723.
5. 1866.	10. 58,343.	15. 8.0808.	20. 98.871.

646. From § 633, we deduce the following rules for writing the decimal point in the *antilogarithm* of a logarithm; that is, in the number corresponding to a logarithm:

RULE 1. *If the characteristic of the given logarithm is positive, make the number of figures in the integral part of its antilogarithm one more than the number of units in the characteristic of the logarithm.*

RULE 2. *If the characteristic of the given logarithm is negative, make the number of zeros between the decimal point and the first significant figure of its antilogarithm one less than the number of units in the characteristic of the logarithm.*

Thus, the number corresponding to the logarithm 4.7542 contains five figures in its integral part; the decimal corresponding to the logarithm 7.2816 − 10, that is $\bar{3}.2816$, contains two zeros between the decimal point and the first significant figure.

N	0	1	2	3	4	5	6	7	8	9
10	0000	0043	0086	0128	0170	0212	0253	0294	0334	0374
11	0414	0453	0492	0531	0569	0607	0645	0682	0719	0755
12	0792	0828	0864	0899	0934	0969	1004	1038	1072	1106
13	1139	1173	1206	1239	1271	1303	1335	1367	1399	1430
14	1461	1492	1523	1553	1584	1614	1644	1673	1703	1732
15	1761	1790	1818	1847	1875	1903	1931	1959	1987	2014
16	2041	2068	2095	2122	2148	2175	2201	2227	2253	2279
17	2304	2330	2355	2380	2405	2430	2455	2480	2504	2529
18	2553	2577	2601	2625	2648	2672	2695	2718	2742	2765
19	2788	2810	2833	2856	2878	2900	2923	2945	2967	2989
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598
23	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784
24	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962
25	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133
26	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298
27	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456
28	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609
29	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757
30	4771	4786	4800	4814	4829	4843	4857	4871	4886	4900
31	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038
32	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172
33	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302
34	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428
35	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551
36	5563	5575	5587	5599	5611	5623	5635	5647	5658	5670
37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786
38	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899
39	5911	5922	5933	5944	5955	5966	5977	5988	5599	6010
40	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117
41	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222
42	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325
43	6335	5345	6355	6365	6375	6385	6395	6405	6415	6425
44	6435	6444	6454	6464	6474	6484	6493	6503	6513	6522
45	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618
46	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712
47	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803
48	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893
49	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067
51	7076	7084	7093	7101	7110	7118	7126	7135	7143	7152
52	7160	7168	7177	7185	7193	7202	7210	7218	7226	7235
53	7243	7251	7259	7267	7275	7284	7292	7300	7308	7316
54	7324	7332	7340	7348	7356	7364	7372	7380	7388	7396

N	0	1	2	3	4	5	6	7	8	9
55	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474
56	7482	7490	7497	7505	7513	7520	7528	7536	7543	7551
57	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627
58	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701
59	7709	7716	7723	7731	7738	7745	7752	7760	7767	7774
60	7782	7789	7796	7803	7810	7818	7825	7832	7839	7846
61	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917
62	7924	7931	7938	7945	7952	7959	7966	7973	7980	7987
63	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055
64	8062	8069	8075	8082	8089	8096	8102	8109	8116	8122
65	8129	8136	8142	8149	8156	8162	8169	8176	8182	8189
66	8195	8202	8209	8215	8222	8228	8235	8241	8248	8254
67	8261	8267	8274	8280	8287	8293	8299	8306	8312	8319
68	8325	8331	8338	8344	8351	8357	8363	8370	8376	8382
69	8388	8395	8401	8407	8414	8420	8426	8432	8439	8445
70	8451	8457	8463	8470	8476	8482	8488	8494	8500	8506
71	8513	8519	8525	8531	8537	8543	8549	8555	8561	8567
72	8573	8579	8585	8591	8597	8603	8609	8615	8621	8627
73	8633	8639	8645	8651	8657	8663	8669	8675	8681	8686
74	8692	8698	8704	8710	8716	8722	8727	8733	8739	8745
75	8751	8756	8762	8768	8774	8779	8785	8791	8797	8802
76	8808	8814	8820	8825	8831	8837	8842	8848	8854	8859
77	8865	8871	8876	8882	8887	8893	8899	8904	8910	8915
78	8921	8927	8932	8938	8943	8949	8954	8960	8965	8971
79	8976	8982	8987	8993	8998	9004	9009	9015	9020	9025
80	9031	9036	9042	9047	9053	9058	9063	9069	9074	9079
81	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133
82	9138	9143	9149	9154	9159	9165	9170	9175	9180	9186
83	9191	9196	9201	9206	9212	9217	9222	9227	9232	9238
84	9243	9248	9253	9258	9263	9269	9274	9279	9284	9289
85	9294	9299	9304	9309	9315	9320	9325	9330	9335	9340
86	9345	9350	9355	9360	9365	9370	9375	9380	9385	9390
87	9395	9400	9405	9410	9415	9420	9425	9430	9435	9440
88	9445	9450	9455	9460	9465	9469	9474	9479	9484	9489
89	9494	9499	9504	9509	9513	9518	9523	9528	9533	9538
90	9542	9547	9552	9557	9562	9566	9571	9576	9581	9586
91	9590	9595	9600	9605	9609	9614	9619	9624	9628	9633
92	9638	9643	9647	9652	9657	9661	9666	9671	9675	9680
93	9685	9689	9694	9699	9703	9708	9713	9717	9722	9727
94	9731	9736	9741	9745	9750	9754	9759	9763	9768	9773
95	9777	9782	9786	9791	9795	9800	9805	9809	9814	9818
96	9823	9827	9832	9836	9841	9845	9850	9854	9859	9863
97	9868	9872	9877	9881	9886	9890	9894	9899	9903	9908
98	9912	9917	9921	9926	9930	9934	9939	9943	9948	9952
99	9956	9961	9965	9969	9974	9978	9983	9987	9991	9996

To Find the Antilogarithm of a Given Logarithm.

647. If the given mantissa can be found in the table, the first two figures of the required number are in the column headed **N** on the same line with the mantissa, and the third figure is at the top of the column that contains the mantissa.

The position of the decimal point is determined by the characteristic (§ 646).

Thus, the number corresponding to the logarithm

2.9736 is 941, 0.8169 is 6.56,
6.0899 is 1,230,000, 7.8739 − 10 is 0.00748.

648. If the given mantissa cannot be found in the table, find in the table the two adjacent mantissas between which the given mantissa lies, and the three figures corresponding to the smaller of these two mantissas are the first three significant figures of the required number. The fourth figure is found as in the following

Examples. 1. Find the antilogarithm of the logarithm 3.7936.

Solution. The two adjacent mantissas of the table between which the given mantissa 7936 lies are 7931 and 7938. The corresponding numbers are 621 and 622. The smaller of these, 621, contains the first three significant figures of the required number.

The difference between the two adjacent mantissas is 7, and the difference between the corresponding numbers is 1.

The difference between the smaller of the two adjacent mantissas, 7931, and the given mantissa, 7936, is 5. Therefore, the number to be annexed to 621 is $\frac{5}{7}$ of $1 = 0.71$, and the fourth significant figure of the required number is 7.

Hence, the required number is 6217.

2. Find the antilogarithm of the logarithm 7.3884 − 10.

Solution. The two adjacent mantissas of the table between which the given mantissa 3884 lies are 3874 and 3892. The corresponding numbers are 244 and 245. The smaller of these, 244, contains the first three significant figures of the required number.

The difference between the two adjacent mantissas is 18, and the difference between the corresponding numbers is 1.

The difference between the smaller of the two adjacent mantissas, 3874, and the given mantissa, 3884, is 10. Therefore, the number to be annexed to 244 is $\frac{10}{18}$ of $1 = 0.55$, and the fourth significant figure of the required number is 6.

Hence, the required number is 0.002446.

Exercise 152.

Find the antilogarithms of the following logarithms:

1. 3.9017.
2. 1.2076.
3. 0.4442.
4. 1.0090.
5. 4.8697.
6. 1.9214.
7. 2.9850.
8. 4.5388.
9. 0.8550.
10. 9.9992 − 10.
11. 7.0016 − 10.
12. 9.2618 − 10.
13. 8.7324 − 10.
14. 9.5555 − 10.
15. 6.0216 − 10.
16. 7.0080 − 10.
17. 8.2361 − 10.
18. 9.4513 − 10.

649. Since every factor of a product may be expressed as a power of ten (§ 628),

The logarithm of a product is equal to the sum of the logarithms of its factors (§ 69).

650. Example. Find by logarithms the product of $908.4 \times 0.05392 \times 2.117$.

Solution.

$$\begin{aligned} \log 908.4 &= 2.9583 \\ \log 0.05392 &= 8.7318 - 10 \\ \log 2.117 &= \underline{0.3257} \\ &\ 2.0158 = \log 103.7. \end{aligned}$$

Therefore, the required product is 103.7.

Exercise 153.

Find by logarithms the value of:

1. 948.22×0.4387.
2. 1.9704×0.0786.
3. 380.25×0.00673.
4. 270.05×0.0087.
5. 11.163×0.3333.
6. 777.78×0.0787.

7. 216.21 × 0.76312.
8. 0.56127 × 1.2312.
9. 0.86311 × 56.371.
10. 59.795 × 0.7955.
11. 2.6537 × 0.2313.
12. 37.587 × 12.371.
13. 89.313 × 2.3781.
14. 9.1765 × 0.089.
15. 4786 × 5.4187 × 0.00218 × 0.8652.
16. 3.1416 × 7.77 × 184 × 0.01865.
17. 0.7854 × 129.6 × 63.45 × 0.0021.
18. 1842.65 × 9.876 × 0.843 × 0.0265.
19. 12.48 × 44.63 × 32.78 × 0.004587.
20. 0.9876 × 0.8765 × 0.7654 × 0.6543.

651. Any required power of a given power of a number is found by multiplying the exponent of the given power by the exponent of the required power.

Thus, the cube of $10^2 = 10^{2\times 3} = 10^6$; the fifth power of $10^4 = 10^{4\times 5} = 10^{20}$. Hence,

The logarithm of a power of a number is found by multiplying the logarithm of the number by the exponent of the power.

652. Example. Find by logarithms the cube of 0.0497.

SOLUTION.

$$\begin{aligned} \log 0.0497 &= 8.6964 - 10 \\ & \quad\;\; \underline{3} \\ \log 0.0497^3 &= 6.0892 - 10 = \log 0.0001228. \end{aligned}$$

Therefore, $0.0497^3 = 0.0001228$.

NOTE. The real product is 26.0892 − 30. Removing 20 from the 26 and − 20 from the − 30, we have 6.0892 − 10.

EXERCISE 154.

Find by logarithms the value of :

1. 5.06^3.
2. 2.501^5.
3. 1.716^7.
4. 1.178^{10}.
5. 7.6821^6.
6. 0.7685^6.
7. 0.9611^3.
8. 0.0231^2.
9. 0.8567^3.
10. 0.5438^5.
11. 2.861415^4.
12. 3.79125^6.
13. 0.021875^5.
14. 0.87152^7.
15. 0.95956^3.

653. Any required root of a given power of a number is found by dividing the exponent of the power by the index of the root.

Thus, the sixth root of 10^7, that is, $\sqrt[6]{10^7} = 10^{\frac{7}{6}}$. Hence,

The logarithm of a root of a number is found by dividing the logarithm of the number by the index of the root.

654. Examples. 1. Find the cube root of 271.

SOLUTION.	$\log 271 =$	2.4330
Divide by 3,		$3\overline{)2.4330}$
		$0.8110 = \log 6.471.$

Therefore, $\sqrt[3]{271} = 6.471.$

2. Find the fifth root of 0.4654.

SOLUTION.	$\log 0.4654 =$	$9.6679 - 10$
Add,		$40. \quad -40$
Divide by 5,		$5\overline{)49.6679 - 50}$
		$9.9336 - 10 = \log 0.8582.$

Therefore, $\sqrt[5]{0.4654} = 0.8582.$

If the given number is less than 1, its logarithm, when written in the ordinary form, will have a -10 annexed. In this case, the form of the logarithm should be changed as in Example 2, so that, when the logarithm is divided by the index of the root, the negative number of the quotient shall be -10.

EXERCISE 155.

Find by logarithms the value of :

1. $13^{\frac{1}{3}}$.
2. $29^{\frac{1}{3}}$.
3. $471^{\frac{1}{4}}$.
4. $288^{\frac{1}{6}}$.
5. $1019^{\frac{1}{7}}$.
6. $1281^{\frac{1}{8}}$.
7. $1862^{\frac{1}{9}}$.
8. $879^{\frac{1}{10}}$.
9. $0.609^{\frac{1}{4}}$.
10. $0.8716^{\frac{1}{3}}$.
11. $0.021641^{\frac{1}{6}}$.
12. $0.9825^{\frac{1}{7}}$.
13. $0.42184^{\frac{1}{8}}$.
14. $0.02187^{\frac{1}{9}}$.
15. $93.73^{\frac{1}{3}}$.
16. $21.97^{\frac{5}{6}}$.
17. $7.935^{\frac{5}{7}}$.
18. $0.815^{\frac{3}{4}}$.
19. $2.8145^{\frac{3}{5}}$.
20. $0.04165^{\frac{2}{11}}$.
21. $4,516,298^{\frac{1}{12}}$.

655. Since a quotient is equal to the dividend divided by the divisor,

The logarithm of a quotient is equal to the logarithm of the dividend minus the logarithm of the divisor (§ 108).

656. Example. Divide 905.6 by 38.45.

SOLUTION.

$$\begin{aligned} \log 905.6 &= 2.9569 \\ \log 38.45 &= \underline{1.5849} \\ &\ 1.3720 = \log 23.55. \end{aligned}$$

Therefore, $905.6 \div 38.45 = 23.55$.

657. The Cologarithm of a Number. The logarithm of the reciprocal of a number is called the *cologarithm* of the number.

Thus, $\quad \text{colog } 30 = \log \frac{1}{30} = \log 1 - \log 30.$
Since $\log 1 = 0$, $\log 1 - \log 30 = -\log 30$.

Now, in the expression $-\log 30$, the minus sign affects the entire logarithm, both the characteristic and the mantissa. To avoid a negative mantissa, we can substitute for $-\log 30$ its equivalent

$$(10 - \log 30) - 10.$$

Therefore, the cologarithm of a number is found by subtracting the logarithm of the number from 10, and annexing -10 to the remainder.

NOTE. The best way to perform the subtraction is to begin at the left, subtract from 9 each figure except the last significant figure, and subtract the last significant figure from 10.

658. Examples. **1.** Find the cologarithm of 4007.

SOLUTION.

$$\begin{aligned} &\ 10. \qquad -10 \\ \log 4007 &= \underline{3.6028 \qquad\quad} \\ \text{colog } 4007 &= 6.3972 - 10 \end{aligned}$$

2. Find the cologarithm of 0.004007.

SOLUTION.

$$\begin{aligned} &\ 10. \qquad -10 \\ \log 0.004007 &= \underline{7.6028 - 10} \\ \text{colog } 0.004007 &= 2.3972 \end{aligned}$$

659. By using cologarithms, the inconvenience of subtracting the logarithm of the divisor is avoided.

Thus, the example of § 656 is usually solved as follows:

$$\begin{aligned} \log 905.6 &= 2.9569 \\ \text{colog } 38.45 &= \underline{8.4151 - 10} \\ & 1.3720 \qquad = \log 23.55. \end{aligned}$$

Therefore, $905.6 \div 38.45 = 23.55$.

660. Example. By the use of cologarithms find the value of $\dfrac{7.56 \times 4667 \times 567}{899.1 \times 0.00337 \times 23435}$.

SOLUTION.

$$\begin{aligned} \log 7.56 &= 0.8785 \\ \log 4667 &= 3.6690 \\ \log 567 &= 2.7536 \\ \text{colog } 899.1 &= 7.0462 - 10 \\ \text{colog } 0.00337 &= 2.4724 \\ \text{colog } 23{,}435 &= \underline{5.6301 - 10} \\ & 2.4498 \qquad = \log 281.7. \end{aligned}$$

NOTE. Log 899.1 is 2.9538, and its cologarithm is $(10-2.9538)-10$, or $7.0462 - 10$; log 0.00337 is $7.5276 - 10$, and subtracting this from $10 - 10$ we obtain for its cologarithm 2.4724; log 23,435 is 4.3699, and its cologarithm is $(10 - 4.3699) - 10$, or $5.6301 - 10$.

EXERCISE 156.

Find by logarithms the value of:

1. $\dfrac{56.407}{13.045}$.

2. $\dfrac{857.06}{3079.8}$.

3. $\dfrac{0.9387}{598.6}$.

4. $\dfrac{3069}{0.7891}$.

5. $\dfrac{75.46 \times 0.0765}{93.08 \times 98.071}$.

6. $\dfrac{98 \times 537 \times 0.0079}{67309 \times 0.0947}$.

7. $\dfrac{314 \times 7.18 \times 8132}{519 \times 827 \times 3.215}$.

8. $\dfrac{212 \times 2.16 \times 8002}{536 \times 351 \times 7.256}$.

9. $(\frac{61}{73})^4$.

10. $(\frac{13}{71})^3$.

11. $(5\frac{5}{11})^2$.

12. $(4\frac{4}{31})^3$.

13. $(\frac{412}{617})^5$.

14. $(\frac{83}{97})^3$.

15. $(\frac{507}{822})^3$.

16. $(\frac{1741}{1816})^3$.

17. $\dfrac{19.258 \times 3.1416 \times 812.72}{716.4 \times 8.002 \times 21.465}$.

18. $\dfrac{2018 \times 0.00261 \times 1728}{1412 \times 0.0965 \times 0.08621}$.

19. $\dfrac{44{,}816 \times 17.265 \times 181}{28{,}754 \times 1.2871 \times 206.45}$.

20. $\dfrac{216.1 \times 5280 \times 144.2}{187.42 \times 4622.6 \times 156.8}$.

21. $\dfrac{5982.55 \times 0.02987 \times 0.9852}{42.875 \times 34.62 \times 28.47}$.

22. $\dfrac{14.718 \times 48.67 \times 96.542}{2746.2 \times 0.0467 \times 2.1876}$.

23. $\sqrt{\dfrac{83.25 \times 4267 \times 0.008576}{0.0327 \times 687.5 \times 0.005003}}$.

24. $\sqrt[3]{\dfrac{4.163^2 \times 17.74^4 \times 0.7183^{\frac{1}{2}}}{3.013^2 \times 34.34 \times 0.08137^{\frac{1}{3}}}}$.

25. $\sqrt[4]{\dfrac{0.7132 \times 9.245 \times 0.5477^2}{76.93 \times 0.000173^{\frac{1}{2}} \times 0.01}}$.

26. $\sqrt[5]{\dfrac{65.02^2 \times 0.002753 \times 97.98^{\frac{1}{2}}}{7.298 \times 0.04754 \times 8.156^2}}$.

27. $\sqrt[6]{\dfrac{23.79^2 \times 0.00756 \times 0.4648^{\frac{1}{3}}}{4723^{\frac{1}{2}} \times 0.6571 \times 0.8246^{\frac{1}{3}}}}$.

28. $\sqrt[7]{\dfrac{0.6012 \times 0.6012^{\frac{1}{2}} \times 0.6012^{\frac{1}{3}}}{0.5926 \times 0.5926^{\frac{1}{2}} \times 0.5926^{\frac{1}{3}}}}$.

29. $\left(\dfrac{0.03214 \times 3.718^3 \times 0.07824^{\frac{1}{2}}}{0.05142 \times 0.4728^{\frac{1}{2}} \times 1.239^3}\right)^{\frac{3}{4}}$.

30. $\left(\dfrac{0.07986 \times 0.7555^{\frac{1}{3}} \times 0.5557^{\frac{1}{2}}}{0.06897 \times 0.5777^{\frac{1}{3}} \times 0.05698^2}\right)^{\frac{3}{7}}$.

31. $\left(\dfrac{0.07543 \times 0.7689^{\frac{1}{2}} \times 0.8965^2}{0.06987 \times 0.07986^{\frac{1}{2}} \times 0.9867^{\frac{1}{3}}}\right)^{\frac{5}{6}}$.

CHAPTER XX.

APPLICATIONS OF LOGARITHMS.

Compound Interest Problems.

661. In compound interest, if P represents the principal, r the rate per cent, n the number of years, A the amount,

then $(1+r)$ represents the amount of \$1 for 1 year.
$(1+r)^2$ represents the amount of \$1 for 2 years.
$(1+r)^3$ represents the amount of \$1 for 3 years.
$(1+r)^n$ represents the amount of \$1 for n years.
$P\times(1+r)^n$ represents the amount of \$$P$ for n years.

Therefore, $$A = P\times(1+r)^n,$$
and $$\log A = \log P + n\times\log(1+r).$$

662. Examples. 1. Find the amount of \$150 for 6 years at 4% compound interest.

SOLUTION. Here $P=150$, $r=0.04$, $n=6$.

Therefore, $\log A = \log 150 + 6\times\log 1.04.$

$$\begin{aligned}\log 150 &= 2.1761\\ 6\times\log 1.04 &= \underline{0.1020}\\ \log A &= 2.2781 = \log 189.7.\end{aligned}$$

Hence, the required amount is \$189.70.

2. What principal will amount to \$500 in 5 years at $4\frac{1}{2}$% compound interest?

SOLUTION. Here $A=500$, $r=0.045$, $n=5$.

Therefore, $500 = P\times(1.045)^5$; and $P=\dfrac{500}{1.045^5}\cdot$

$$\begin{aligned}\log 500 &= 2.6990\\ 5\times\operatorname{colog}1.045 &= \underline{9.9045-10}\\ \log P &= 2.6035 \quad = \log 401.4.\end{aligned}$$

Hence, the required principal is \$401.40.

3. At what rate of interest will \$360 amount to \$481.80 in 5 years at compound interest?

Solution. Here $A = 481.80$, $P = 360$, $n = 5$.

Therefore,
$$481.80 = 360 \times (1+r)^5,$$
$$(1+r)^5 = \frac{481.80}{360}, \text{ and } 1 + r = \sqrt[5]{\frac{481.80}{360}}.$$
$$\begin{aligned} \log 481.80 &= 2.6828 \\ \text{colog } 360 &= 7.4437 - 10 \\ & 5\overline{)0.1265} \\ \log(1+r) &= 0.0253 = \log 1.06. \end{aligned}$$

Hence, the required rate of interest is 6%.

4. In what time at $3\frac{1}{2}\%$ compound interest will \$540 amount to \$619.40?

Solution. Here $A = 619.40$, $P = 540$, $r = 0.035$.

Therefore,
$$\log 619.40 = \log 540 + n \times \log 1.035,$$
$$n \times \log 1.035 = \log 619.40 + \text{colog } 540,$$
and
$$\begin{aligned} n &= \frac{\log 619.40 + \text{colog } 540}{\log 1.035} \\ &= \frac{2.7920 + 7.2676 - 10}{0.0149} \\ &= \frac{0.0596}{0.0149} = 4. \end{aligned}$$

Hence, the required time is 4 years.

Exercise 157.

1. Find the compound interest on \$1280 for 7 years at $4\frac{1}{2}\%$.

2. Find the compound interest on \$2645 for 5 years at $3\frac{1}{2}\%$.

3. Find the amount of \$848 for 6 years at 5% compound interest.

4. Find the amount of \$3600 for 5 years at $5\frac{1}{2}\%$ compound interest.

5. What principal will amount to \$720 in 4 years at 6% compound interest?

6. What principal will amount to \$1640 in 6 years at 3% compound interest?

7. At what rate of interest will \$648 amount to \$788.20 in 5 years at compound interest?

8. At what rate of interest will \$2415 amount to \$3237 in 6 years at compound interest?

9. In what time at $4\frac{1}{2}\%$ compound interest will \$1265 amount to \$1576?

10. In what time at 5% compound interest will \$1845 amount to \$2413?

663. The table on the following page has been made with the aid of logarithms. It shows the amount of \$1 at compound interest at various per cents for from 1 to 20 years. The compound interest on \$1 is found by subtracting 1 from the proper number shown in the table.

664. Examples. 1. What principal will in 10 years at 6% compound interest yield \$1898.04 interest?

Solution. The interest on \$1 for 10 yr. at 6% is \$0.79085.
Since \$0.79085 is the interest on \$1,

$$\$1898.04 \text{ is the interest on } \$\frac{1898.04}{0.79085}, \text{ or } \$2400.$$

2. In what time will \$1600 at $4\frac{1}{2}\%$ compound interest yield \$1000 interest?

Solution. Since \$1600 yields \$1000, \$1 will yield $\frac{1}{1600}$ of \$1000, or \$0.625, in the same time, and \$1 will amount to \$1.625. By the table, \$1 will in 11 yr. at $4\frac{1}{2}\%$ amount to \$1.62285, and in 12 yr. to \$1.69588. Hence, the required time is a little more than 11 yr.

3. At what rate, compound interest, will \$1500 yield \$1201.41 interest in 15 years?

Solution. Since \$1500 yields \$1201.41 interest in 15 yr., \$1 in 15 yr. will yield $\frac{1}{1500}$ of \$1201.41, or \$0.80094, and \$1 will amount in 15 yr. to \$1.80094. In the table, opposite 15 yr., we find in the 4% column the amount of \$1 is \$1.80094. Therefore, the rate is 4%.

665. TABLE.

Showing the amount of $1 at compound interest for:

YR.	2 PER CENT.	2½ PER CENT.	3 PER CENT.	3½ PER CENT.	4 PER CENT.
1	1.02000	1.02500	1.03000	1.03500	1.04000
2	1.04040	1.05063	1.06090	1.07123	1.08160
3	1.06121	1.07689	1.09273	1.10872	1.12486
4	1.08243	1.10381	1.12551	1.14752	1.16986
5	1.10408	1.13141	1.15927	1.18769	1.21665
6	1.12616	1.15969	1.19405	1.22926	1.26532
7	1.14869	1.18869	1.22987	1.27228	1.31593
8	1.17166	1.21840	1.26677	1.31681	1.36857
9	1.19509	1.24886	1.30477	1.36290	1.42331
10	1.21899	1.28009	1.34392	1.41060	1.48024
11	1.24337	1.31209	1.38423	1.45997	1.53945
12	1.26824	1.34489	1.42576	1.51107	1.60103
13	1.29361	1.37851	1.46853	1.56396	1.66507
14	1.31948	1.41297	1.51259	1.61870	1.73168
15	1.34587	1.44830	1.55797	1.67535	1.80094
16	1.37279	1.48451	1.60471	1.73399	1.87298
17	1.40024	1.52162	1.65285	1.79468	1.94790
18	1.42825	1.55966	1.70243	1.85749	2.02582
19	1.45681	1.59865	1.75351	1.92250	2.10685
20	1.48595	1.63862	1.80611	1.98979	2.19112

YR.	4½ PER CENT.	5 PER CENT.	5½ PER CENT.	6 PER CENT.	7 PER CENT.
1	1.04500	1.05000	1.05500	1.06000	1.07000
2	1.09203	1.10250	1.11303	1.12360	1.14490
3	1.14117	1.15763	1.17424	1.19102	1.22504
4	1.19252	1.21551	1.23882	1.26248	1.31080
5	1.24618	1.27628	1.30696	1.33823	1.40255
6	1.30226	1.34010	1.37884	1.41852	1.50073
7	1.36086	1.40710	1.45468	1.50363	1.60578
8	1.42210	1.47746	1.53469	1.59385	1.71819
9	1.48610	1.55133	1.61909	1.68948	1.83846
10	1.55297	1.62889	1.70814	1.79085	1.96715
11	1.62285	1.71034	1.80209	1.89830	2.10485
12	1.69588	1.79586	1.90121	2.01220	2.25219
13	1.77220	1.88565	2.00577	2.13293	2.40985
14	1.85194	1.97993	2.11609	2.26090	2.57853
15	1.93528	2.07893	2.23248	2.39656	2.75903
16	2.02237	2.18287	2.35526	2.54035	2.95216
17	2.11338	2.29202	2.48480	2.69277	3.15882
18	2.20848	2.40662	2.62147	2.85434	3.37993
19	2.30786	2.52695	2.76565	3.02560	3.61653
20	2.41171	2.65330	2.91776	3.20714	3.86968

EXERCISE 158.

1. A man deposits $60 in a savings bank, and draws out his whole account at the end of 8 years, with 4% compound interest. What amount does he receive?

2. What will $100 amount to in 7 years with interest at 8% per annum, compounded semi-annually?

3. In how many years will a sum of money double itself at 6%, compounded annually?

4. In how many years will a sum of money treble itself at 6%, compounded annually?

5. In how many years will $87 amount to $99 at 3%, compounded annually?

6. In how many years will $100 amount to $175 at 4%, compounded annually?

7. At what rate per cent will a sum of money double itself in 12 years, compound interest?

8. At what rate will a sum of money treble itself in 19 years, compound interest?

9. At what rate will $80 at compound interest amount to $110 in 8 years?

10. What sum must be invested at 5%, compound interest, to amount to $1200 in 7 years?

11. What sum must be invested at 4%, compound interest, to amount to $2000 in 10 years? To amount to $5000 in 8 years?

12. At what rate compound interest will $462.50 yield $277.98 interest in 12 years?

13. What principal will in 10 years at 6% amount to $3612.22, interest being compounded semi-annually?

14. In what time at 5% will $1250 amount to $2000, interest being compounded semi-annually?

15. At what rate per annum will $500 amount to $779.83 in 9 years, interest being compounded semi-annually?

Annuities.

666. An **annuity** is a sum of money to be paid at regular intervals of time, as years, half years, quarter years.

667. A **perpetual annuity** is an annuity that continues forever.

668. A **certain annuity** is an annuity that begins at a specified time and ends at a specified time.

669. A **contingent annuity** is an annuity that depends upon some particular event, as the death of an individual.

Life insurance, dowers, and pensions are examples.

670. The **final value** of an annuity is the sum to which all its payments at compound interest will amount at the end of the annuity.

671. The **present value** of an annuity is the sum which at compound interest will amount to its final value.

672. A **sinking fund** is the *final value* of sums of money set apart at regular intervals of time, and invested at compound interest, to pay a debt due at a stated time.

673. To Find the Final Value of an Annuity.

If S represents the amount paid each year, R the amount of $1 at interest for 1 year, n the number of years, and A the final value, then,

The amount at the end of the

$$\begin{aligned} &\text{1st year} = S, \\ &\text{2d year} = S + SR, \\ &\text{3d year} = S + SR + SR^2, \\ &n\text{th year} = S + SR + SR^2 + \cdots + SR^{n-1}. \end{aligned}$$

That is, $$A = S + SR + SR^2 + \cdots + SR^{n-1}. \quad (1)$$

Multiplying (1) by R, we have

$$AR = SR + SR^2 + SR^3 + \cdots + SR^{n-1} + SR^n. \quad (2)$$

Subtracting (1) from (2), we have

$$AR - A = SR^n - S,$$

or

$$A(R-1) = S(R^n - 1).$$

Therefore,

$$A = \frac{S(R^n-1)}{R-1}.$$

Here, $R-1$ is the interest on \$1 for 1 year, and $R^n - 1$ is the compound interest of \$1 for n years.

674. To Find the Present Worth of an Annuity.

If P represents the present worth of the annuity, then the amount of P for n years $= A$, the final value of the annuity in n years.

The amount of P for n years

$$= P(1+r)^n = PR^n, \qquad (\S\ 661)$$

and

$$A = \frac{S(R^n-1)}{R-1}. \qquad (\S\ 673)$$

Therefore,

$$PR^n = \frac{S(R^n-1)}{R-1},$$

and

$$P = \frac{S(R^n-1)}{R^n(R-1)} = \frac{S}{R-1} \times \frac{R^n-1}{R^n}.$$

The value of P may be written $\frac{S}{R-1} \times \left(1 - \frac{1}{R^n}\right)$.

If the annuity is *perpetual*, n is too large to be counted; therefore, R^n is too large to be counted and $\frac{1}{R^n} = 0$.

Hence,

$$P = \frac{S}{R-1} = \frac{S}{r}.$$

675. Examples. 1. What is the value of a sinking fund, if \$6000 is set apart annually for 6 years and put at 4% compound interest?

SOLUTION. $A = \frac{S(R^n-1)}{R-1} = \frac{\$6000 \times (1.04^6 - 1)}{0.04}$. (§ 673)

By the use of logarithms, A is found to be \$39,750.

2. Find the present value of an annuity of \$500 for 5 years, if money is worth 4%.

SOLUTION. $P = \frac{S}{R-1} \times \frac{R^n - 1}{R^n} = \frac{500}{0.04} \times \frac{1.04^5 - 1}{1.04^5}$. (§ 674)

$$\begin{aligned} \log 1.04 &= 0.0170 \\ &\quad\underline{5} \\ \log 1.04^5 &= 0.0850 = \log 1.216. \end{aligned}$$

Therefore, $P = \frac{500}{0.04} \times \frac{0.216}{1.216}$.

$$\begin{aligned} \log 500 &= 2.6990 \\ \log 0.216 &= 9.3345 - 10 \\ \text{colog } 0.04 &= 1.3979 \\ \text{colog } 1.216 &= \underline{9.9150 - 10} \\ \log P &= 3.3464 \qquad = \log 2220. \end{aligned}$$

Hence, the present value of the annuity is \$2220.

3. Find the present value of a perpetual annuity of \$500, if money is worth 4%.

SOLUTION. $P = \frac{S}{r} = \frac{\$500}{0.04} = \$12,500.$ (§ 674)

EXERCISE 159.

1. Find the present value of an annuity of \$300 for 6 years, if money is worth 5%.
2. Find the present value of an annuity of \$600 for 4 years, if money is worth $5\frac{1}{2}$%.
3. Find the present value of an annuity of \$800 for 5 years, if money is worth 6%.
4. Find the present value of a perpetual scholarship of \$900, if money is worth $3\frac{1}{2}$%.
5. Find the present value of a perpetual fellowship of \$3200, if money is worth $4\frac{1}{2}$%.
6. What is the value of a sinking fund, if \$25,000 is set apart yearly for 7 years at $4\frac{1}{2}$% compound interest?
7. What is the value of a sinking fund, if \$18,000 is set apart yearly for 5 years at $3\frac{1}{2}$% compound interest?

676. The table at the bottom of the page shows the average number of years persons live after the ages indicated. This table is known as the Carlisle Table because it is based upon the rate of mortality, as carefully observed at Carlisle, England. Several other tables of Expectancy of Life have been compiled from other data and are in common use with insurance companies.

The table on the following page has been made with the aid of logarithms. It shows the present value of an annuity of $1 per annum at compound interest from 1 to 40 years at 3½% and at 4%.

677. *Carlisle Table of Expectancy of Life.*

AGE.	EXPECTANCY.	AGE.	EXPECTANCY.	AGE.	EXPECTANCY.	AGE.	EXPECTANCY.
0	38.72	26	37.14	52	19.68	78	6.12
1	44.68	27	36.41	53	18.97	79	5.80
2	47.55	28	35.69	54	18.28	80	5.51
3	49.82	29	35.00	55	17.58	81	5.21
4	50.76	30	34.34	56	16.89	82	4.93
5	51.25	31	33.68	57	16.21	83	4.65
6	51.17	32	33.03	58	15.55	84	4.39
7	50.80	33	32.36	59	14.92	85	4.12
8	50.24	34	31.68	60	14.34	86	3.90
9	49.57	35	31.00	61	13.82	87	3.71
10	48.82	36	30.32	62	13.31	88	3.59
11	48.04	37	29.64	63	12.81	89	3.47
12	47.27	38	28.96	64	12.30	90	3.28
13	46.51	39	28.28	65	11.79	91	3.26
14	45.75	40	27.61	66	11.27	92	3.37
15	45.00	41	26.97	67	10.75	93	3.48
16	44.27	42	26.34	68	10.23	94	3.53
17	43.57	43	25.71	69	9.70	95	3.53
18	42.87	44	25.09	70	9.18	96	3.46
19	42.17	45	24.46	71	8.65	97	3.28
20	41.46	46	23.82	72	8.16	98	3.07
21	40.75	47	23.17	73	7.72	99	2.77
22	40.04	48	22.50	74	7.33	100	2.28
23	39.31	49	21.81	75	7.01	101	1.79
24	38.59	50	21.11	76	6.69	102	1.30
25	37.86	51	20.39	77	6.40	103	0.83

678. TABLE.

Showing the present value of an annuity of $1 per annum, at compound interest from 1 to 40 years at 3½% and at 4%.

YR.	3½ PER CENT.	4 PER CENT.	YR.	3½ PER CENT.	4 PER CENT.
1	0.96618	0.96154	21	14.69797	14.02916
2	1.89969	1.88610	22	15.16713	14.45112
3	2.80164	2.77509	23	15.62041	14.85684
4	3.63708	3.62990	24	16.05837	15.24696
5	4.51505	4.45182	25	16.48152	15.62208
6	5.32855	5.24214	26	16.89035	15.98277
7	6.11454	6.00206	27	17.28537	16.32959
8	6.87396	6.73275	28	17.66702	16.66306
9	7.60769	7.43533	29	18.03577	16.98372
10	8.31661	8.11090	30	18.39205	17.29203
11	9.00155	8.76048	31	18.73628	17.58849
12	9.66333	9.38507	32	19.06887	17.87355
13	10.30274	9.98565	33	19.39021	18.14765
14	10.92052	10.56312	34	19.70068	18.41120
15	11.51741	11.11839	35	20.00066	18.66461
16	12.09412	11.65230	36	20.29049	18.90828
17	12.65132	12.16567	37	20.57053	19.14258
18	13.18968	12.65930	38	20.84109	19.36786
19	13.70984	13.13394	39	21.10250	19.58449
20	14.21240	13.59033	40	21.35507	19.79277

679. Examples. 1. Find the present value of an annuity for $500 for 5 yr. at 4%.

SOLUTION. The present value of $1 for 5 yr. at 4% by the table is $4.45182; and of $500 is 500 × $4.45182, or $2225.91.

2. A person 41 years of age pays $9797.75 for a life annuity. If interest is reckoned at 4%, find the amount of the annuity.

SOLUTION. According to the table on page 367, the expectancy of life for a person 41 years of age is about 27 years.

The present value of an annuity of $1 for 27 yr. at 4% is $16.32959.

Hence, the amount of the annuity is $ $\frac{9797.75}{16.32959}$, or $600.

Exercise 160.

1. Find the present value of an annuity of $900 for 15 years at 4%.

2. Find the present value of an annuity of $1500 for 12 years at 4%.

3. Find the present value of an annual pension of $144 for 10 years at 3½%.

4. Find the present value of a scholarship of $200 for 25 years at 3½%.

5. Find the present value of an annuity of $2500 for 30 years at 4%.

6. Find the present value of an annuity of $250 for 12 years at 3½%.

7. A person 22 years old has a life annuity of $750. Find its present value at 4%.

8. A person 35 years old has a life annuity of $1800. Find its present value at 4%.

9. A person 53 years old has a life annuity of $500. Find its present value at 4%.

10. A person 75 years old has a life annuity of $2400. Find its present value at 3½%.

11. A boy 15 years old has a life annuity of $3250. Find its present value at 4%.

12. A person 22 years old pays $4948.19 for a life annuity. If interest is 4%, find the amount of the annuity.

13. A person 29 years old pays $7465.84 for a life annuity. If interest is 4%, find the amount of the annuity.

14. A person 35 years old pays $9368.14 for a life annuity. If interest is 3½%, find the amount of the annuity.

15. A person 44 years old pays $5933.35 for a life annuity. If interest is 3½%, find the amount of the annuity.

Coöperative Banks.

680. A **coöperative bank** is a *mutual corporation* with the object of the accumulation of a capital to be loaned to its members, especially for the purchase of homes.

681. Shares. The capital stock is usually divided into shares of final value $200 each, that are paid for in monthly instalments of $1 each. The number of shares that any member may purchase is limited, usually to twenty-five. Each shareholder pays $1 a month per share until his shares are worth $200 each. The shares are then said to be *matured.* At maturity the shareholder receives $200 in money for each share he holds.

If no *profits* were added to the value of the shares, it would take 200 months, that is, 16 years 8 months to mature the shares; but the profits generally reduce this time to between 10 and 12 years.

682. Loans. Any shareholder may borrow $200 on each share he holds, provided he furnishes the security required by law. Security may be by mortgage upon real estate or upon the shares themselves. If the shares are offered as security, no shareholder is allowed to borrow more than the present value of his shares.

The amount of a loan is usually limited to $2000.

683. When the accumulation of the various payments has reached a certain sum, these funds are offered at auction and loaned to the shareholder who offers proper security and bids the highest *premium,* in addition to interest at the rate of 6% per annum. This interest and premium is added to the general fund, and at stated times is credited equally among the various shares.

684. Fines. To prevent payments falling into arrears, a fine, usually 2 cents a month on every dollar not paid

when due, is imposed on delinquent shareholders, whether the delinquency is the monthly instalment, interest, or premium.

685. Examples. 1. Find the cost at compound interest of a coöperative bank share, if the share matured in 11 years, and money was worth 4%.

SOLUTION. 11 yr. = 132 mo. $1 was paid monthly for 132 mo. The rate of interest was 4% yearly, or ⅓% monthly.

$$A = \frac{S(R^n - 1)}{R - 1} = \frac{\$1 \times (1.003\tfrac{1}{3}^{132} - 1)}{1.003\tfrac{1}{3} - 1}. \qquad (§ 673)$$

By the four-place table of logarithms, the value of this fraction is found to be $163.80; by a seven-place table, the value is $165.46. The seven-place table gives a very close approximation.

2. Find the cost at compound interest of a loan of $200 from a coöperative bank, if the borrower pays $1 per month interest. The shares are worth $60, and mature in 7 years, and money is worth 4%.

SOLUTION. The borrower pays monthly $2 for 7 yr., that is, 84 mo. The final value of $2 deposited monthly for 84 mo. at 4% is found by § 673 to be $191.40, and the compound amount of $60 at 4% for 7 yr. is $78.96. Hence, the cost of the loan is $191.40 + $78.96 = $270.36.

EXERCISE 161.

Find the cost at compound interest of a:

1. Coöperative bank share that matured in 10 years, when money was worth 4½%.

2. Coöperative bank share that matured in 11½ years, when money was worth 5%.

3. How much more does it cost to borrow $2000 from a coöperative bank, monthly interest being $12, and the shares maturing in 10 years, than to borrow $2000 at compound interest for 10 years, if money is worth 5% in both cases?

CHAPTER XXI.

MISCELLANEOUS PROBLEMS.

In solving these problems, logarithms should be used whenever they can be used with advantage.

1. Make six different numbers with the digits 1, 2, 3, and find their sum.

2. Make six different numbers with the digits 2, 3, 5, and find, by logarithms, their continued product.

3. Make six different numbers with the digits 8, 7, 3, and find, by logarithms, their continued product.

4. Find, by logarithms, the missing term in each of the following proportions:

(i) 7.13 : 3.57 :: 4.18 : ?. (iii) 7.37 : ? :: 86.1 : 43.7.

(ii) 5.89 : 76.3 :: ? : 38.7. (iv) ? : 69.7 :: 3.79 : 29.4.

5. Find, by logarithms, the value of $0.08^{\frac{1}{3}}$; $2734^{\frac{1}{5}}$; $21.97^{\frac{1}{3}}$; $7^{3.6}$; $9.71^{\frac{2}{5}}$; $7.936^{\frac{4}{7}}$.

6. Find the value of $\sqrt[5]{\dfrac{4.79^2 \times 3.1416 \times 12.72}{0.5236 \times 14.28}}$.

7. If the air-line distance between two points is 1534 ft., and the difference of level is 34 ft., what is the horizontal distance between the two points?

8. If the road distance is 1 mi., and the rise 347 ft., find the horizontal distance.

9. If the road distance is half a mile, and the horizontal distance 2513 ft., find the difference of level.

10. The diagonal of a rectangular floor is 34.6 ft., and the width is 17.8 ft. Find the length of the floor.

11. The height of a tower on the bank of a river is 55 ft., and the length of a line from the top of the tower to the opposite bank is 78 ft. Find the breadth of the river.

12. The number of seamen at Portsmouth is 800, at Charlestown 404, and at Brooklyn 756. A ship is commissioned whose complement is 490 seamen. Determine the number to be drafted from each place to obtain a proportionate number from each.

13. Show, without division, that 36,432 contains 8, 9, 11 as factors.

14. Find the smallest multiplier that will make 47,250 a perfect cube.

15. Find the proper fraction that, when reduced to a continued fraction, has for quotients 1, 3, 5, 7, 2, 4.

16. If the meter is equal to 1.09362 yd., find a series of four fractions that will express more and more nearly the true ratio of the meter to the yard.

17. Find the square factors contained in 33,075.

18. The height of St. Peter's, Rome, is $\frac{8}{110}$ of a mile, and that of St. Paul's, London, is $\frac{17}{264}$ of a mile. How many feet higher is St. Peter's than St. Paul's?

19. How many days elapsed between the annular eclipse of May 15, 1836, and that of March 15, 1858?

20. In a gale, a flagstaff 60 ft. high snaps 28.8 ft. from the bottom; and, not being wholly broken off, the top touches the ground. If the ground is level, how far is the top from the bottom?

21. Seventeen trees are standing in a straight line, 20 yd. apart; a man walks from the first to the second and back, then to the third and back, and so on. How far does he walk?

22. A canal is $14\frac{3}{4}$ mi. long and 48 ft. wide. At one end is a lock 80 ft. by 24 ft., with a fall of 8 ft. 6 in. How many barges can pass through the lock before the water in the canal is lowered 1 in.?

23. Find the capacity, in liters and in bushels, of a box 1.7^{m} long, 87^{cm} wide, and 31^{cm} deep.

24. Find the number of kilograms of olive oil, specific gravity 0.915, required to fill a rectangular vessel 2.3^{m} long, 1.8^{m} wide, and 74^{cm} deep.

25. How many tons in a block of marble 4 ft. long, 34 in. wide, 17.3 in. thick, specific gravity 2.73?

26. Find the surface of a sphere 18.3 in. in diameter.

27. Find the number of acres in a circular field 213 yd. 2 ft. in diameter.

28. How many cubic inches in a 10-inch globe? in a 20-inch globe? What is the ratio of their volumes?

29. How many balls 3 in. in diameter can be cast from a pig of iron 7 ft. long, 6.7 in. wide, 3.8 in. thick, if the waste in melting and casting is reckoned at $3\frac{1}{4}\%$?

30. Find the difference in length, at 80° F., of a glass rod and a steel rod, each 3 ft. long at 0° C., if the expansion at 100° C. is 0.00085 for glass and 0.0012 for steel.

31. A grain of gold is beaten into leaf to cover 56 sq. in. What weight will be required to gild the faces of a cube whose edge is $3\frac{1}{2}$ ft.?

32. What premium must be paid, at the rate of $4\frac{3}{8}\%$, for insuring a vessel worth $100,000, in order that in the event of loss the owner may receive both the value of the ship and the premium?

33. By selling goods at 60 cents a pound, 8% is lost. What advance must be made in the price to gain 15%?

34. The sharpest grade on Mt. Washington Ry. is 1980 ft. to the mile. What fraction of a foot is the rise for each foot? What is the per cent of grade?

35. Find the square root, to four decimal places, of the reciprocal of 0.0043.

36. The population of a city in 1890 was 12,298, showing a decrease of $8\frac{1}{8}\%$ on its population in 1880; in 1880 there was an increase of $7\frac{1}{2}\%$ on the census of 1870. What was its population in 1870?

37. Find the increase of income obtained by transferring 25 shares of 3% stock at $94\frac{3}{4}$ to 4% stock at $104\frac{3}{4}$, brokerage $\frac{1}{4}$ on each transaction.

38. Each person in breathing spoils the air of a closed room at the rate of about 8 cu. ft. a minute. An audience of 400 persons enter a closed hall 70 ft. by 40 ft., and 20 ft. high. How long will it take them to spoil the air?

39. How long can the windows and doors of a schoolroom be safely kept closed when occupied by 50 children, if the room is 25 ft. by 20 ft. and 10 ft. high?

40. A pays B \$230 as the present value of \$300 due in 5 years. Which gains by the payment, and how much, if interest is reckoned at 5% compound interest?

41. Find the quantity of coal required by a steamer for a voyage of 4043 mi., if her rate per hour is 14.04 knots, and her consumption of coal 87 long tons per day.

42. Find the area of a circular ring whose inner and outer diameters are 7.36 in. and 10.64 in., respectively.

43. A and B can do a piece of work in $13\frac{1}{3}$ days; A and C in $10\frac{2}{3}$ days; A, B, and C in $7\frac{1}{2}$ days. In how many days can A do the work alone?

44. If 3 men working 11 hours a day can reap 20 A. in 11 days, how many men working 12 hours a day can reap a field 360 yd. long and 320 yd. broad in 4 days?

45. Find the area of a triangle whose sides are 12 in., 5 in., and 13 in., respectively.

46. The four sides of a field measured in succession are 237 ft., 253 ft., 244 ft., and 261 ft., and the diagonal measured from the end of the first side to the end of the third side is 351 ft. Find the area of the field.

47. The four sides of a field measured in succession are 361 ft., 561 ft., 443 ft., and 357 ft., and the distance from the beginning of the first side to the end of the second side is 682 ft. Find the area of the field.

48. Find the altitude of a triangle, if each side is 1000 ft.

49. Find the three altitudes of a triangle, if its sides are 17.8^{mm}, 23.6^{mm}, and 31.5^{mm}, respectively.

50. How many square inches in the surface of a sphere that has a radius of 12.37 in.?

51. Find the area of the surface of the largest globe that can be turned out from a joist 4 in. by 6 in.

52. How many cubic inches in a globe that has a diameter of 10 in.?

53. If a tree is round, and its girth is 17 ft. 6 in., find its diameter. Find the area of a cross section, and also the number of cubic feet in the largest sphere that can be cut from it.

54. Find the weight in kilograms and in pounds of an iron ball 21.5^{cm} in diameter, specific gravity 7.47; of a tin ball 13^{cm} in diameter, specific gravity 7.29; of a lead ball 17.3^{cm} in diameter, specific gravity 11.35; of a silver ball 1.31^{cm} in diameter, specific gravity 10.47.

55. A slab of cast iron 4 ft. $2\frac{1}{2}$ in. long, 17 in. wide, and $8\frac{1}{8}$ in. thick, specific gravity 7.31, is cast into 2-lb. balls. If there is a loss of 5% in melting, how many balls are obtained, and what is the diameter of each?

56. How many pounds will a ball of iron 30 in. in diameter weigh, if the specific gravity of the iron is 7.31?

57. If the specific gravity of ice is 0.930, find the weight and the surface of each of three spheres of ice whose diameters are 1^{cm}, 10^{cm}, and 1^{m}.

58. Find the capacity in gallons of a round cistern 13 ft. in diameter and 9 ft. deep.

59. A cylinder is 10 in. in diameter and 12 in. long. Find the area of each end, the lateral surface, the total surface, and the contents in gallons.

60. What must be the diameter of a cylinder 10 in. deep that it may hold 1 gallon?

61. Find the volume of a cylinder 8 in. in diameter and 11 in. high.

62. Find the dimensions of three cylinders that have the diameters equal to the heights, and hold 1 gallon, 1 quart, and 1 liter, respectively.

63. How many cubic yards in a pyramid 123 ft. high, with a square base 210 ft. on a side?

64. Find the capacity of a cup, whose mouth is 4 in. square, and whose sides are four equilateral triangles.

65. The largest of the Egyptian pyramids is 147^{m} high, with a base 231^{m} square. Find its volume in cubic meters.

66. The slant depth of a conical cup is 93^{mm}, and the diameter at the top 8^{cm}. What is its capacity?

67. The volume of a cone is 1^{cbm}; its height is equal to the radius of its base. Find the dimensions of the cone.

68. Find the capacity in pints of a cylinder, diameter 1.9375 in., height 2.4375 in.; of a cylinder, diameter $3\frac{1}{8}$ in., height $3\frac{5}{8}$ in.; of a cylinder, diameter $3\frac{13}{16}$ in., height $5\frac{1}{16}$ in.

69. Find the capacity in pecks of a cylinder, diameter 15.865 in., height 12.5 in.; of a cylinder, diameter 9.25 in., height 4.25 in.; of a cylinder, diameter 18.5 in., height 8 in.

70. What must be the diameter of a circle to contain 78.54 sq. ft.? to contain 314.16 sq. ft.?

71. What must be the diameter of a circle to contain 1 A.? to contain 9 A.?

72. What must be the diameter of a circle to contain 1^{ha}? to contain 25^{ha}?

73. Divide $1270 into parts proportional to $4\frac{1}{3}$, $5\frac{1}{6}$, $6\frac{2}{7}$.

74. How much water will a hemispherical bowl hold that is 10 in. in diameter?

75. At 50 cents a square foot, what will it cost to gild a hemispherical dome 10 ft. in diameter?

76. If the moon is a sphere 2170 miles in diameter, how many million bushels would it hold if hollow?

77. If the earth is 7920 miles in diameter, and the air is 40 miles deep, how many cubic miles of air are there?

78. What is the difference between 2 feet square and 2 square feet? between a foot square and a square foot? between half a foot square and 6 in. square?

79. Find the volume of a frustum of a right pyramid whose lower base is a square 3 ft. on a side, upper base a square 2 ft. on a side, and height 4 ft.

80. Find the capacity in liquid quarts of a tin pan 10 in. in diameter at the top, 8 in. in diameter at the bottom, and 4 in. deep.

81. How many hektoliters will a circular vat hold 5^m in diameter at the top, 4.57^m in diameter at the bottom, and 1.17^m deep?

82. If 4 cu. in. of iron weigh 1 lb. avoirdupois, what is the weight in grains of 1 cu. in. of iron? What is the specific gravity of the iron?

83. If 4 cu. in. of iron weigh 1 lb., what is the diameter of a 6-lb. ball? of a 32-lb. ball?

84. At $\frac{1}{4}$ lb. to the cubic inch, what is the weight of a rectangular block of iron 17.36 in. by 8.7 in. by 1.76 in.? What would be its diameter if cast into a ball, if 11% is allowed for waste?

85. At $\frac{1}{4}$ lb. to the cubic inch, what is the weight of a rectangular block of iron 71.4 in. by $8\frac{3}{8}$ in. by $3\frac{1}{8}$ in.? What would be its diameter if cast into a ball, if 11% is allowed for waste?

86. What is the diameter of a cylinder 11 in. long that will hold 2 gallons?

87. What is the diameter of a cylinder 9 in. long that will hold 2 gallons?

88. What is the diameter of a cylinder 30^{cm} long that will hold 10 liters?

89. Find the circumference of a globe, if the number of square centimeters in its surface is three times the number of cubic centimeters in its volume.

90. Find the diameter of a circle, if the number of inches in its circumference is equal to the number of square feet in its area.

91. How many times does a carriage wheel 3 ft. 2 in. in diameter turn in going a mile on a smooth road?

92. A point in the tire moves, while the wheel turns once, just four times the diameter of the wheel. How far does a spike head in the tire travel while a wheel, 3 ft. 2 in. in diameter, travels 1 mi.?

93. An oil can is formed of two cylinders connected by a frustum of a cone. The upper cylinder, or neck, is 6^{cm} in diameter, and 75^{mm} high; the lower cylinder is 13^{cm} in diameter, and 153^{mm} high; the total length of the can is 30^{cm}. Find the capacity of the can in liters.

94. A common tunnel is formed of a frustum of a cone terminated with a cylinder. The height of the frustum is 14^{cm}, and the diameters of the two bases are 175^{mm} and 16^{mm}, respectively. The cylinder is 8^{cm} long. Find the capacity of the tunnel in liters.

95. A pan in the form of a frustum of a cone is 10^{cm} deep, 12^{cm} across the bottom, and 23^{cm} across the top. Find the capacity of the pan in liters.

96. Find the number of square centimeters of sheet iron in a stovepipe 4^{m} long, 26^{cm} in diameter, and 1^{mm} thick, if the edges lap one centimeter. Find the weight of the pipe, if the specific gravity of the sheet iron is 7.8.

97. A steam boiler is formed of a cylinder terminated at each end by a hemispherical cap of the same diameter. The length of the cylinder is 3.4^{m}, interior diameter 0.8^{m}. Find the number of hektoliters of water required to fill the boiler half full.

98. A spherical bomb is 32^{cm} in diameter, and the sides 38^{mm} thick. If the specific gravity of the metal is 7.2, what is the weight of the bomb and its capacity?

99. The diameters of a lamp shade are 25^{cm} and 7^{cm}, and its slant height is 134^{mm}. Find its curved surface in square centimeters.

100. A niche is formed like a half-cylinder surmounted by a quarter of a sphere. The height of the cylinder is 1.2^{m}, the diameter 0.8^{m}. Find the volume of the niche, and the area of its interior surface.

101. What is the expense, at 30 cents a square yard, of painting the walls and ceiling of a room 22 ft. 6 in. long, 13 ft. 6 in. wide, and 10 ft. high?

102. In what time will an empty cistern be filled by three pipes whose diameters are $\frac{1}{2}$ in., $\frac{3}{4}$ in., and 1 in., if the largest alone would fill it in 40 min.? The rates of flow are proportional to the squares of the diameters.

103. How many gallons of water are contained in a length of 50 yd. of a canal, if its width at the top is 8 yd. and at the bottom 7 yd., and its depth 5 ft.?

104. A man who rows 4 miles an hour in still water takes 1 hr. 12 min. to row 4 miles up a river. How long will it take him to row down again?

105. How long must a ladder be to reach a window 40 ft. from the ground, if the distance of the foot of the ladder from the wall is 9 ft.?

106. If 3 oz. of gold 15 carats fine are mixed with 7 oz. 12 carats fine, what will be the fineness of the compound? What must be the fineness of 11 oz. that, when added to this compound, the whole may be 14 carats fine?

107. Find the surface of each face of a cube whose volume is 14 cu. ft. 705.088 cu. in.

108. Determine the depth of conical wineglasses $2\frac{1}{2}$ in. across the top that 60 of them may hold a gallon.

109. What must be the length of spermaceti candles $\frac{7}{8}$ of an inch in diameter that six of them may weigh a pound, if the specific gravity of spermaceti is 0.943?

110. A cylinder 10 in. across and 10 in. high contains 0.3927 cu. ft. of water. How many shot 0.1 in. in diameter must be poured in to raise the water to the top?

111. How deep must a round cistern 4 ft. in diameter be made to be lined with the same amount of lead as a cubical cistern 4 ft. on an edge? Compare their capacities.

112. The material for lining a cubical cistern cost $10. Find the cost of the material for lining two similar cisterns which shall each hold one half as much.

113. If 5 excavators sink a circular shaft 8 ft. in diameter and 125 fathoms deep in 100 days of 10 hr. each, how many nights of 7 hr. each will 4 excavators be in sinking a shaft 6 ft. in diameter and 75 fathoms deep, if the difficulty of working by night is one seventh greater than by day, and the hardness of the ground in the smaller shaft is to that in the larger shaft as 7 is to 5?

114. Find the number of dry quarts a tub will hold that is 22 in. across the top, 20 in. across the bottom, and 18 in. deep.

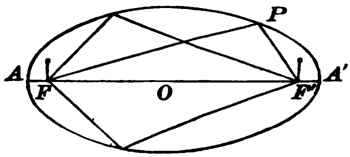
115. Find the number of dry quarts a cylinder will hold that is 28 in. long and has a diameter of 18 in.

116. How high will 2 quarts of milk stand in a cylindrical pail 7 in. in diameter? How high will 2 quarts of oats stand in the same pail?

117. Find the capacity in gallons of a cylindrical boiler 1 ft. in diameter and 4 ft. 10 in. long; of a cylindrical boiler 1 ft. 6 in. in diameter and 3 ft. 6 in. long; of a cylindrical boiler 2 ft. 8 in. in diameter and 5 ft. 6 in. long.

118. Find the capacity of a tumbler $3\frac{1}{4}$ in. across the bottom, $3\frac{1}{2}$ in. across the top, and $3\frac{1}{2}$ in. deep; of a cylindrical tumbler $3\frac{1}{2}$ in. in diameter and $3\frac{1}{2}$ in. deep.

686. Ellipse. If a point moves continuously so that the sum of its distances from two fixed points, called the *foci*, always remains the same, the point traces a curve called an *ellipse*.



Two common examples of an ellipse are the shadow of a circular plate and a section of a right cylinder not parallel to the bases.

687. To Find the Area of an Ellipse,

Multiply the product of its longest and shortest diameters by 0.7854 (¼ of 3.1416).

119. Find the area of an ellipse whose longest and shortest diameters are 11 in. and 8 in., respectively.

120. The ends of a rope 100 ft. long are fastened to stakes placed 80 ft. apart on level ground. A ring, to which a kid is tied, plays freely on the rope. How far from a straight line joining the stakes can the ring be pulled?

121. If the stakes of Ex. 120 are placed 25 ft. apart, by how many per cent is the kid's pasturage increased, provided he can graze 18 in. beyond the rope when stretched?

122. A cylindrical log, 11 in. in diameter, is sawed off at such a slant that the pieces are 8 in. longer on the longest than on the shortest side. Find the diameters of the ellipse thus made, and its area.

123. Find the area of an ellipse, if its longest diameter is 12 in. and its shortest diameter 9 in.

688. To Find the Capacity of Any Round Vessel, like a Cup, Saucer, Bowl, or Tunnel,

If spherical, the vessel holds two thirds as much as a cylinder of the same diameter and depth; if conical, one third as much; if like a coffee cup, one half as much.

124. Find the number of quarts a conical tunnel will hold if it is 9 in. across the top and 8 in. deep.

125. Find the number of pints a spherical bowl will hold if it is 5 in. across the top and $2\frac{1}{2}$ in. deep.

126. Find the number of pints a spherical bowl will hold if it is 4 in. across the top and $3\frac{1}{2}$ in. deep.

127. Find the capacity in pints of a coffee cup 3 in. across the top and 3 in. deep.

128. Find the capacity in liters of a spherical wash bowl 30^{cm} in diameter and 5^{cm} deep.

129. Find the capacity in liters of the basin of a fountain 89^{cm} in diameter and 31^{cm} deep.

130. Find the capacity in quarts of a bowl 10 in. in diameter and 4 in. deep.

131. Find the capacity in pints of a saucer 6 in. across and $1\frac{1}{2}$ in. deep; of a bowl 7 in. across and 3 in. deep.

132. How many gallons will a spherical basin 5 ft. in diameter and 2 ft. deep hold?

133. How many gallons will a spherical bowl 30 in. in diameter and 1 ft. deep hold?

134. Find the capacity in pints of a saucer 5 in. across and 2 in. deep.

135. Find the capacity in gallons of a paraboloid (shaped like a coffee cup) boiler 25 in. across and 14 in. deep.

136. Find the capacity in quarts of a conical funnel 9 in. across and 7 in. deep.

689. To Find the Number of Gallons a Cask will Hold,

Multiply by 0.65 the difference between the bung and head diameters expressed in inches, and add the product to the head diameter for the mean diameter.

Divide the product of the length of the cask expressed in inches and the square of the mean diameter by 294; the quotient is the number of gallons the cask will hold.

137. Find the number of gallons contained in a full cask whose bung diameter is 24 inches, head diameter 22 inches, and length 30 inches.

138. Find the number of gallons contained in a full cask whose bung diameter is 22 inches, head diameter 20 inches, and length 28 inches.

139. Find the number of gallons contained in a full cask whose bung diameter is 20 inches, head diameter 18 inches, and length 28 inches.

690. Sound travels in still air at 32° F. 1090 ft. a second, and 1.1 ft. a second faster for every degree Fahrenheit increase in temperature.

Sound travels in still air at 0° C. 332^{m} a second, and 60.9^{cm} a second faster for every degree Centigrade increase in temperature.

140. The flash of a gun is seen $7\frac{1}{2}$ sec. before the report of the gun is heard; there is no wind, and the temperature is 73° F. How far off is the gun?

141. A meteor was seen to burst; the report followed in 4 min. 17 sec. What was its distance, if the average temperature of the intervening air was 50° F.?

142. How long will it take for an explosion at the equator to be heard at the antipodes of the place, if the circumference of the earth at the equator is reckoned at $40,000^{km}$, and the average temperature at the equator at 23° C.?

143. If an explosion at the equator occurs at sunset and the average temperature east of the spot is 22° C., and that to the west 24° C., how far from the antipodes will the sound waves meet?

144. How far off is the lightning when the thunder follows in 13 sec., the temperature being 76° F.?

145. How long would it take sound to go through a whispering tube 3 mi. long, temperature 61° F.?

146. Sound travels in iron about $10\frac{1}{2}$ times as fast as in air. How long, then, after seeing the blow of a sledge hammer given on the other end of an iron pipe $1\frac{1}{2}$ mi. long, may I expect to hear the sound by the iron; and how long after, to hear the sound through the air in the pipe; thermometer 63° F.?

147. Two gunners fire at each other simultaneously from forts $1\frac{1}{2}$ mi. apart; the wind, at 70° F., blows steadily from one fort to the other, at 11 mi. an hour. How soon will each hear the report of the other's gun? Suppose one ball flies on the average 987 ft. a second, the other 818 ft. a second; when will each receive the other's shot?

148. Sound travels in water about 4.26 times as fast as in air. How many seconds sooner would the sound of a torpedo exploded under water 2 mi. off reach you by water than by air, at 68° F.?

691. In the average state of the atmosphere, the distance at which an object is visible at sea may be found by the following relation:

The square of the distance in miles is seven fourths the height of the object in feet.

The square of the distance in kilometers is fifteen times the height in meters.

Hence, $\log \text{miles} = 0.1215 + \frac{1}{2} \log \text{feet}.$

$\log \text{kilometers} = 0.5880 + \frac{1}{2} \log \text{meters}.$

149. A hill 482 ft. high is 8 mi. from the shore. How many miles out at sea is it visible?

150. A sailor at the topmast 80 ft. above the sea can just see a sailor at the topmast of a similar ship. How many miles apart are the vessels?

151. How far is a mountain 1000^{m} high visible? a mountain 2000^{m} high?

152. If a man stands on a bluff that raises his eyes 11^m above the sea, how far can he see from the shore?

153. A sailor at sea is at a distance of 171^{km} from a mountain when the top of the mountain is just visible. How high is the mountain?

154. A vessel approaching Valparaiso at daybreak just makes out the peak of Aconcagua, 22,427 ft. high and 140 mi. back from the coast. How far is the vessel from land if the eye of the observer is 30 ft. above the water?

155. If Mount Washington is 6293 ft. high and 76 mi. in an air line from Cape Elizabeth, how far out from the Cape will its peak be visible in the ordinary state of the atmosphere?

156. How many acres of water can a man see if he stands on a raft with his eyes just 6 ft. above the water, and no land is in sight?

157. How far would a mountain 29,000 ft. high be visible? one 5000 ft. high? one 1000 ft. high?

158. How high must a mountain be in order to be visible at sea level 50 miles? 100 miles? 150 miles?

159. What distance can be seen from the top of a mountain 4 miles high?

692. Pendulum. A body suspended by a straight line from a fixed point so as to swing freely is called a *pendulum.*

693. *The number of vibrations that pendulums make in a given time is inversely as the square root of their lengths.*

A pendulum that passes its central point of rest once every mean solar second is 39.138 in. long.

160. Find the length of a pendulum that beats half-seconds; of a pendulum that beats quarter-seconds.

161. How many centimeters long is a pendulum that swings 80 times a minute? a pendulum that swings 30 times a minute?

162. If a cannon ball is suspended by a fine wire 176 ft. long in the central well of the Bunker Hill Monument, how many times a minute will it swing?

163. How long is a pendulum that swings three times in two seconds? that swings five times in two seconds?

694. A body falling from rest in a vacuum falls $16\frac{1}{12}$ ft. or 4.903^m in the first second; it then has acquired a velocity of $32\frac{1}{6}$ ft. or 9.806^m.

A falling body increases its velocity in proportion to the time it is falling; and the distance fallen is in proportion to the square of the number of seconds of time it is falling.

Thus, a body falling from rest in a vacuum in half a second has fallen $4\frac{1}{48}$ ft. or 1.2257^m, and has acquired a velocity of $16\frac{1}{12}$ ft. or 4.903^m per second; in 3 sec. it has fallen $144\frac{3}{4}$ ft. or 44.127^m, and has acquired a velocity of $96\frac{1}{2}$ ft. or 29.418^m per second.

The velocity of heavy bodies falling short distances in air will not be much less than in a vacuum.

164. What velocity in meters a second will a cannon ball acquire in falling three quarters of a second? in falling three and a quarter seconds?

165. How long will it take a leaden ball, rolling off a table 29 in. high, to reach the floor?

166. What velocity will a crowbar attain in falling endwise from a balloon 2000^m high? How long will it be in coming down?

167. What velocity will a crowbar attain in falling endwise from a balloon one mile and a quarter high? How long will it be coming down?

168. How long will it take a ball, rolling off a table, to drop 1^{cm}? 1 in.? 10^{cm}? 6 in?

169. If Carisbrook Well is 210 ft. deep, how long after a pebble is dropped will it be heard to strike the bottom, if the velocity of sound is 1120 ft. a second?

170. How long after a pebble is dropped will it be heard to strike the bottom of a ventilating shaft 1600 ft. deep, if the temperature is 68° F.?

171. If a rock dropped over a precipice strikes the bottom in $7\frac{1}{2}$ sec., how high is the precipice?

172. How long after a pebble dropped down a shaft 133 ft. deep will it be heard to strike the bottom, if the temperature is 59° F.?

695. *If a plunger fits tightly in a small cylinder, and by it water is forced into a large cylinder, a plunger in the large cylinder is lifted with a force nearly equal to the product of the force with which the little plunger is driven in multiplied by the square of the ratio of the diameters of the two cylinders.*

173. Find the lifting power of a hydraulic press, the plunger being 1^{cm} in diameter and driven with a force of 100^{kg}, if the lifting piston is 1^{m} in diameter.

174. If the plunger is $\frac{1}{2}$ in. in diameter, and is driven with a force of 1000 lb., how much can it lift with a lifting piston 4 ft. in diameter?

175. If the plunger is 2 in. in diameter, and is driven with a force of 1000 lb., how much can it lift with a lifting piston 2 ft. in diameter?

176. The water stands in a fissure in a rock 10^{m} high and 12^{m} long. What pressure is exerted to split the rock on the lowest meter's width? on the highest meter's width? in the whole fissure?

NOTE. This pressure is found by multiplying the surface upon one side by the height of water above the centre line, counting the product as volume of water, and then finding the weight of this volume of water. The principle is precisely the same as in the hydraulic press.

177. A dam is 100 ft. long and 10 ft. deep, and the water is just flowing over it. What pressure is exerted on the lowest two feet of the dam?

178. Water is running 2 ft. over a dam that is 180 ft. long and 12 ft. deep. Find the pressure on the dam.

179. Water is running 9 in. deep over a dam that is 78 ft. long and 8 ft. deep. Find the pressure on the dam.

696. *The velocity with which water will flow out of a hole in the side of a reservoir is nearly proportional to the square root of the depth of the hole below the surface of the water; and is about 32 ft. a second at the depth of 16 ft.*

180. With what velocity will water flow through a hole 9 ft. below the surface?

181. With what velocity will water leave a fountain having free play, and a head of 25 ft.? a head of 100 ft.?

182. If a hole in the side of a cistern 4 ft. below the surface of the water is delivering 10 gal. an hour, how many gallons would it deliver with 5 ft. more head?

183. If a pipe 2 in. in diameter, and 1 ft. long, inserted in a dam, the head of water being kept constant, delivers 4 gallons of water a minute, how many gallons a minute may be expected when another pipe of the same length, but $2\frac{1}{2}$ in. in diameter, is substituted for the two-inch pipe?

184. If a one-inch pipe, 20 in. long, is substituted for the two-inch pipe, 1 ft. long, in Example 183, and the flow is found to be 5 pints a minute, what part of the decrease of flow is due to the smaller area of the orifice, and what part to the increased friction on the sides of the longer pipe?

697. *The quantity of water issuing from a hole is in proportion to the square root of the head; and the velocity is in proportion to the square root of the head.*

The work which the water can do is in proportion to the quantity multiplied by the square of the velocity; that is,

The work is in proportion to the square root of the cube of the head.

185. A miller is using water flowing through the gateway under 4 ft. head. How much more work could he do if the head was raised to 9 ft.? how much more if the head was raised to 25 ft.?

698. Work is the act of changing the position of a body by overcoming the resistance to the change.

699. Units of Work. The unit of work is the work done in raising a weight of one pound through a distance of one foot. This unit is called a *foot-pound.*

The corresponding metric unit is the *kilogram-meter.*

700. Horse Power. The rate at which work is done is called *power.* A *horse power* is the power to do 33,000 foot-pounds of work per minute, or 550 foot-pounds per second.

186. A cross section of a stream of water is a rectangle 6 ft. by $2\frac{1}{2}$ ft.; the velocity is 40 ft. per minute. There is a fall of 10 ft. where a water wheel is erected that utilizes 70% of the work. Find the horse power of the wheel.

187. Find the horse power of the wheel of Example 186, if the fall of the water is 14 ft.

188. A cross section of a stream of water is a rectangle 5 ft. by 4 ft.; the velocity is 50 ft. per minute. There is a fall of 12 ft. where a water wheel is erected that utilizes 65% of the work. Find the horse power of the wheel.

189. Find the horse power of the wheel of Example 188, if the fall of the water is 16 ft.

190. A cross section of a stream of water is a trapezoid whose altitude is $3\frac{1}{2}$ ft., and parallel sides 6 ft. and 5 ft., respectively; the velocity is 150 ft. per minute. There is a fall of 9 ft. where a water wheel is erected that utilizes 75% of the work. Find the horse power of the wheel.

701. *When a body is moving in a circle, the centrifugal force is about 1.227 of the continued product of the weight of the body, the number of feet in the radius of the circle, and the square of the number of revolutions in a second.*

Thus, a body going round a circle of 5 ft. radius once a minute presses away from the centre with a force equal to $1.227 \times 5 \times \frac{1}{60^2}$ of the weight of the body.

NOTE. When the radius is measured in meters, the multiplier 4.025 must be used in place of 1.227.

191. If a top 3 in. in diameter is making 200 revolutions a second, with what force does the outer layer pull away from the centre?

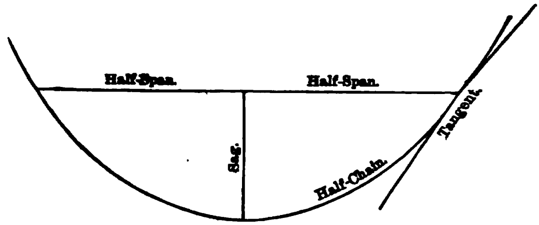
192. If a sling 30 in. long contains a stone that weighs $\frac{1}{2}$ lb., and is whirled round 80 times a minute, what is the force pulling on the string?

193. With what force does a locomotive that weighs 60 tons running 30 mi. an hour, on a curve of 800 ft. radius, bear against the outer rail? If the locomotive is running 60 mi. an hour, with what force does it bear on the outer rail?

194. If washed wool is put wet into a wire basket 1.2^{m} in diameter, and the basket is set to spinning at the rate of 180 revolutions a second, with what force is water wrung out of the wool?

195. If steel pens are revolved in a basket 32^{cm} in diameter, 17 revolutions a second, with what force is the oil drained from them?

196. The top of a wheel is at each instant moving with twice the velocity of the carriage, and is moving in a curve whose centre, at the instant, is as far below ground as the point is above ground. What, then, is the force exerted to separate the mud from the top of a wheel 3 ft. 2 in. in diameter, when the carriage is moving at the rate of 10 miles an hour?



702. When a chain of uniform thickness hangs from two points not in the same vertical line, it hangs in a curve called a **common catenary.**

The length of the chain from the lowest point to any point selected may be called **half-chain.** The height of the point selected above the lowest point is called **sag.** The horizontal distance of the point selected from the lowest point is called **half-span.** The horizontal force with which the point selected is drawn inward is called **tension.** The radius of the circle which will fit the curve at its lowest point is called **radius.** The straight line which touches the curve at the point selected is called **tangent.**

703. 1. *The tension is equal to the weight of a piece of the same chain as long as the radius.*

2. *The radius is equal to the sum of the half-chain and sag multiplied by the difference of the half-chain and sag, and divided by twice the sag.*

3. *The log of half-span is equal to the log of sum of half-chain and sag plus log of their difference, plus log of the difference of these two logarithms plus colog of sag plus 0.0612.*

4. *Radius divided by half-chain measures the "batter" at the point selected; that is, measures the horizontal falling back for every unit of vertical ascent in a straight line tangent at that point.*

197. How strong a horizontal pull on a chain, weighing half a pound to the yard, is required to make the lowest part curve with an 18-in. radius? with a 6-ft. radius?

198. A $\frac{3}{4}$-in. rope, weighing $\frac{1}{4}$ lb. to the yard, is fastened at one end to a staple, and near the other end, on the same level, runs over a pulley, and has a 25-lb. weight hung to it. What is the radius of its curvature at the middle?

199. A shower wets the rope of Example 198, and increases its weight 40%; what does its radius now become?

200. A steam tug, in attempting to move a ship, straightened the hawser until the radius of the lowest point was 1980 ft. The rope was wet, and weighed $3\frac{1}{4}$ lb. to the yard. With what force was it stretched?

201. A chain 31 ft. long hangs between points on a level, and sags 4 ft. What is the radius at the lowest point?

202. The whole chain, in Example 201, weighs 18 lb. What is the horizontal tension? What is the distance between the points? What is the slant, or batter, of the end of the chain?

203. A chain weighing 1^{kg} to the meter is suspended from points on a level; the length of chain is 31^{m}, and it sags 1.3^{m}. Find all the conditions, and find how much it falls below a level at 10^{cm} from each end.

204. A chain 100^{m} long, weighing 14 oz. to the foot, is suspended from points on a level 80^{m} apart, and sags 26.55^{m}. What is the radius at the middle, the batter at the ends, and the horizontal tension?

205. If the chain of Example 204 is shortened 5^{m}, the sag is decreased 4^{m}. What is the radius and tension?

704. A **lever** is a rigid bar that can be moved about a fixed axis called the fulcrum.

The *perpendicular* from the fulcrum to the line in which the power acts is the *power arm* of a lever.

The *perpendicular* from the fulcrum to the line in which the weight acts is the *weight arm* of a lever.

705. *The ratio of the power to the weight raised by a lever is equal to the ratio of the weight arm to the power arm.*

206. How heavy a rock placed 6 in. from the fulcrum can a man, who weighs 180 lb., raise with a crowbar 5 ft. 6 in. long?

207. Two weights of 30 lb. and 20 lb., respectively, at the ends of a horizontal lever 5 ft. long balance. Find how far and in which direction the fulcrum must be moved for the weights to balance when each is increased by 5 lb.

208. A man who weighs 160 lb., wishing to raise a rock, leans with his whole weight on a horizontal crowbar 5 ft. long, which is propped at the distance of 4 in. from the end in contact with the rock. Find the force he exerts on the rock, and the pressure the prop has to sustain, if the weight of the crowbar is not reckoned.

209. A child weighing 56 lb. is seated at one end of a plank 16 ft. long, and a child weighing 72 lb. is at the other end. Find the distance of each child from the fulcrum when the plank is used for a seesaw.

210. In a pair of nutcrackers if the nut is placed at a distance of 1 in. from the hinge, and the hand presses at a distance of 8 in. from the hinge, find the pressure upon the nut for every ounce of pressure exerted by the hand.

211. A body is weighed in both arms of a false balance, and its apparent weights are 2.56 lb. and 2.25 lb. Find its true weight.

212. In a steelyard the weight of the beam is 15 lb., and the distance of its centre of gravity from the fulcrum is 3 in. Find the distance from the fulcrum a weight of 6 lb. must be placed to balance the beam.

706. *With the Wheel and Axle, the ratio of the power to the weight to be raised is equal to the ratio of the radius of the axle to the radius of the wheel.*

213. A cask weighing 160^{kg} is attached to a rope wound on an axle 19^{cm} in diameter; at one end of the axle is a wheel 175^{cm} in diameter. With what force must a man pull down on a rope passing over the wheel to raise the cask?

214. A rope passes over a single pulley. How much force is required to raise 180 lb. attached to one end of a rope if 1% of the force is required to overcome friction?

215. If the radius of the wheel is four times that of the axle, and the string round the wheel can support a weight of 50 lb. only, find the greatest weight that can be lifted.

216. Find the ratio of the radii of a wheel and axle that a force of 100 lb. may just support a weight of 1 ton.

217. The radius of a wheel is 80^{cm} and the radius of the axle is 12^{cm}. What weight can be supported by a force of 30^{kg}? Find the work done if the weight is raised 60^{cm}.

707. *With the Screw, the ratio of the power to the weight is equal to the ratio of the distance between two consecutive threads to the circumference described by the end of the power arm.*

218. The power arm of a screw is 16 in. long, and by one turn of the screw the head advances one eighth of an inch. If the power is 3 lb., find the weight lifted.

219. In a screw used to raise a load of 10 tons, the power is 50 lb., acting by an arm 4 ft. long. Find the distance between two consecutive threads.

220. The lever of a screw is 1 ft. 9 in. long, and the power applied at the end is 100 lb. What must be the distance between the threads that a pressure of 5000 lb. may act on the press board?

221. The lever of a screw is 3 ft. 6 in. long, and the distance between the threads is $\frac{1}{8}$ in. What power must be applied at the end of the lever to produce a pressure of 10 tons on the press board?

708. Chemists employ initial letters to signify fixed proportional quantities. Multiples of these are indicated by small figures placed after them; and multiples of compounds are indicated by figures placed before them.

Thus, if the gram is the unit, H signifies 1^g of hydrogen, and O signifies 16^g of oxygen; then H_2O means 18^g of water, and 2 H_2O means 36^g of water; while $H_2 + O$ would mean 2^g of hydrogen and 16^g of oxygen mixed, but not chemically united. An electric spark passed through $H_2 + O$ would produce H_2O, with an explosion.

709. The chemical symbols and numerical proportions of a few common elements are as follows:

Element.	Symbol.	Numerical Equivalent.
Hydrogen	H	1
Oxygen	O	16
Sulphur	S	32
Iron (ferrum)	Fe	56
Calcium	Ca	40
Sodium (natron)	Na	23
Carbon	C	12
Boron	B	11

222. What per cent of water is oxygen? what per cent hydrogen?

223. What per cent of quicklime, CaO, is oxygen?

224. What per cent of water in slacked lime, CaO_2H_2?

225. What per cent of pure marble, $CaCO_3$, is oxygen?

226. What per cent of gypsum, called plaster of Paris, $CaSO_4 + 2\ H_2O$, is sulphur?

227. What per cent of washing soda, $Na_2CO_3 + 10\ H_2O$, is carbon?

228. In 118 lb. of Glauber salts, $Na_2SO_4 + 10\ H_2O$, how many ounces of sulphur?

229. How many ounces of soda, $Na_2O + H_2O$, in 7 lb. of borax, $Na_2B_4O_7 + 10\ H_2O$?

230. What per cent of pure alcohol, C_2H_6O, is carbon? What per cent of pure white marble, $CaCO_3$, is carbon?

231. What per cent of pure acetic acid (the acid of vinegar) is carbon, the formula being $C_2H_4O_2$?

232. How much acetic acid can be obtained from 12 lb. of alcohol, C_2H_6O, if there is no waste?

233. How many grains of carbon in 1 oz. avoirdupois of oxalic acid, $C_2H_2O_4 + 2\ H_2O$?

234. How many milligrams of carbon in 3^g of tartaric acid, $C_4H_6O_6$?

235. How many kilograms of carbon in 95^{kg} of white sugar, $C_{12}H_{22}O_{11}$?

236. The formula of camphor is $C_{10}H_{16}O$. How many grams of carbon in 14^{kg} of camphor?

237. In 20^{kg} of oil of vitriol, H_2SO_4, how many grams of sulphur?

238. What per cent of oil of vitriol is water? what per cent sulphuric acid, SO_3?

239. In 3.5^g of black oxide of iron, FeO, how many milligrams of iron?

240. Red iron-rust consists of 70% iron and 30% oxygen. Find its formula.

241. The choking vapor of burning sulphur is sulphur and oxygen in equal parts. Find its formula.

242. Copperas is 28.9% sulphuric acid, 25.7% oxide of iron, 45.4% water. Find its formula.

Solution. Water being 18, oxide of iron 72, and sulphuric acid 80, first seek multiples of 72 and 80, in the ratio of 25.7 to 28.9; that is, of 0.8892 to 1. But 72 and 80 are in almost exactly that ratio. This gives $FeSO_4$ + water; and it remains to find a multiple of 18 which is to 152 as 45.4 is to 54.6; that is, which is 0.8315 of 152, or 126.4. But $7 \times 18 = 126$; and the addition of seven parts of water gives $FeSO_4 + 7\ H_2O$.

243. Spirits of turpentine is 11.76% hydrogen and 88.24% carbon. Find its formula. What per cent of oxygen combined with spirits of turpentine are required to make camphor, $C_{10}H_{16}O$?

710. Ohm and Ampère. The unit of the resistance of the conductor to an electrical current is an *ohm.* The unit of the *strength* of an electrical current through a conductor is an *ampère.* These units have been arbitrarily chosen.

NOTE. The strength of the electrical current for an ordinary arc lamp is about 10 ampères.

711. A volt is the force required to send an electrical current of one ampère through a conductor that offers a resistance of one ohm.

712. *The resistance of the conductor to an electrical current is directly proportional to the length of the conductor, and inversely proportional to the area of its cross section.*

713. *The strength of an electrical current in ampères is equal to the force in volts divided by the resistance in ohms.*

244. If the resistance of 1 mile of wire 2^{mm} in diameter is 4.72 ohms, what is the resistance of 3 miles of wire of the same material 3^{mm} in diameter?

245. What length of copper wire 1^{mm} in diameter has the same resistance as 720^{m} of copper wire 4^{mm} in diameter?

246. The conductivity of iron is $\frac{1}{7}$ that of copper. If the resistance of a copper wire 1 mile long and $\frac{1}{8}$ in. in diameter is 6.8 ohms, what is the resistance of an iron wire $\frac{1}{16}$ in. in diameter and 5 miles long?

247. If 50 volts force 54.8 ampères of electrical current through a lamp, what is the resistance?

248. If the resistance of an electric lamp is 2.8 ohms when a current of 10 ampères is passing through it, what is the voltage?

249. Five arc lamps on a circuit have each a resistance of 2.35 ohms. The resistance of the wires is 1.2 ohms and of the dynamo is 0.75 ohm. What voltage is required to send a current of 15 ampères through the circuit?

Table of Specific Gravities.

NAME OF SUBSTANCE.	SPECIFIC GRAVITY.	WEIGHT OF CUBIC FOOT IN POUNDS.
Acid, acetic, strongest	1.062	66.4
" nitric, "	1.583	98.94
" sulphuric, "	1.841	115.
Air	0.001292	0.0808
Alcohol, pure	0.792	49.43
" of commerce	0.834	52.1
Aluminum, lightest	2.560	160.
Brass, average	7.611	475.7
Brick, common	2.000	125.
" pressed	2.400	150.
Cedar, dry	0.350 to 0.600	22 to 37.5
Cherry, dry, average	0.715	44.7
Coal, bituminous, average	1.250	78.1
" anthracite,	1.500	93.75
Copper, cast	8.788	549.3
Cork	0.240	15.
Glass, average	2.760	172.5
Gold, pure, cast	19.258	1204.
Granite, average	2.720	170.
Gypsum, heaviest	2.288	143.
Hydrogen	0.0000893	0.00558
Ice	0.930	58.1
Iron, cast, average	7.150	447.
" wrought, average	7.770	486.
Lead, cast	11.350	709.4
" white	7.235	452.
Lignum vitæ	1.333	83.3
Lime	0.804	50.25
Marble, average	2.720	170.
Mercury	13.580	849.
Milk	1.032	64.5
Nitrogen	0.001250	0.0781
Oil, linseed	0.940	58.8
Oak, dry white	0.830	51.9
Oxygen	0.001429	0.0893
Pine, dry white	0.400	25.
Platinum, hammered	21.841	1365.
Salt	2.130	133.
Sand, average	1.650	103.
Silver, hammered	10.500	656.2
Slate, lightest	2.110	132.
Steel	7.816	488.5
Tin, cast	7.291	456.
Water, sea	1.026	64.13
Zinc	7.190	449.4

Table of Selected Constants.

	CONSTANT.	NUMBER.	LOGARITHM.
ROOTS.	Diagonal of square in terms of side	1.4142136	0.1505150
	Side in terms of diagonal	0.7071068	9.8494850
	Cube root of 2	1.2599210	0.1003433
	Cube root of ½	0.7937002	9.8996567
	Square root of 20	4.4721359	0.6505150
	Square root of 3	1.7320508	0.2385606
	Square root of ⅓	0.5773503	9.7614394
	Square root of 30	5.4772256	0.7385606
	Square root of 5	2.2360680	0.3994850
	Extreme and mean ratio	0.3819660	9.5820247
	Extreme and mean ratio	0.6180340	9.7910124
CIRCLE AND SPHERE.	Circumference in terms of diameter .	3.1415927	0.4971499
	Diameter in terms of circumference .	0.3183099	9.5028501
	Circle in terms of diameter square .	0.7853982	9.8950899
	Square in terms of the circle . . .	1.2732395	0.1049101
	Sphere in terms of diameter cube .	0.5235988	9.7189986
	Cube in terms of sphere	1.9091406	0.2810014
LENGTHS.	One meter in feet	3.2808693	0.5159889
	One foot in meters	0.3047973	9.4840111
	One meter in yards	1.0936231	0.0388677
	One yard in meters	0.9143918	9.9611323
	One centimeter in inches	0.3937043	9.5951702
	One inch in centimeters	2.5399772	0.4048298
	One mile in kilometers	1.6093295	0.2066450
	One kilometer in miles	0.6213768	9.7933550
SURFACES.	One square foot in square meters .	0.0929014	8.9680221
	One square meter in square feet . .	10.7641036	1.0319779
	One sq. inch in square centimeters .	6.4514847	0.8096596
	One square centimeter in sq. inches .	0.15500308	9.1903404
	One acre in hektars	0.4046784	9.6071100
	One hektar in acres	2.4710982	0.3928900
VOLUMES.	One cubic foot in cubic centimeters .	28,316.085	4.4520332
	One cubic meter in cubic feet . . .	35.315617	1.5479668
	One U.S. bushel in cubic inches . .	2150.42	3.3325232
	One U.S. bushel in cu. centimeters .	35,238.117	4.5470127
	One cubic foot in U.S. bushels . .	0.8035640	9.9050204
	One U.S. gallon in liters	3.7853103	0.5781015
	One liter in U.S. gallons	0.2641791	9.4218985
WEIGHTS.	One gram in grains	15.4323487	1.1884320
	One grain in grams	0.0647990	8.8115680
	One pound in kilograms	0.4535927	9.6566660
	One kilogram in pounds	2.2046212	0.3433340